SDLV122 - Extrapolation of local measurements on a complete model (3D)

Summary:

It is about a linear test of dynamics $3D$.

The goal is to test the order PROJ_MESU_MODAL in the case of a system $3D$. This order makes it possible to project experimental dynamic transitory answers in a certain number of points on a modal basis of a digital modeling.

This test contains 2 modelings:

- projection (of constraints) is done on a basic concept modal of type [mode_meca],
- projection (of constraints) is done on a basic concept modal of type [base_modale].

For 2 modelings, provided experimental measurements are identical and make it possible to test the research of the nodes in opposite and the taking into account of a local orientation.

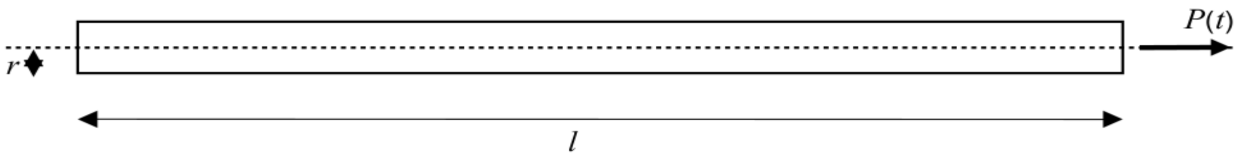
In both cases, the reference solution is obtained by a direct calculation with *Code_Aster* ; projection is carried out in the successful outcome where the number of modes is equal to the number of measurements. The answers in constraint obtained after projection are identical to the constraints of reference provided in data.

For modeling A, the answers in displacements and deformation obtained after projection are in perfect adequacy with the reference solutions. The values speeds and the accelerations deduced from the identified modal contributions are close to those obtained by direct calculation. The weak noted variations are due to the errors of approximation generated by the determination via a linear diagram in time of speeds and accelerations.

1 Problem of reference

1.1 Geometry

Let us consider the embed-free slim cylindrical bar described below:



Length : $l=4m$
Ray : $r=0.1m$
End 1 : embedded ($x=0$)
End 2 : free ($x=l$)

1.2 Properties of materials

The characteristics of material are the following ones:
Young modulus: $E=2.1\,10^{11}Pa$
Poisson's ratio: $\nu=0.3$
Density: $\rho=7800\,kg/m^3$

1.3 Boundary conditions and loading

The boundary condition is the embedding of the end 1 bar. This embedding is of standard beam to allow the effects Fish on the section.

The loading applied for the calculation of answer is a thrust load, constant in traction, distributed on the section of the end 2 :

$$P(t)=\begin{cases} 0 & \text{if } t<0 \\ P_0=10^6 N & \text{if } t\geq 0 \end{cases}$$

2 Reference solutions

2.1 Method of calculating used for the reference solution

- Analytical solution:
  An analytical solution of this problem exists. It is described in [1]:

  In this case, the solution by modal superposition of this problem is written:

$$u(x,t)=\frac{P_0}{EA}x-\frac{8P_0 I}{\pi^2 EA}\sum_{s=1}^{\infty}\left[\frac{(-1)^{s-1}}{(2s-1)^2}\sin\left((2s-1)\frac{\pi x}{2I}\right)\cos\left((2s-1)\frac{\pi c}{2I}t\right)\right]$$

with: $c=\sqrt{\frac{EA}{m}}$: speed of wave propagation in the bar

- Adopted reference solution:
  The analytical solution utilizing an infinite sum on the modes, it is preferable that the reference solution and the solution with projection correspond to the same configuration, with the same number of modes.
  Moreover, to avoid problems involved in the discretization of the digital grid, the adopted reference solution is the answer provided by the direct calculation carried out with *Code_Aster* with the order `DYNA_TRAN_MODAL`. One simulates displacement, speed and the constraint in some points of structure. The simulated answers were versed in a file with the universal format (IDEAS).

2.2 Results of reference

For modeling A, the comparison of the results relates to displacements, speeds, accelerations, deformations and forced along the axis x , nodes $N2$ and $N4$ at 3 different moments. $N4$ corresponds to a node of measurement and $N2$ is not one. It is also checked if one obtains exactly the component of the field measured after expansion of measurement on the digital model. One compares for that the component of the extended field following the direction of observation (displacement $N3$ according to **DX**, speed $N5$ according to **DX** and **SIXX** with the node $N4$).

For modeling B, the comparison of the results relates to the constraints of the nodes $N3$ and $N4$ at 3 different moments.

2.3 Uncertainty on the solution

The selected reference makes it possible to draw aside uncertainties related to the discretization of the digital grid. The number of modes of the base of projection is equal to the number of measurements, therefore the solution of the inversion is exact (in opposition to an approximate solution of a generalized opposite problem).

If projection is done on a concept of the type [`mode_meca`], the modal bases of the reference solution and the solution obtained by projection are identical, the answers in displacements, deformations and constraints obtained must thus be similar to the answers of reference. Some errors of approximation can appear on speeds and accelerations which are determined by a linear diagram in time.
If projection is done on a concept of the type [`base_modale`], the modal bases of the reference solution and the solution obtained by projection contain the same number of modes but are different. The calculation of reference not being possible on a concept of the type [`base_modale`], the comparison of the results relates only to answers corresponding to provided measurements.

2.4 Bibliography

1) MR. GERADIN, D. RIXEN: Theory of the vibrations - Application to the dynamics of the structures - Edition MASSON 1993

3 Modeling A

3.1 Characteristics of modeling and the grids

- This modeling tests the reading of dataset 58 of a file to the universal format (IDEAS), containing at the same time fields of displacement, speed and constraint. One also carries out the expansion on the digital model by exploiting mixed measurement (displacement, speed and constraint).
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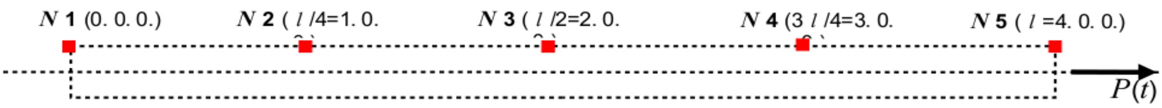
- Digital grid:
  The digital grid is carried out with I-DEAS version Master degree Series 5. It comprises 2667 nodes and 3328 meshs of the type $3D$ linear.

- Experimental grid :
  The grid of measurement understands only 5 specific elements and 5 positioned nodes as indicates it the following figure:



3.2 Characteristics of measurements

Provided experimental measurements are:

- axial stresses σ_{xx} , according to the direction x with the nodes $N3$, $N4$ and $N5$,
- axial displacement with the node $N3$,
- axial speed with the node $N5$.

The sampling of time is constant: initial time is $0\,s$, the step of time is $10^{-5}\,s$ and the number of moments is 1001 (i.e until a final time of $0.01\,s$).

The values result from the direct calculation carried out with *Code_Aster*.

3.3 Characteristics of the modal base

The modes are stored in a concept of the type [`mode_meca`], containing the first three dynamic modes of traction. These modes are obtained by blocking transverse displacements (i.e according to DY and DZ) nodes of fibre of neutral and nodes of the higher line ($x=0.$ with $4.$ $y=0.1$ and $z=0.$). Their Eigen frequencies ($326.5\,Hz$, $980.0\,Hz$ and $1634.5\,Hz$) are close to the Eigen frequencies of traction analytically calculated ($324.3\,Hz$, $972.9\,Hz$ and $1621.5\,Hz$).

3.4 Sizes tested and results

Identification			Reference
		with T = 9. 10^{-4}S	2,686 10^{-4}
`DEPL_X`	with the node $N2$	with T = 17. 10^{-4}S	3,074 10^{-4}
(m)		with T = 25. 10^{-4}S	1,446 10^{-5}
		with T = 9. 10^{-4}S	5,793 10^{-4}

DEPL_X	with the node $N4$	with T = 17. 10⁻⁴S	9,160 10^{-4}
(m)		with T = 25. 10⁻⁴S	3,095 10^{-4}
		with T = 9. 10⁻⁴S	6,221 10^{-1}
VITE_X	with the node $N2$	with T = 17. 10⁻⁴S	– 4,683 10^{-2}
(m/s)		with T = 25. 10⁻⁴S	– 3,542 10^{-1}
		with T = 9. 10⁻⁴S	8,056 10^{-1}
VITE_X	with the node $N4$	with T = 17. 10⁻⁴S	– 3,556 10^{-1}
(m/s)		with T = 25. 10⁻⁴S	– 8,638 10^{-1}
		with T = 9. 10⁻⁴S	– 3,633 10^{+3}
ACCE_X	with the node $N2$	with T = 17. 10⁻⁴S	6,337 10^{+2}
(m/s^2)		with T = 25. 10⁻⁴S	3,801 10^{+3}
		with T = 9. 10⁻⁴S	8,655 10^{+2}
ACCE_X	with the node $N4$	with T = 17. 10⁻⁴S	– 2,387 10^{+3}
(m/s^2)		with T = 25. 10⁻⁴S	– 6,355 10^{+2}
		with T = 9. 10⁻⁴S	1,957 10^{-4}
EPXX	with the node $N2$	with T = 17. 10⁻⁴S	3,015 10^{-4}
(m)		with T = 25. 10⁻⁴S	5,422 10^{-5}
		with T = 9. 10⁻⁴S	1,822 10^{-4}
EPXX	with the node $N4$	with T = 17. 10⁻⁴S	2,611 10^{-4}
(m)		with T = 25. 10⁻⁴S	1,681 10^{-4}
		with T = 9. 10⁻⁴S	5,012 10^{+7}
SIXX	with the node $N2$	with T = 17. 10⁻⁴S	7,717 10^{+7}
(Pa)		with T = 25. 10⁻⁴S	1,390 10^{+7}
		with T = 9. 10⁻⁴S	4,650 10^{+7}
SIXX	with the node $N4$	with T = 17. 10⁻⁴S	6,671 10^{+7}
(Pa)		with T = 25. 10⁻⁴S	4,293 10^{+7}

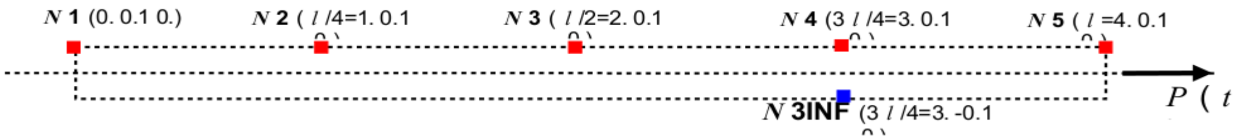
$\sum_i \lvert depl_x(NRES3)(t_i) - depl_x(N3)(t_i) \rvert$ (m)	0
$\sum_i \lvert vite_x(NRES5)(t_i) - vite_x(N5)(t_i) \rvert$ (m/s)	0
$\sum_i \lvert SIXX(NRES4)(t_i) - SIXX(N4)(t_i) \rvert$ (Pa)	0

4 Modeling B

4.1 Characteristics of modeling and the grids

- Digital grid:
  The digital grid is identical to that used in the preceding case. It is carried out with I - DEAS version Master degree Series 5 and comprises 2667 nodes and 3328 meshs of the type $3D$ linear. A group comprising only one node is added to be used as interface.

- Experimental grid :
  The grid of measurement understands only 5 specific elements and 5 positioned nodes as indicates it the following figure:



4.2 Characteristic of measurements

Provided experimental measurements are:

- With the nodes $N3$, $N4$ and $N5$:
  The data are the axial stresses, applied in the direction x .
  The sampling of time is constant: initial time is $0\,s$, the step of time is $10^{-5}\,s$ and the number of moments is 1001 (i.e until a final time of $0.01\,s$).

The values result from the direct calculation carried out with *Code_Aster*.

4.3 Characteristics of the modal base

The modes are stored in a concept of the type [`base_modale`], containing the first two dynamic modes of traction and the static mode with the node $N3INF$ for the degree of freedom DX . The interface is of Craig-Bampton type. The base thus contains on the whole 3 modes.

Note:

The number of modes being very reduced, the solution depends on the modal base. However, the modes determined for this modeling are not the same ones as those of the modal base of reference, and it is not possible to carry out direct calculation with Code_Aster on a concept [`base_modale`]. Only the answers corresponding to provided measurements can thus be validated. No comparison can be realized on the other answers.

4.4 Sizes tested and results

Identification			Reference
		with T = 9. 10^{-4} S	3,416 10^{+7}
`SIXX`	with the node $N3$	with T = 17. 10^{-4} S	8,046 10^{+7}
(Pa)		with T = 25. 10^{-4} S	4,251 10^{+7}

		with T = 9. 10^{-4}S	4,650 10^{+7}
SIXX	with the node $N4$	with T = 17. 10^{-4}S	6,671 10^{+7}
(Pa)		with T = 25. 10^{-4}S	4,293 10^{+7}

5 Summary of the results

For two modelings, the answers in constraints obtained after projection are identical to the constraints of reference obtained by direct calculation with *Code_Aster* and provided in data.

For modeling A, the answers in displacements and deformation are in perfect adequacy with the reference solutions. The values speeds and the accelerations obtained after projection are close to those obtained by direct calculation. The weak noted variations are due to the errors of approximation generated by the determination by a linear diagram in time of speeds and accelerations.

The cases where the number of modes is not equal to the number of measurements are not tested (generalized opposite problem); in particular, the method of regularization of Tikhonov is not tested.