SDLS106 - Modal calculation of a rectangular plate simply pressed on all its edges

Summary:

This test of the field of the modal analysis implements the calculation of clean modes of inflection of a rectangular plate simply pressed on all its edges.

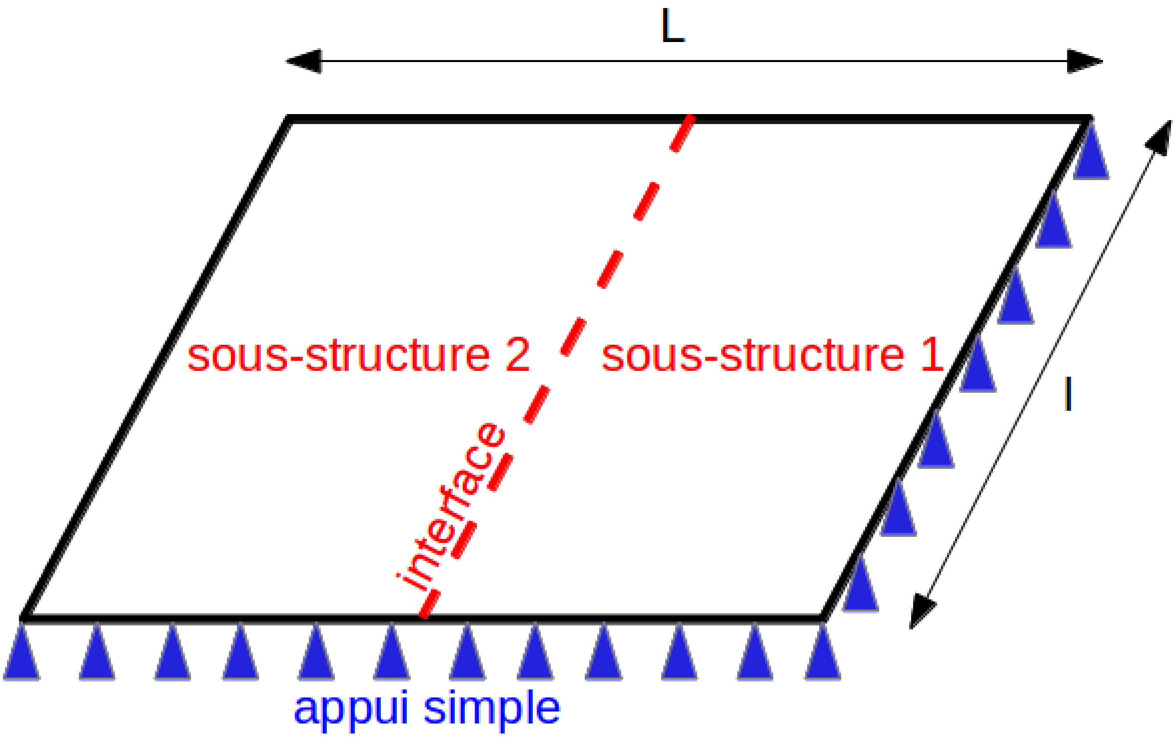
Is calculated

- maybe under - structuring on the basis of modal Ritz, with an interface of the type Craig-Bampton or Mac Neal;
- maybe in a direct way.

The reference solution is analytical, at the same time for the Eigen frequencies and the modal deformations.

1 Problem of reference

1.1 Geometry



$L=2\,m$

$l=1{,}5\,m$

1.2 Properties of the structure

$\rho_v=7800\,kg/m^3$ $E=2.10^{11}\,Pa$ $\nu=0.3$ thickness $h=1\,mm$.

1.3 Boundary conditions and loadings

The plate is in simple support on its four edges.

Note: in the case of calculations by under-structuring (modelings A with G), the interface of each substructure is embedded (interface of the Craig-Bampton type).

2 Reference solution

2.1 Reference solution

According to [bib1], the Eigen frequencies of vibration of a rectangular plate pressed on all its edges are given by the analytical formula:

$$f_{ij}=\frac{\lambda_{ij}^2}{2\pi L^2}\left[\frac{E h^3}{12\rho_s(1-\nu^2)}\right]^{\frac{1}{2}}$$

with

$$\lambda_{ij}^2=\pi^2\left[i^2+\left(\frac{L}{l}\right)^2 j^2\right]$$

where
i and j are the number of half-waves of the modal deformation along the main roads and small axis of the plate. ρ_s is the mass per unit of area.

That is to say

$$\begin{aligned} f_{11}&=17{,}13\,Hz\\ f_{21}&=35{,}63\,Hz\\ f_{12}&=50{,}01\,Hz\\ f_{31}&=66{,}46\,Hz\\ f_{22}&=68{,}51\,Hz \end{aligned}.$$

The modal deformations are also calculated analytically: modal displacement z_{ij} perpendicular with the plate, for the mode (i, j), according to the point of coordinates (x, y), is given by:

$$z_{ij}(x,y)=\sin\left(\frac{i\pi x}{L}\right)\sin\left(\frac{j\pi y}{l}\right) .$$

2.2 Case of calculations by under-structuring: reference solution of each substructure

Each substructure is a plate length $l=1{,}5\,m$ and of width $\frac{L}{2}=1\,m$, supported on three dimensioned and embedded on a long side, vibrating in inflection.
It is shown [bib1] that the Eigen frequencies are worth:

$$f_{ij}=\frac{\lambda_{ij}^2}{2\pi l^2}\left[\frac{E h^3}{12\rho_s(1-\nu^2)}\right]^{\frac{1}{2}}$$

with $\lambda_{11}^2=42{,}53$, $\lambda_{21}^2=69{,}00$, $\lambda_{31}^2=116{,}30$, $\lambda_{12}^2=121{,}00$

what gives for the first frequencies:

$$f_{11}=47,26\,Hz$$
$$f_{21}=76,57\,Hz$$
$$f_{31}=129,24\,Hz$$
$$f_{12}=134,47\,Hz$$

2.3 Bibliographical reference

1.BLEVINS R.D: Formulated for natural frequency and shape mode. ED. Krieger 1984.

3 Modeling A

3.1 Characteristics of modeling

For each of the two substructures: 600 meshs `QUAD4`.

3.2 Sizes tested on the complete structure

One tests the Eigen frequencies:

Identification	Reference	Tolerance
N°11 mode frequency	$17.13\,Hz$	0.5%
N°21 mode frequency	$35.63\,Hz$	0.5%
N°12 mode frequency	$50.01\,Hz$	0.5%
N°31 mode frequency	$66.46\,Hz$	0.5%
N°22 mode frequency	$68.51\,Hz$	0.5%

One tests also the modal deformations of the third and fourth modes (located by the indices $(1,2)$ and $(3,1)$ respectively), by recovering the component DZ deformation according to three lines: two median axes and the diagonal of the plate. Each line is discretized by 10 points.

Identification	Reference	Tolerance (relative, except contrary mention)
N°12 mode		
Deformation along the median main roads	cf paragraph 2.1	0.1 (in absolute)
Deformation according to petir it median axis	"	1 %
Deformation according to the diagonal	"	2 %
N°31 mode		
Deformation along the median main roads	"	4 %
Deformation according to petir it median axis	"	1 %
Deformation according to the diagonal	"	4 %

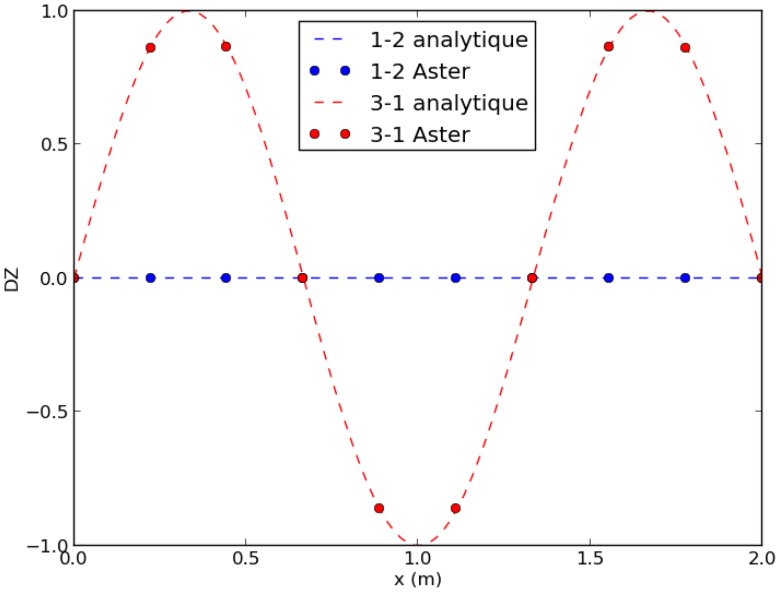


Figure 3.2-1 - Modal deformations along the median main roads.

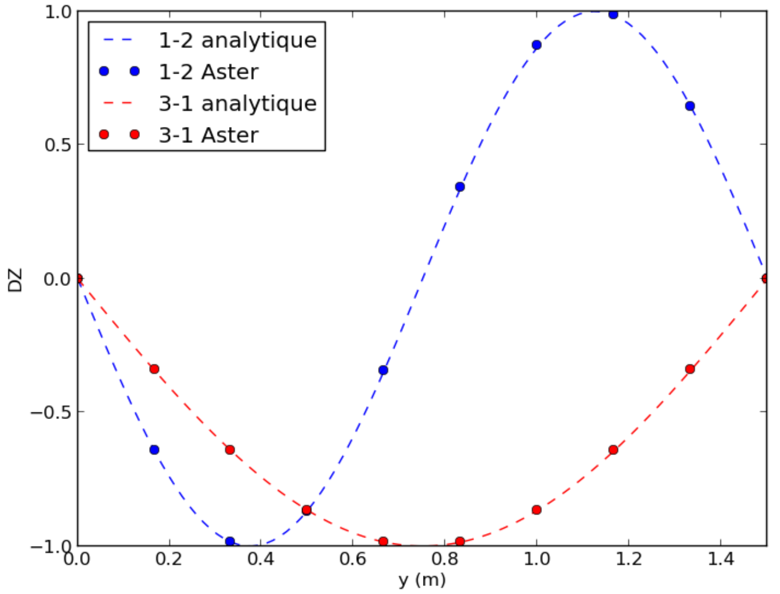


Figure 3.2-2 - Modal deformations along the small median axis.

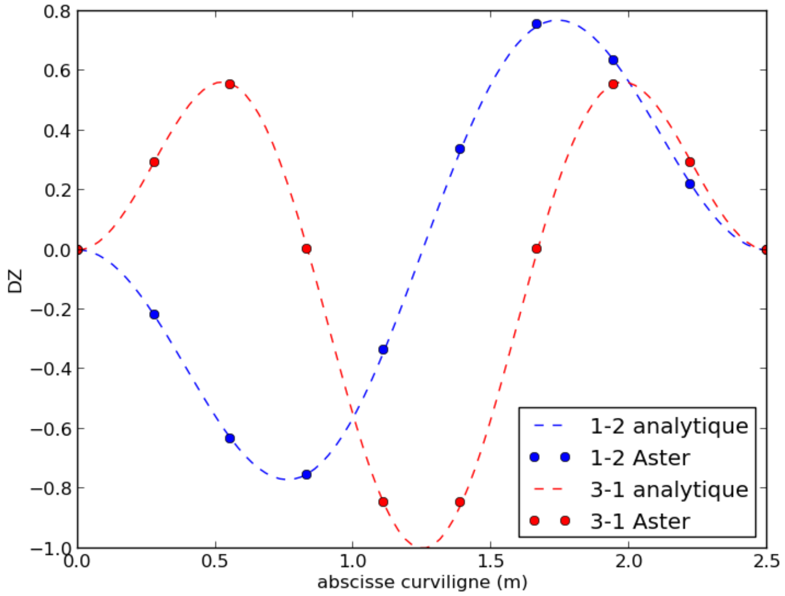


Figure 3.2-3 - Modal deformations according to the diagonal.

The results are very satisfactory.

4 Modeling B

4.1 Characteristics of modeling

Under structure 1: 600 meshs QUAD4.
Under structure 2: 509 meshs QUAD4.
For each under structure: 6 modes with interface fixes + 20 modes of coupling.

4.2 Sizes tested on the complete structure

Identification	Reference	Tolerance
N°11 mode frequency	$17.13\ Hz$	1.25%
N°21 mode frequency	$35.63\ Hz$	1.25%
N°12 mode frequency	$50.01\ Hz$	1.25%
N°31 mode frequency	$66.46\ Hz$	1.25%
N°22 mode frequency	$68.51\ Hz$	1.25%

5 Modeling C

5.1 Characteristics of modeling

One releases the supports to test the existence of the 6 modes of rigid body.

Under structure 1: 600 meshs QUAD4.
Under structure 2: 509 meshs QUAD4.
For each under structure: 6 modes with interface fixes + 20 modes of coupling.

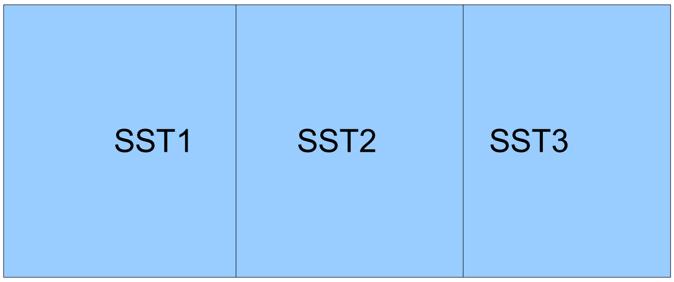
5.2 Sizes tested on the complete structure

Identification	Reference	Tolerance
N°1 mode frequency	$0.0\,Hz$	$0.1\,Hz$
N°2 mode frequency	$0.0\,Hz$	$0.1\,Hz$
N°3 mode frequency	$0.0\,Hz$	$0.1\,Hz$
N°4 mode frequency	$0.0\,Hz$	$0.1\,Hz$
N°5 mode frequency	$0.0\,Hz$	$0.1\,Hz$

6 Modeling D

6.1 Characteristics of modeling

The structure is cut out in 3 pennies structures, all pressed on the external edges. The complete model presents two interfaces, one of type Craig&Bampton, the other of MacNeal type. One tests the possibility of having under structure having two different types of interfaces:



Under structure 1 and 3: 360 meshs `QUAD4`.
Under structure 2: 480 meshs `QUAD4`.
For each under structure: 6 modes with interface fixes + 20 modes of coupling.

6.2 Sizes tested on the complete structure

Identification	Reference	Tolerance
N°11 mode frequency	$17.13\,Hz$	1.25%
N°21 mode frequency	$35.63\,Hz$	1.25%
N°12 mode frequency	$50.01\,Hz$	1.25%
N°31 mode frequency	$66.46\,Hz$	1.25%
N°22 mode frequency	$68.51\,Hz$	1.25%

7 Modeling E

7.1 Characteristics of modeling

Identical to modeling B, but use of the operators CREA_ELEM_SSD and ASSE_ELEM_SSD.

7.2 Sizes tested on the complete structure

Identification	Reference	Tolerance
N°11 mode frequency	$17.13\,Hz$	1.25%
N°21 mode frequency	$35.63\,Hz$	1.25%
N°12 mode frequency	$50.01\,Hz$	1.25%
N°31 mode frequency	$66.46\,Hz$	1.25%
N°22 mode frequency	$68.51\,Hz$	1.25%

8 Modeling F

8.1 Characteristics of modeling

Identical to modeling D, but use of the operators CREA_ELEM_SSD and ASSE_ELEM_SSD.

8.2 Sizes tested on the complete structure

Identification	Reference	Tolerance
N°11 mode frequency	$17.13\ Hz$	1.25%
N°21 mode frequency	$35.63\ Hz$	1.25%
N°12 mode frequency	$50.01\ Hz$	1.25%
N°31 mode frequency	$66.46\ Hz$	1.25%
N°22 mode frequency	$68.51\ Hz$	1.25%

9 Modeling G

9.1 Characteristics of modeling

Identical to modeling A, but of use a method of under structuring by free modes only (MODELE_GENE with OPTION "REDUCED", NUME_DDL_GENE with method ELIMINATE to ensure the compatibility of the interfaces). The results bad, like are expected by using this kind of approach, from where the use of important tolerances.

9.2 Sizes tested on the complete structure

Identification	Reference	Tolerance
N°11 mode frequency	$17.13\ Hz$	100%
N°21 mode frequency	$35.63\ Hz$	100%
N°12 mode frequency	$50.01\ Hz$	100%
N°31 mode frequency	$66.46\ Hz$	100%
N°22 mode frequency	$68.51\ Hz$	100%

10 Modeling H

10.1 Characteristics of modeling

The plate is modelled with a complete grid (contrary to the under-structuring). The grid comprises 1200 meshs QUAD4, which corresponds to the double of modeling A.

10.2 Sizes tested on the complete structure

One tests the Eigen frequencies:

Identification	Reference	Tolerance
N°11 mode frequency	$17.13\,Hz$	0.5%
N°21 mode frequency	$35.63\,Hz$	0.5%
N°12 mode frequency	$50.01\,Hz$	0.5%
N°31 mode frequency	$66.46\,Hz$	0.5%
N°22 mode frequency	$68.51\,Hz$	0.5%

One tests also the modal deformations of the third and fourth modes (located by the indices $(1,2)$ and $(3,1)$ respectively), by recovering the component DZ deformation according to three lines: two median axes and the diagonal of the plate. Each line is discretized by 10 points.

Identification	Reference	Tolerance (relative, except contrary mention)
N°12 mode		
Deformation along the median main roads	cf paragraph 2.1	0.1 (in absolute)
Deformation according to petir it median axis	"	1 %
Deformation according to the diagonal	"	1 %
N°31 mode		
Deformation along the median main roads	"	1 %
Deformation according to petir it median axis	"	1 %
Deformation according to the diagonal	"	1 %

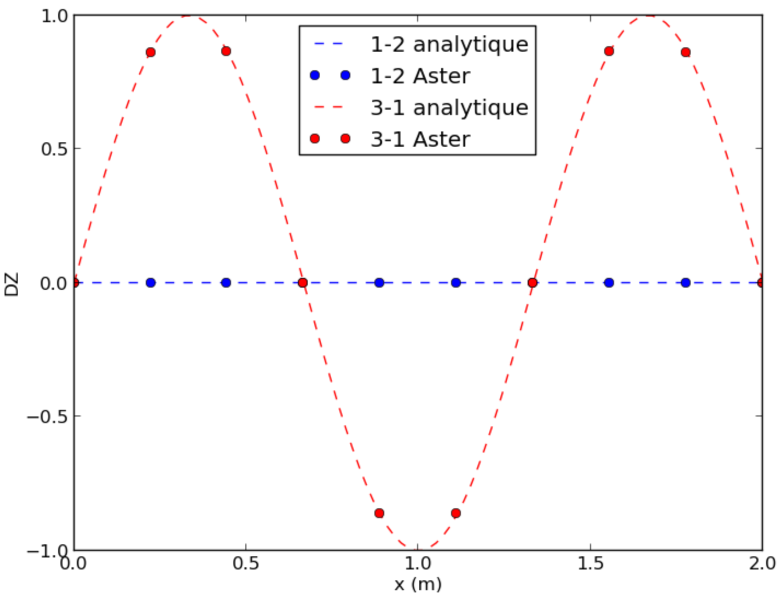


Figure 10.2-1 - Modal deformations along the median main roads.

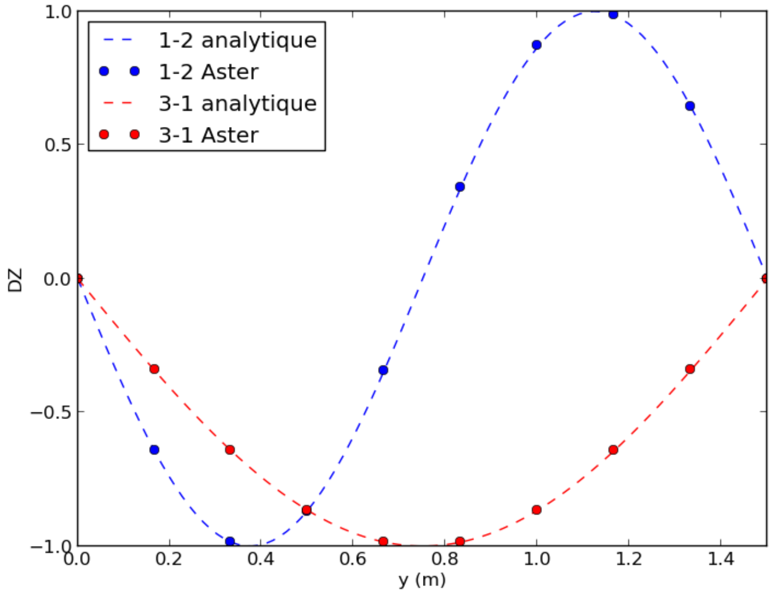


Figure 10.2-2 - Modal deformations along the small median axis.

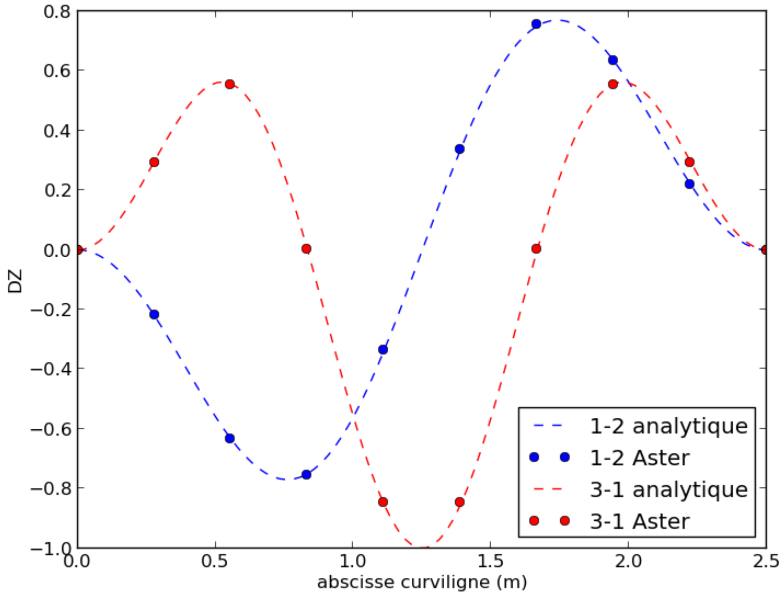


Figure 10.2-3 - Modal deformations according to the diagonal.

The results are very satisfactory.

11 Summary of the results

Modal calculation was validated on the clean modes of inflection of a plate simply pressed on its four edges, as well for the Eigen frequencies as the modal deformations.

The results are satisfactory, that is calculated direct manner or by under-structuring with modal base of type “Ritz”.