SDLL06 - Transitory answer of a post embed-free

Summary

In this case test, one analyzes the transitory answer of a not deadened embed-free beam, modelled by a system masses - arises and subjected to an unspecified dynamic loading.

One tests the discrete element in inflection, the calculation of the clean modes by the method of Lanczos and the calculation of the transitory answer by modal recombination of the structure subjected either to a accélérogramme (modeling A) or with an equivalent imposed force (modeling B).

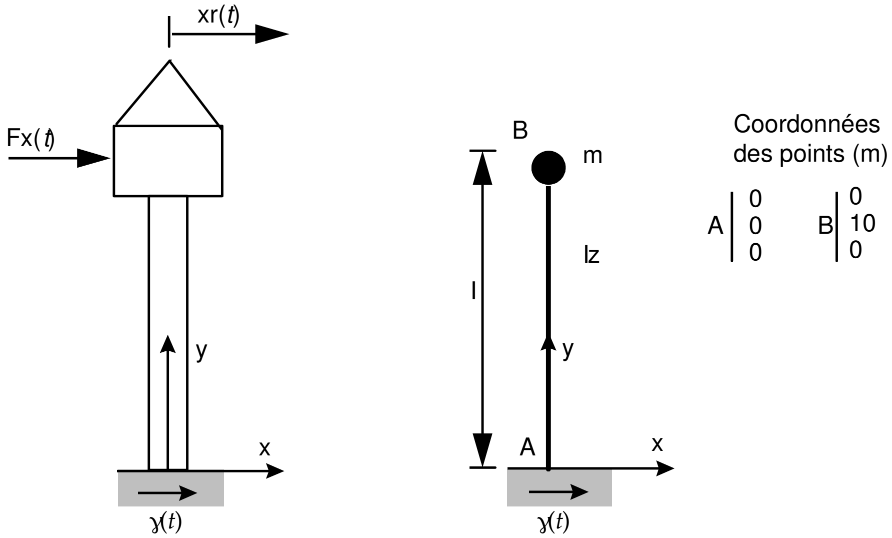
The diagram of Euler is used.

The got results are in concord with the results of reference (analytical results).

1 Problem of reference

1.1 Geometry

It is a problem suggested initially in the reference [bib1] and contained in [bib2].



- beam AB : beam hurled without mass length AB , $l=10\ m$ and of moment of inertia $I_Z=0{,}3285\ m^4$.
- specific mass in B : $m=43{,}8\ 10^3\ kg$

1.2 Properties of materials

Young modulus: $E=4.10^{10}\ Pa$
Density: $\rho=0\ kg/m^3$

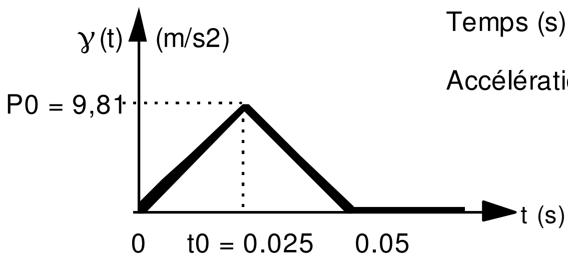
1.3 Boundary conditions and loadings

Boundary conditions:

Only authorized displacements are the translations according to the axis x .
The point A is embedded: $dx=dy=dz=drx=dry=drz=0$.

Loadings:

- modeling a: transverse acceleration at point a: $\gamma(t)$



Temps (s)	0	0.025	0.05
Accélération suivant x (ms-2)	0	9.81	0

- modeling b: forces transverse at the point B : $Fx(t)$ with $Fx(t)=-m.\gamma(t)$

1.4 Initial conditions

The system is at rest: with $t=0$, $dx(0)=0$, $dx/dt(0)=0$ in any point.

2 Reference solution

2.1 Method of calculating used for the reference solution

With the problem is dealt by a model with a degree of freedom. The post is regarded as a slim not deadened and nonheavy beam of rigidity $k=3EI_Z/l^3=3{,}942.10^7\ N/m$. The superstructure located at the top of the post is modelled by a specific mass $m=43{,}8\ 10^3\ kg$.

The two loading cases lead to the calculation of the response of a system to a degree of freedom subjected to an acceleration $\gamma(t)$ of an unspecified form:

$\ddot{x}_r+\omega^2 x_r=-\gamma(t)$ with $\omega=\sqrt{\frac{k}{m}}=\sqrt{\frac{3EI_z}{ml^3}}$ the Eigen frequency of the system and x_r the relative displacement of the point B compared to the point A . The solution is obtained by integration of the integral of Duhamel [bib3]:

$$x_r(t)=-\frac{m}{\omega}\int_0^t \gamma(t)\sin\omega(t-\tau)\,d\tau$$

2.2 Results of reference

Displacement relating to the point B .

For a triangular imposed acceleration, one can calculate the integral of Duhamel analytically [bib3]:

$$t<t_0 \quad : \quad x_r=-\frac{P_0}{\omega^2 t_0}\left(t-\frac{\sin\omega t}{\omega}\right)$$

$$t_0<t<2t_0 \quad : \quad x_r=-\frac{P_0}{\omega^2 t_0}\left(2t_0-t-\frac{2\sin\omega(t-t_0)}{\omega}-\frac{\sin\omega t}{\omega}\right)$$

$$t_0<t<2t_0 \quad : \quad x_r=-\frac{P_0}{\omega^3 t_0}\left(2\sin\omega(t-t_0)-\sin\omega(t-2t_0)-\sin\omega t\right)$$

2.3 Uncertainty on the solution

No if one calculates the integral of Duhamel analytically [bib3]. About the precision of the method of integration digital employed to calculate the integral of Duhamel ([bib1], [bib2]): method of Simpson with 40 points per period.

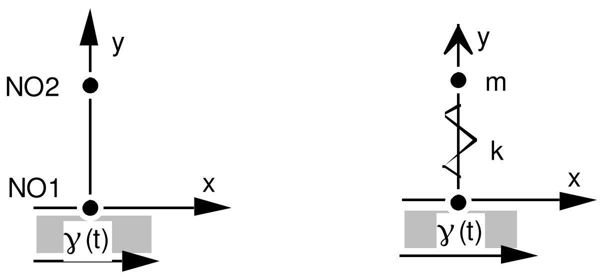
2.4 Bibliographical references

1) R.W. Clough and J. Penzien: Dynamics of New York structures, Mac Graw-Hill, 1975, p. 102 - 105
2) Technical guide VPCS AFNOR - 1990
3) J.S. Przemieniecki: Theory of matrix structural analysis New York, Mac Graw-Hill, 1968, p. 351-357

3 Modeling A

3.1 Characteristics of modeling

The elements are modelled by discrete elements with 6 degrees of freedom `DIS_TR`.



The node $NO1$ is subjected to an imposed acceleration $\gamma(t)$. One calculates the relative displacement of the node $NO2$ compared to the displacement of the node $NO1$ and one it compared to analytically calculated displacement.
Temporal integration is carried out with the algorithm of Euler (not of time: $5.10^{-4} s$).

3.2 Characteristics of the grid

The grid consists of 2 nodes and a discrete element (`DIS_TR`).

3.3 Sizes tested and results

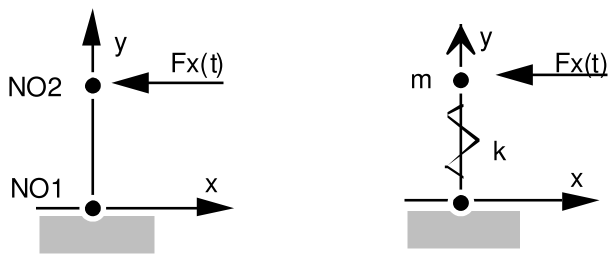
Relative displacement of node NO1 (in meters).

Time (S)	Analytical calculation	*Code_Aster*	Error (%)
0.010	– 6,511E-05	– 6,495E-05	0
0.015	– 2,185E-04	– 2,183E-04	0
0.020	– 5,139E-04	– 5,136E-04	– 0.058
0.024	– 8,809E-04	– 8,806E-04	– 0.039
0.026	– 1,115E-03	– 1,115E-03	– 0.041
0.030	– 1,679E-03	– 1,679E-03	– 0.014
0.035	– 2,523E-03	– 2,523E-03	– 0.004
0.040	– 3,457E-03	– 3,457E-03	0
0.045	– 4,412E-03	– 4,412E-03	0.004
0.049	– 5,143E-03	– 5,143E-03	0.005
0.051	– 5,485E-03	– 5,485E-03	0.005
0.055	– 6,109E-03	– 6,109E-03	0.005
0.060	– 6,765E-03	– 6,765E-03	0.005
0.065	– 7,269E-03	– 7,269E-03	0.005
0.070	– 7,610E-03	– 7,610E-03	0.005
0.075	– 7,779E-03	– 7,780E-03	0.005
0.080	– 7,774E-03	– 7,775E-03	0.004
0.085	– 7,595E-03	– 7,595E-03	0.004

4 Modeling B

4.1 Characteristics of modeling

The elements are modelled by discrete elements with 6 degrees of freedom `DIS_TR`.



The node $NO2$ is subjected to an imposed force $Fx(t)$. One calculates the relative displacement of the node $NO2$ compared to the displacement of the node $NO1$ and one it compared to displacement calculated in the references [bib1] and [bib2].
Temporal integration is carried out with the algorithm of Euler (not of time: $10^{-3} s$).

4.2 Characteristics of the grid

It is the same grid as for modeling A.

4.3 Sizes tested and results

Relative displacement of the node $NO1$ (in meters).

Time (s)	References [bib1], [bib2]	*Code_Aster*	Error (%)
0.01	– 6,500E-05	– 6,447E-05	– 0.82
0.02	– 5,130E-04	– 5,127E-04	– 0.064
0.03	– 1,679E-03	– 1,678E-03	– 0.037
0.04	– 3,457E-03	– 3,457E-03	0.013
0.05	– 5,316E-03	– 5,317E-03	0.022
0.06	– 6,764E-03	– 6,766E-03	0.035
0.07	– 7,609E-03	– 7,611E-03	0.027
0.08	– 7,774E-03	– 7,776E-03	0.024
0.09	– 7,244E-03	– 7,246E-03	0.028
0.1	– 6,068E-03	– 6,069E-03	0.014
0.12	– 2,242E-03	– 2,242E-03	– 0.017
0.14	2,367E-03	2,369E-03	0.071
0.16	6,149E-03	6,152E-03	0.041
0.18	7,783E-03	7,785E-03	0.029
0.2	6,698E-03	6,699E-03	0.018

5 Summary of the results and general remarks

The simplified model presented in this case test makes it possible to validate the method of resolution digital. To deal with the real physical problem, it would be necessary to take into account the effects of inertia (mass of the post, effect of inertia of rotation around B of the superstructure) and of compression of the post (actual weight).

For modeling A, the mistake made with a step of time of $5.10^{-4}s$ is about $0{,}01\%$; for modeling B (not of time of $10^{-3}s$) it is about $0{,}6\%$.

One will be able to supplement this case test by checking the convergence of the results for other step values of time and by comparing the results got with other diagrams of integration.