SDLD105 - Transitory answer of a system mass-springs with an earthquake mono-support with static correction

---

Summary

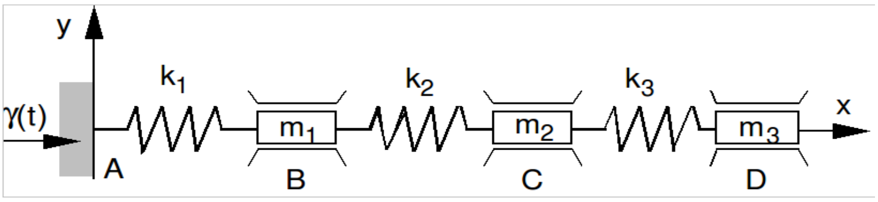
This case test, inspired by the case test VPCS SDLD04, consists in calculating the not deadened transitory answer of a embed-free system mass-springs linear, subjected to an imposed acceleration mono-support.

Its objective is to show the interest of the static correction, that it is carried out *a priori* by addition of static modes to the base of the dynamic modes, or *a posteriori*.

1 Problem of reference

1.1 Geometry

One calculates the response of a linear system composed of three masses and three springs to an acceleration imposed on the level of his point of anchoring (A) :



1.2 Properties of materials

- stiffnesses of connection: $k_1=1000\,N/m$ and $k_2=k_3=100\,N/m$;
- specific masses: $m=m_1=m_2=m_3=1\,kg$.
- modal damping of $5\,\%$ for all modes

1.3 Boundary conditions and loadings

Boundary conditions

Only authorized displacements are the translations according to the axis x .
The point A is embedded: $dx=dy=dz=drx=dry=drz=0$.

Loading

The point of anchoring A is subjected to a harmonic acceleration of frequency f_{ex}. Is calculated 0 with $20\,s$.

1.4 Initial conditions

The system is initially at rest: with $t=0$, $dx(0)=0$ and $dx/dt(0)=0$ in any point.

2 Reference solution

The objective of the case test is to show the effect of modal truncation on a very simple example of seismic calculation and to illustrate the interest of the static correction, that it is carried out *a priori* by addition of static modes to the base of the dynamic modes, or *a posteriori*. The reference solution is the transitory calculation on complete modal basis carried out with the operator `DYNA_VIBRA`.

The system mass-springs has three degrees of freedom. It is thus associated with three modes whose calculated frequencies are:
$f_1=0{,}946\,Hz$, $f_2=2{,}533\,Hz$ and $f_3=5{,}305\,Hz$.

The frequency of excitation of the harmonic seismic signal was selected with $f_{ex}=2\,Hz$ to keep in the modal base only the first two modes. One complies with the rule to twice retain the modes until the maximum frequency of excitation.

The table of the unit effective modal masses gives interesting information:

Unit effective mass			
NUME_MODE	FREQUENCY	MASS_EFFE_UN_DX	CUMUL_DX
1	9.48538E-01	6.82972E-01	6.82972E-01
2	2.53344E+00	5.03369E-02	7.33309E-01
3	5.30513E+00	2.66691E-01	1.00000E+00

It is observed that the third mode will have a negligible dynamic response because its Eigen frequency is worth $f_3=5{,}305\,Hz$, beyond $2\times f_{ex}=4{,}0\,Hz$. On the other hand its unit effective modal mass in the direction x is worth $26{,}7\,\%$. It is thus not negligible and this mode can affect the answer of the system mass-springs by its quasi-static contribution. It is the goal of the static correction which to take it into account.

If one looks at now the geometry of mode 3, one note that it is mainly on the node $N02$ that one will be able to observe the effect of the mode and thus the effect of the static correction.

	Mode 1	Mode 2	Mode 3
Node	DX	DX	DX
$N01$ (not A)	0.00000E+00	0.00000E+00	0.00000E+00
$N02$ (not B)	5.08430E-02	9.84653E-02	9.93841E-01
$N03$ (not C)	5.41213E-01	8.33623E-01	-1.10279E-01
$N04$ (not D)	8.39347E-01	-5.43487E-01	1.09069E-02

Indeed the table, upon reading of the modal components, it appears that the component of mode 3 on all the nodes is minority compared to the other modes, except for node $N02$.

2.1 Results of reference

One takes for results of reference the results given by `DYNA_VIBRA` with the modal base supplements with the nodes $N02$ and $N04$ at the moment $t=19{,}4\,s$.

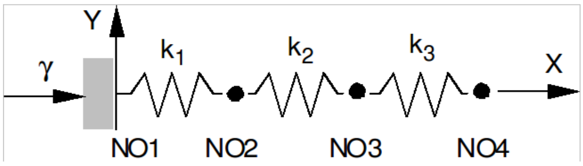
2.2 Uncertainty on the solution

Precision on integration in time in `DYNA_VIBRA`

3 Modeling A

3.1 Characteristics of modeling

The springs and specific masses are modelled by discrete elements with 3 degrees of freedom `DIS_T`:



The node $NO1$ is embedded and subjected to an imposed acceleration $\gamma(t)$.

3.2 Characteristics of the grid

Many nodes: 4
Many meshs and types: 3 `DIS_T`

3.3 Sizes tested and results

Eigen frequencies (in Hz) system:

Number of the mode	***Code_Aster***
1	0.94853
2	2.53344
3	5.30513

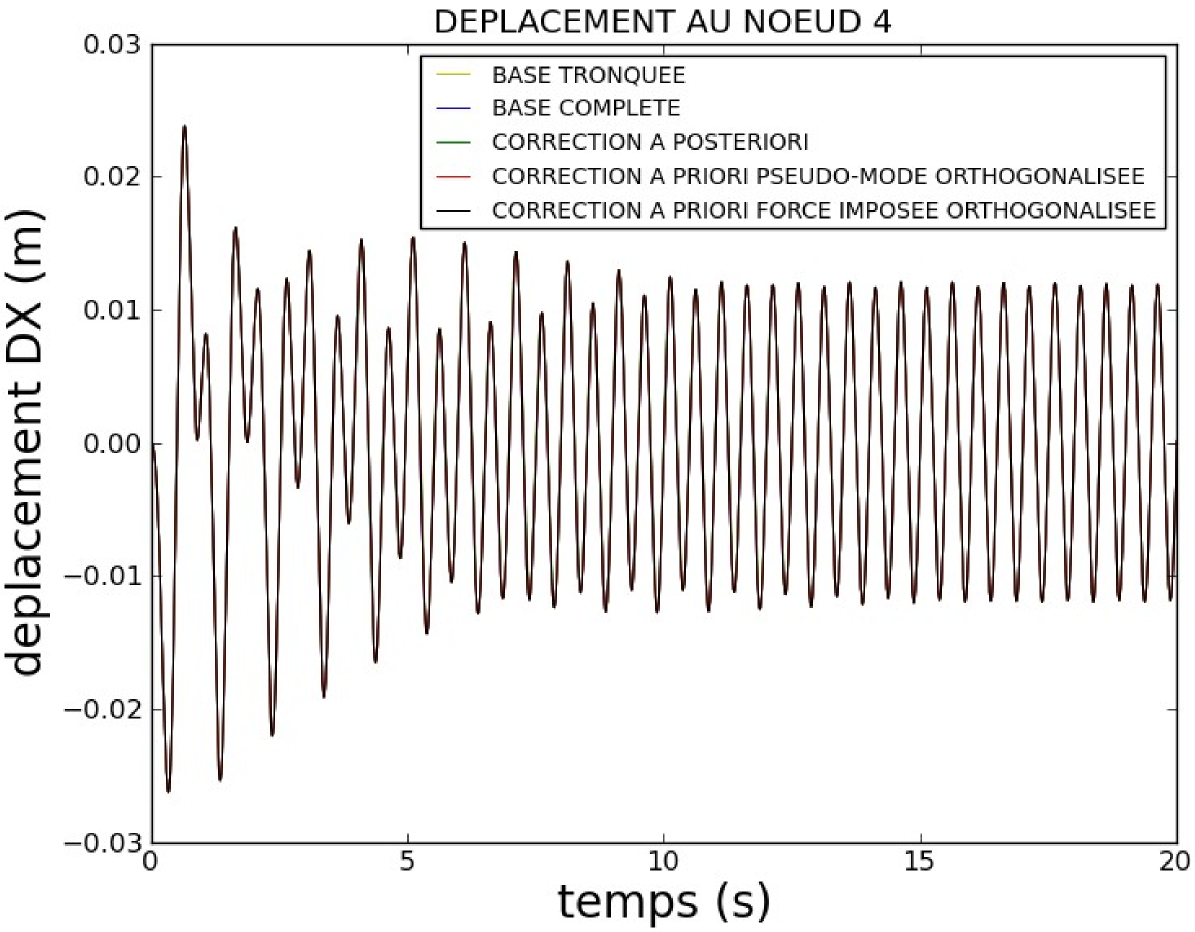
Transitory calculation by modal synthesis
One tests the taking into account of a loading in the form of vector project on the modal basis, in the form of modal component, vector project and of modal component simultaneously, as well as the taking into account of the modes neglected by the static correction.
The static correction is taken into account according to the various following possibilities:

- static correction *a posteriori* ;
- static correction *a priori*, by supplementing the base of the dynamic modes by the static modes with imposed force, with D-orthogonalisation of the base thus supplemented;
- static correction *a priori*, by supplementing the base of the dynamic modes by the static modes with imposed acceleration (pseudo-mode), with D-orthogonalisation of the base thus supplemented;
- static correction *a priori*, by supplementing the base of the dynamic modes by the static modes with force imposed and the static modes on imposed acceleration (pseudo-mode), with D-orthogonalisation of the base thus supplemented.

Values of the relative displacement of the node $NO4$ at the moment $t=19{,}4\,s$ (m):

base modal	Reference	Tolerance
base truncated	-0.011349	0.1%
base complete	-0.011349	1.e-04%
static correction *a posteriori*	-0.011349	0.1%
static correction *a priori*, pseudo-mode, with D-orthogonalisation	-0.011349	1.e-04%

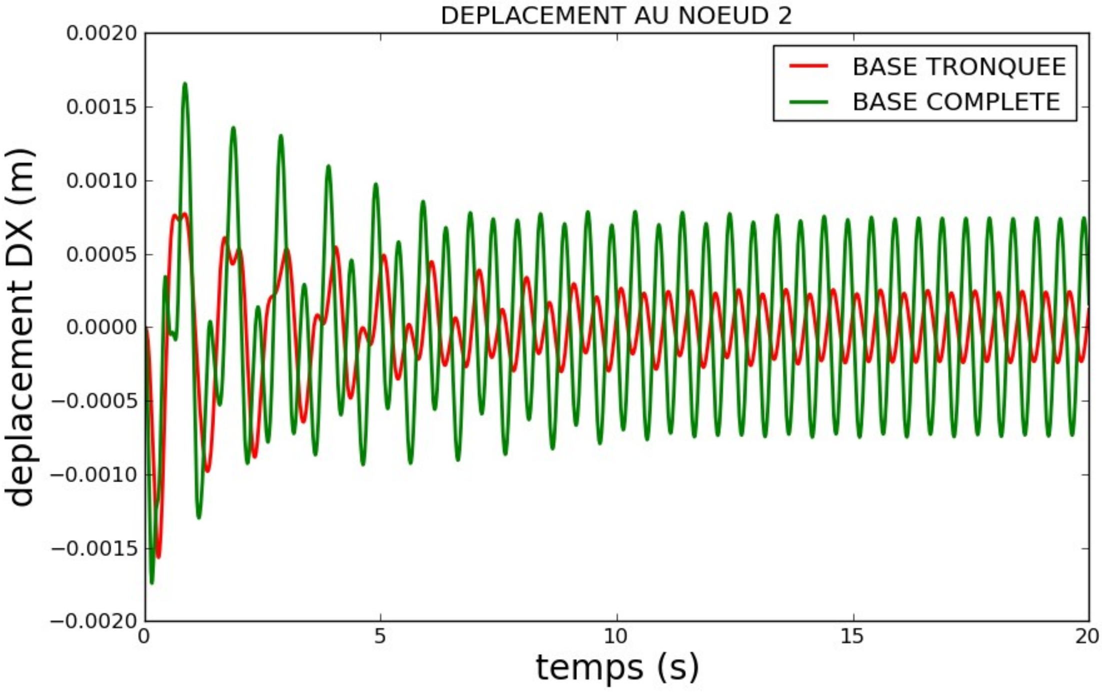
static correction *a priori*, modes statics with imposed force, with D-orthogonalisation	-0.011349	1.e-04%
static correction *a priori*, pseudo-mode, static modes with imposed force, with D-orthogonalisation	-0.011349	1.e-04%

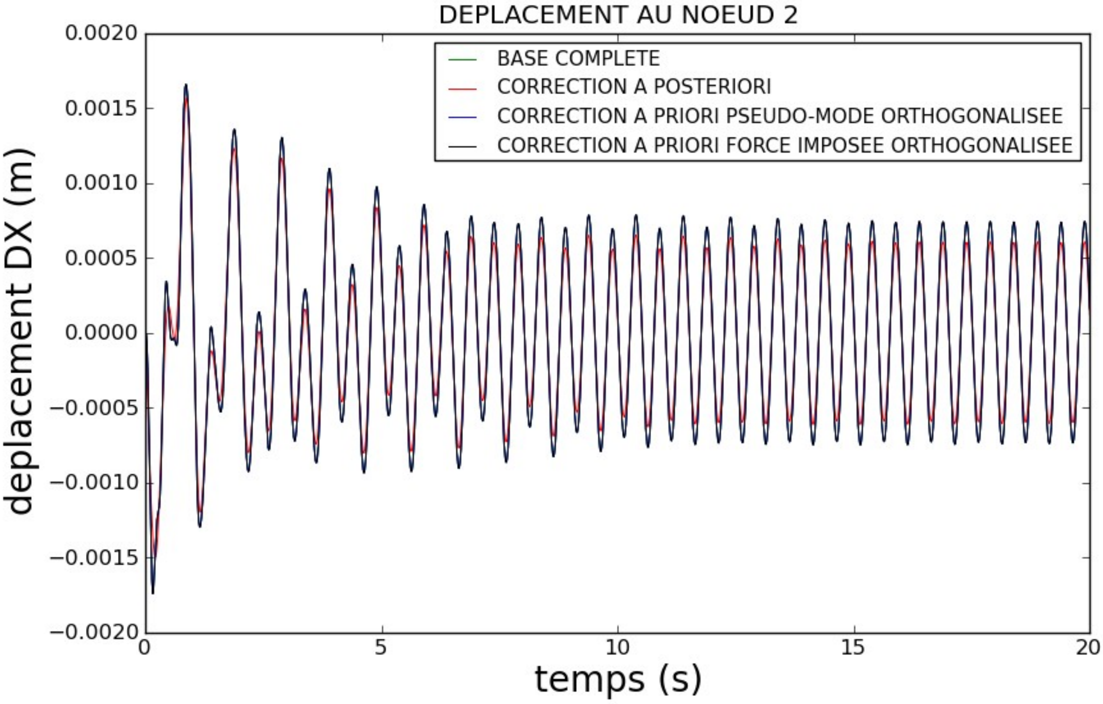


Values of the relative displacement of the node $NO2$ at the moment $t=19{,}4\,s$ (m) :

base modal	Reference	Tolerance
base truncated	7.340082E-04	123.0%
base complete	7.340082E-04	1.0E-04%
static correction *a posteriori*	7.340082E-04	19.5%
static correction *a priori*, pseudo-mode, with D-orthogonalisation	7.340082E-04	1.0E-04%

static correction *a priori*, modes statics with imposed force, with D-orthogonalisation	7.340082E-04	1.0E-04%
static correction *a priori*, pseudo-mode, static modes with imposed force, with D-orthogonalisation	7.340082E-04	1.e-04%





One illustrated Ci - above the interest of the static correction: as envisaged with the reading of the modal deformations, the static correction is not visible for $NO4$ but plays an important role for $NO2$. Without static correction with $NO2$, displacement is out of phase and its amplitude undervalued of 50% . With $NO2$, with static correction *a posteriori*, the error remains visible (lower than 20%), but calculation remains realistic; with static correction *a priori*, whatever the nature of the static modes considered (with acceleration or imposed force), the modal truncation error is completely compensated, making it possible to find the displacement of reference exactly. On a less caricatural calculation, the error will be less sensitive. It is also noticed that the amplitude of displacement with the node $NO2$ is of two orders smaller than that with the node $NO4$.

Summary of the results

The cas-test is an example of implementation of the static correction, that it is carried out *a priori* by addition of static modes to the base of the dynamic modes, or *a posteriori*. On a system especially calculated to show its effects, it shows that the static correction can reduce in an important way the error due to the effect of modal truncation on the “high frequencies” (static correction a posteriori), even to cancel it completely (static correction *a priori*), on condition that D-orthogonaliser the base of Ritz thus obtained. On an industrial study, one can expect a less visible effect.