SDLD102 - Under transitory structuring: System 3 masses-4 springs

Summary:

The scope of application of this test relates to the dynamics of the structures. It makes it possible to validate the diagrams of integration to step of adaptive time 'ADAPT_ORDRE2' and 'RUNGE_KUTTA_54' of the operator of transitory calculation on modal basis as well as the calculation of transitory answer linear on a modal basis calculated by under-structuring (for the 5 diagrams of integration : 'EULER', 'DEVOGE', 'NEWMARK', 'RUNGE_KUTTA_32' and 'ADAPT_ORDRE2'). In particular, the case of the application of a damping reduced to the dynamic modes of the bases of projection of the substructures is treated.

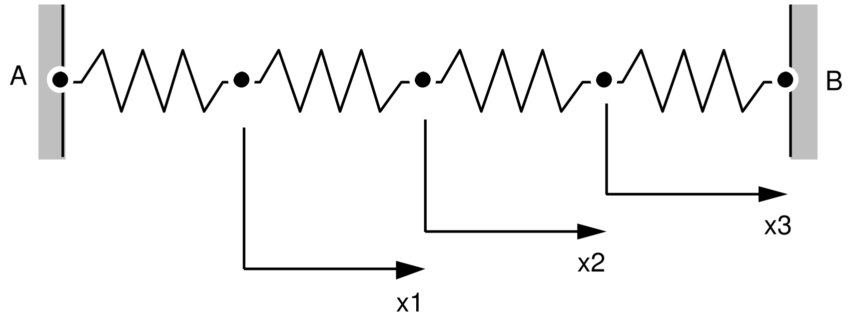
It is a question of determining the answer transitory of a system made up of 3 masses and 4 springs, embedded at its ends and subjected to a constant force as from the initial moment. The springs are modelled by elements of the type 'DIS_TR' and masses by elements of the type 'DIS_T'.

Six modelings are proposed. In the third modeling, the structure is deadened. The methods of calculating transient by under-structuring with interfaces of the Craig-Bampton type ('CRAIGB') and Mac Neal ('MNEAL') are tested. The taking into account of the option is also tested RELA_EFFO_DEPL in the case of a calculation of answer per dynamic under-structuring. The results of reference which are associated for them result from an analytical calculation. In the third, one imposes a reduced damping of 1% with the dynamic modes of the bases of projection of under - structures. The transitory equation checked by the complete structure was obtained analytically. Its resolution, which serves as reference, was carried out by the Maple software.

1 Problem of reference

1.1 Geometry

The studied system is composed of 3 masses (m) and 4 springs (k). The unit is embedded at its ends.

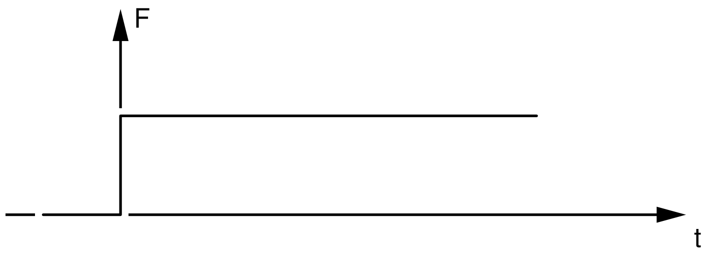


1.2 Material properties

Stiffness of the springs: $k=1\ N/m$.

Specific masses: $m=1\ kg$.

1.3 Boundary conditions and loadings



Points A and B embedded.

Application to the point $x1$ of a constant force $F=1\ N$, as from the moment $t=0\ s$.

1.4 Initial conditions

Structure initially at rest.

2 Reference solution

2.1 Method of calculating used for the reference solution

2.1.1 Not deadened structure

In this case, the reference solution can be obtained analytically:

$$m\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}\ddot{x}_1\\\ddot{x}_2\\\ddot{x}_3\end{pmatrix}+k\begin{pmatrix}2&-1&0\\-1&2&-1\\0&-1&2\end{pmatrix}\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\begin{pmatrix}1\\0\\0\end{pmatrix}$$

The own pulsations of the system mass-arises are worth:

$$\omega_1^2=(2-\sqrt{2})\frac{k}{m}\quad \omega_2^2=2\frac{k}{m}\quad \omega_3^2=(2+\sqrt{2})\frac{k}{m}$$

respective modal deformations:

$$\phi_1=\begin{pmatrix}\sqrt{2}\\2\\\sqrt{2}\end{pmatrix}\quad \phi_2=\begin{pmatrix}1\\0\\-1\end{pmatrix}\quad \phi_3=\begin{pmatrix}-\sqrt{2}\\2\\-\sqrt{2}\end{pmatrix}$$

Projected on the basis of clean mode, the transitory equation becomes η_i with like generalized coordinates:

$$m\begin{pmatrix}8&0&0\\0&2&0\\0&0&8\end{pmatrix}\begin{pmatrix}\ddot{\eta}_1\\\ddot{\eta}_2\\\ddot{\eta}_3\end{pmatrix}+4\mathrm{k}\begin{pmatrix}4-\sqrt{2}&0&0\\0&1&0\\0&0&4+2\sqrt{2}\end{pmatrix}\begin{pmatrix}\eta_1\\\eta_2\\\eta_3\end{pmatrix}=\begin{pmatrix}+\sqrt{2}\\2\\-\sqrt{2}\end{pmatrix}$$

The system can be solved analytically. One obtains:

$$\{\eta(t)\}=\frac{1}{2\mathrm{m}}\begin{pmatrix}\dfrac{\sqrt{2}}{4\omega_1^2}(1-\cos\omega_1 t)\\ \dfrac{1}{\omega_2^2}(1-\cos\omega_2 t)\\ \dfrac{\sqrt{2}}{4\omega_3^2}(\cos\omega_3 t-1)\end{pmatrix}$$

The solution on physical basis is obtained by using the transformation of Ritz:

$$x(t)=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\phi\,\eta=\begin{pmatrix}\sqrt{2}&1&-\sqrt{2}\\2&0&2\\\sqrt{2}&-1&\sqrt{2}\end{pmatrix}\begin{pmatrix}\eta_1\\\eta_2\\\eta_3\end{pmatrix}$$

2.1.2 Deadened structure

Damping is applied to the clean modes of the bases of projection of the embedded substructures (reduced damping). In this case, one leads to the transitory equation in generalized coordinates following (feeding-bottle [1]):

$$m\begin{pmatrix}8&0&0\\0&2&0\\0&0&8\end{pmatrix}\begin{pmatrix}\ddot{\eta}_1\\\ddot{\eta}_2\\\ddot{\eta}_3\end{pmatrix}+4\varepsilon\sqrt{2km}\begin{pmatrix}3-2\sqrt{2}&0&-1\\0&1&0\\-1&0&3+2\sqrt{2}\end{pmatrix}\begin{pmatrix}\dot{\eta}_1\\\dot{\eta}_2\\\dot{\eta}_3\end{pmatrix}+4k\begin{pmatrix}4-\sqrt{2}&0&0\\0&1&0\\0&0&4+2\sqrt{2}\end{pmatrix}\begin{pmatrix}\eta_1\\\eta_2\\\eta_3\end{pmatrix}=\begin{pmatrix}+\sqrt{2}\\1\\-\sqrt{2}\end{pmatrix}$$

This system not being uncoupled, it was solved using the Maple software. One obtained $(\varepsilon=0.01)$:

$$\{\eta(t)\}=\frac{1}{2m}\begin{pmatrix}\frac{\sqrt{2}}{4\omega_1^2}(1-e^{-\frac{t}{\tau_1}}\cos\omega_1 t)\\\frac{1}{\omega_2^2}(1-e^{-\frac{t}{\tau_2}}\cos\omega_2 t)\\\frac{\sqrt{2}}{4\omega_3^2}(e^{-\frac{t}{\tau_3}}\cos\omega_3 t-1)\end{pmatrix}$$

with $\tau_1=1.65\,10^3\,s$, $\tau_2=\frac{1}{\varepsilon\omega_2}=\frac{100}{\sqrt{2}}$ and $\tau_3=4.85\,10^1\,s$

One thus obtains a formulation close to the case not deadened, but in which intervene of the exponential terms which characterize damping.

The solution on physical basis is obtained by using the transformation of Ritz:

$$x(t)=\begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}=\phi\,\eta=\begin{pmatrix}\sqrt{2}&1&-\sqrt{2}\\2&0&2\\\sqrt{2}&-1&\sqrt{2}\end{pmatrix}\begin{pmatrix}\eta_1\\\eta_2\\\eta_3\end{pmatrix}$$

2.2 Results of reference

Not deadened structure:

Displacement, speed and acceleration of the node x_2 at the moment $t=80\ s$:

$$\begin{aligned} x_2(80) &= 4.1700\,10^{-1}m \\ \dot{x}_2(80) &= -4.3011\,10^{-1}m.s^{-1} \\ \ddot{x}_2(80) &= 3.3749\,10^{-1}m.s^{-2} \end{aligned}$$

Deadened structure:

Displacement of the node x_2 at the moment $t=80\ s$:

$$x_2(80)=4.9867\,10^{-1}m$$

2.3 Uncertainty on the solution

Case not deadened: analytical solution.

Deadened case: semi-analytical solution.

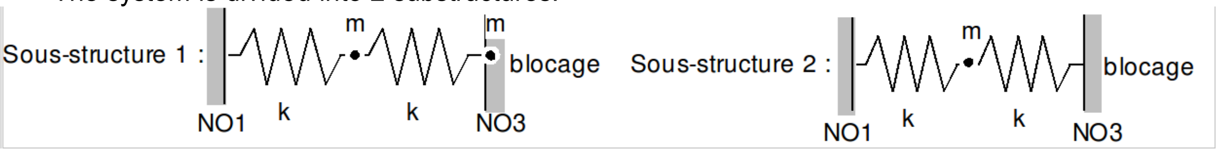
2.4 Bibliographical reference

1. C. VARE - Report HP 61/95/025/A - "Implementation of nonlinear transitory calculation by under-structuring in *Code_Aster*".

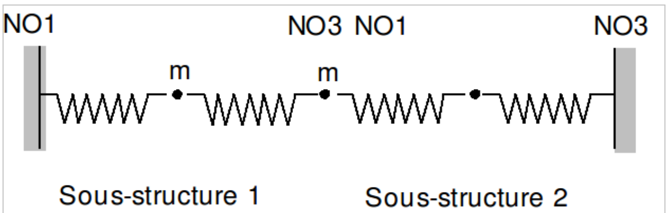
3 Modeling A

3.1 Characteristics of modeling

The system is divided into 2 substructures:



In situation, the two substructures are connected to the level of the 2nd mass. The dynamic interface of the 1st substructure consists of a mass m on the level of the node $NO3$ grid and coincides with the dynamic interface of the 2ème substructure which does not comprise any mass and is simply blocked on the level of the node $NO1$.



The clean modes of the complete system are calculated by using the method of calculating modal by under-structuring with interfaces of the type 'Craig-Bampton' (blocked interfaces). The bases of each substructure are made up of a dynamic mode and a constrained mode.

The transitory answer of the system is calculated on the modal basis calculated by under-structuring.

The steps of times used are equal to: $10^{-2} s$ in 'EULER', $10^{-2} s$ in 'NEWMARK', $10^{-2} s$ in 'DEVOGE', $10^{-1} s$ in 'ADAPT_ORDRE2' (for this last, it is the step of initial time of the algorithm and the step of maximum time is, him, fixed at formula $0{,}15 s$ then formula $0{,}2 s$ to optimize the computing time).

3.2 Characteristics of the grid of the substructure

Many nodes: 3

Many meshs and types: 2 SEG2

3.3 Sizes tested and results

Calculation by modal recombination without under-structuring: Method `ADAPT_ORDRE2` and `RUNGE_KUTTA_54`

Identification	Reference
Method: `ADAPT_ORDRE2`	
Node x_2 , displacement (m)	4.1700 10^{-1}
Node x_2 , speed ($m.s^{-1}$)	– 4.3011 10^{-1}
Node x_2 , acceleration ($m.s^{-2}$)	3.3375 10^{-1}
Method: `RUNGE_KUTTA_54`	
Node x_2 , displacement (m)	4.1700 10^{-1}
Node x_2 , speed ($m.s^{-1}$)	– 4.3011 10^{-1}
Node x_2 , acceleration ($m.s^{-2}$)	3.3375 10^{-1}

Calculation by under-structuring

Method: `EULER`

Node x_2 , displacement (m)	4.1700 10^{-1}
Node x_2 , speed ($m.s^{-1}$)	– 4.3011 10^{-1}
Node x_2 , acceleration ($m.s^{-2}$)	3.3375 10^{-1}

Method: `DEVOGE`

Node x_2 , displacement (m)	4.1700 10^{-1}
Node x_2 , speed ($m.s^{-1}$)	– 4.3011 10^{-1}
Node x_2 , acceleration ($m.s^{-2}$)	3.3375 10^{-1}

Method: `NEWMARK`

Node x_2 , displacement (m)	4.1700 10^{-1}
Node x_2 , speed ($m.s^{-1}$)	– 4.3011 10^{-1}
Node x_2 , acceleration ($m.s^{-2}$)	3.3375 10^{-1}

Method: `RUNGE_KUTTA_32`

Node x_2 , displacement (m)	4.1700 10^{-1}
Node x_2 , speed ($m.s^{-1}$)	– 4.3011 10^{-1}
Node x_2 , acceleration ($m.s^{-2}$)	3.3375 10^{-1}

Method: `ADAPT_ORDRE2`

Node x_2 , displacement (m)	4.1700 10^{-1}

Node x_2 , speed ($m.s^{-1}$)	– 4.3011 10– 1
Node x_2 , acceleration ($m.s^{-2}$)	3.3375 10– 1

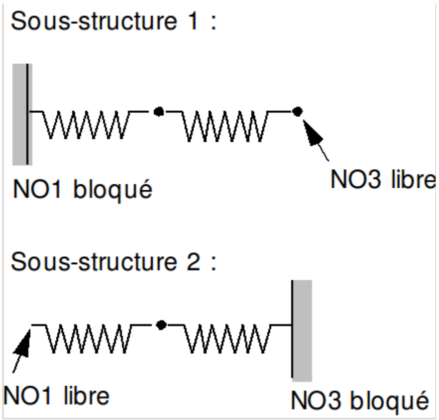
4 Modeling B

4.1 Characteristics of modeling

This modeling is identical to the precedent if it is not that the clean modes of the complete system are calculated by using the method of calculating modal by under-structuring with interfaces of the type 'Mac Neal' (free interfaces). The bases of each substructure are made up of a dynamic mode and a mode of fastener.

The transitory answer of the system is calculated on the modal basis calculated by under - structuring.

More precisely, the studied substructures have their free interfaces:



The steps of times used are worth: $10^{-2} s$ in EULER, $10^{-2} s$ in NEWMARK, $10^{-2} s$ in DEVOGE, $10^{-2} s$ in ADAPT_ORDRE2 (for this last, it is the step of initial time of the algorithm and the step of maximum time is, him, fixed at formula $0,1\ s$ to optimize the computing time).

4.2 Characteristics of the grid of the substructure

Many nodes: 3

Many meshs and types: 2 SEG2

4.3 Sizes tested and results

Identification	Reference
Method: EULER	
Node x_2, displacement (m)	4.1700 10^{-1}
Node x_2, speed ($m.s^{-1}$)	– 4.3011 10^{-1}
Node x_2, acceleration ($m.s^{-2}$)	3.3749 10^{-1}
Method: NEWMARK	
Node x_2, displacement (m)	4.1700 10^{-1}
Node x_2, speed ($m.s^{-1}$)	– 4.3011 10^{-1}

Node x_2 , acceleration ($m.s^{-2}$)	3.3749 10^{-1}

Method: **DEVOGE**

Node x_2 , displacement (m)	4.1700 10^{-1}
Node x_2 , speed ($m.s^{-1}$)	– 4.3011 10^{-1}
Node x_2 , acceleration ($m.s^{-2}$)	3.3749 10^{-1}

Method: ADAPT_ORDRE2

Node x_2 , displacement (m)	4.1700 10^{-1}
Node x_2 , speed ($m.s^{-1}$)	– 4.3011 10^{-1}
Node x_2 , acceleration ($m.s^{-2}$)	3.3749 10^{-1}

5 Modeling C

5.1 Characteristics of modeling

The clean modes of the complete system are calculated by using the method of calculating modal by under-structuring with interfaces of the type 'Craig-Bampton' (blocked interfaces). The bases of each substructure are made up of a dynamic mode and a constrained mode.

With the dynamic mode of each substructure is associated a damping reduced with 1%.

The transitory answer of the deadened system is calculated on the modal basis calculated by under - structuring.

The steps of time taken are equal to: $10^{-2} s$ in EULER, $10^{-2} s$ in NEWMARK, $10^{-2} s$ in ADAPT_ORDRE2 (for this last, it is the step of initial time of the algorithm and the step of maximum time is, him, fixed at formula $0,1 s$ to optimize the computing time).

5.2 Characteristics of the grid of the substructure

Many nodes: 3

Many meshs and types: 2 SEG2

5.3 Sizes tested and results

Identification	Reference
Method: EULER	
Node x_2, displacement (m)	4.9867 10^{-1}
Method : **NEWMARK**	
Node x_2, displacement (m)	4.9867 10^{-1}
Method: ADAPT_ORDRE2	
Node x_2, displacement (m)	4.9867 10^{-1}

Note: the tests of modeling C are duplicated because one wants to test modal calculation on the matrices projected with automatic passage on solvor MUMPS whereas the user asked for the solvor MULT_FRONT, which is not available in this case. The results are identical.

6 Modeling D

6.1 Characteristics of modeling

The characteristics of modeling D are exactly identical to that of modeling A. On the other hand, one selected the method " ELIMINATE " to manage the boundary conditions and the assembly of under structures.

6.2 Sizes tested and results

Identification	Reference
Method: EULER	
Node x_3 , displacement (m)	4.17 10-1
Method: EULER	
Node x_3 , speed ($m.s^{-1}$)	-4.3011 10^{-1}
Method: EULER	
Node x_3 , Acceleration ($m.s^{-2}$)	3.3749 10^{-1}

7 Modeling E

7.1 Characteristics of modeling

The characteristics of modeling E are exactly identical to that of modeling A. On the other hand, one uses the orders CREA_ELEM_SSD and ASSE_ELEM_SSD for the creation and the assembly of the dynamic macronutrients. One also presents how one creates the initial conditions for a transitory calculation on modal basis by dynamic under-structuring. The calculation of the direct answer (without under-structuring) is not included in this modeling.

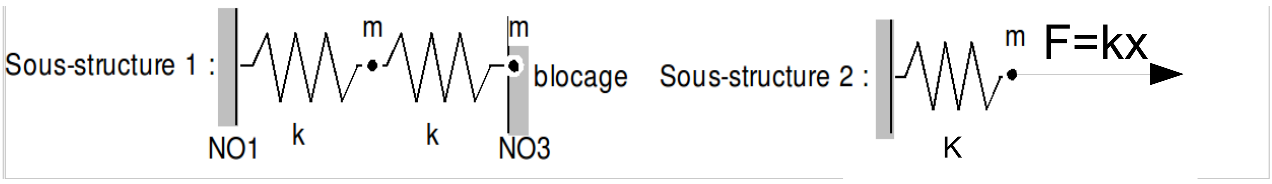
7.2 Sizes tested and results

Identification	Reference
Method: EULER	
Node x_3 , displacement (m)	4.17 10-1
Method: EULER	
Node x_3 , speed ($m.s^{-1}$)	-4.3011 10^{-1}
Method: EULER	
Node x_3 , Acceleration ($m.s^{-2}$)	3.3749 10^{-1}

8 Modeling F

8.1 Characteristics of modeling

The characteristics of modeling F are identical to that of modeling A except for the substructure 2 where one replaced the second discrete element by the relation $F=-kx$. This modeling makes it possible to test the use of the option RELA_EFFO_DEPL in a calculation of type dynamic under-structuring.



The transitory answer of the system is calculated by using interfaces of the type 'Craig-Bampton' (blocked interfaces). One applies the method of integration by centered differences. The calculation of the direct answer (without under-structuring) is not included in this modeling.

8.2 Sizes tested and results

Identification	Reference
Method: DIFF_CENTRE	
Node x_3 , displacement (m)	4.17 10-1
Method: DIFF_CENTRE	
Node x_3 , speed ($m.s^{-1}$)	-4.3011 10^{-1}
Method: DIFF_CENTRE	
Node x_3 , Acceleration ($m.s^{-2}$)	3.3749 10^{-1}

9 Summary of the results

Precision on displacement, the speed and the acceleration of the node x_2 at the moment $t=80\ s$ is excellent (relative error $< 1\%$).

This test thus validates the operators of calculation of transitory answer linear on modal basis calculated by dynamic under-structuring (with and without damping), as well as the diagram of integration to step of adaptive time of order 2 of the operator of transitory calculation on modal basis.