SDLD21 - System mass-arises with 8 degrees of freedom with viscous shock absorber

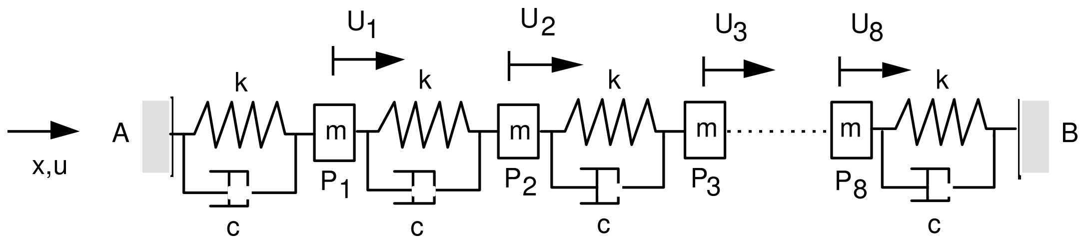
Summary:

This one-way problem consists in carrying out a harmonic analysis of a mechanical structure made up of a set of mass-springs with viscous shock absorbers and subjected to a sinewave excitation. This test of mechanics of the structures corresponds to a dynamic analysis of a discrete model having a linear behavior.

The got results (field of displacement, speed and acceleration for various frequencies of excitation) are in concord with the results of guide VPCS.

1 Problem of reference

1.1 Geometry



Specific masses: $m_{P_1}=m_{P_2}=m_{P_3}=\ldots\ldots=m_{P_8}=m$

Stiffnesses of connection: $k_{AP1}=k_{P1P2}=k_{P2P3}=\ldots\ldots=k_{P8B}=k$

Viscous damping: $c_{AP1}=c_{P1P2}=c_{P2P3}=\ldots\ldots=c_{P8B}=c$

1.2 Material properties

Spring of elastic translation linear	$k=10^5 N/m$
Specific mass	$m=10\,Kg$
One-way viscous damping	$c=50\,N/(m/s)$

1.3 Boundary conditions and loadings

Boundary conditions:
Points A and B : embedded $(u=0)$.

Loading: Sinusoidal concentrated force of variable frequency at the point P_4

Not P_4	$F_{x_4}=F_0 \sin\Omega\, t$	$\Omega=2\pi\, f$ $5\,Hz\le f\le 40\,Hz$ $F_0=constante=1\text{N}$
Other points P_i	$f_{x_i}=0$	

1.4 Initial conditions

Without object for the study of the permanent harmonic mode.

2 Reference solution

2.1 Method of calculating used for the reference solution

The system of differential equations of the second order coupled is form:

$$M\ddot{u}+C\dot{u}+Ku=F$$

with $M=\begin{bmatrix} 10 & & & \\ & 10 & & \\ & & . & \\ & & & 10 \end{bmatrix}$ $C=50\begin{bmatrix} 2 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & . & & \\ & & . & . & . & \\ & & & . & . & -1 \\ & & & & -1 & 2 \end{bmatrix}$

$$K=10^{+5}\begin{bmatrix} 2 & -1 & & & & \\ -1 & 2 & -1 & & & \\ & -1 & 2 & . & & \\ & & . & . & . & \\ & & & . & . & -1 \\ & & & & -1 & 2 \end{bmatrix}$$

The solution ω with a harmonic excitation $\boldsymbol{F}=\boldsymbol{F_0}e^{j\omega t}(j^2=-1)$ is form $u=u_0 e^{j\omega t}$, which leads to: $(\boldsymbol{K}-\boldsymbol{M}\omega^2+j\omega\boldsymbol{C})u_0=\boldsymbol{F_0}$

This system can be solved for all ω, either directly, or by using the modal transformation starting from the real clean modes obtained by the associated conservative system $(\boldsymbol{K}-\boldsymbol{M}\omega^2)\phi=0$.

He admits n clean solutions (8 in this case) ω_i^2 and associated vectors ϕ_i gathered in the spectral matrix $\Lambda=[\omega_i^2]$ and the modal matrix $\Phi=[\phi_i]$.

The modal transformation consists in writing: $\mathbf{u}_0=\Phi\mathbf{q}$ what leads to:

$$[\Lambda-\omega^2\boldsymbol{I}+j\omega\xi]q={}^t\Phi F_0$$

$\boldsymbol{I}$ is the identity,

here ξ is diagonal $\xi=[\xi_{ii}]$ because damping is proportional $(\boldsymbol{C}=\alpha\boldsymbol{K})$.

The answer is written: $$u_0=\sum_{i=1}^{n}\frac{\phi_i^t\phi_i}{\omega_i^2-\omega^2+j\omega\xi_{ii}}F_0$$

One obtains the exact solution by taking all the clean modes.

One from of deduced: $\dot{u}_0=j\omega u_0$ and $\ddot{u}_0=-\omega^2 u_0$

2.2 Results of reference

Displacement according to x point P_4 for certain frequencies.

2.3 Uncertainty on the solution

Semi-analytical solution.

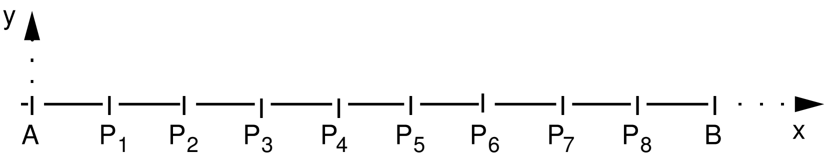
2.4 Bibliographical reference

[1] J. PIRANDA: Note of modal use of analysis software MODAN - Version 0.2 (1990). Laboratory of Mechanics Applied - University of Frank County - Besancon (France).

3 Modeling A

3.1 Characteristics of modeling

Discrete element of rigidity in translation



Characteristics of the elements

`DISCRETE :`	with nodal masses	`M_T_D_N`
	and matrices of rigidity	`K_T_D_L`
	and matrices of damping	`A_T_D_L`

Limiting conditions:

in all the nodes	`DDL_IMPO`	`(TOUT=' OUI'    DY= 0. , DZ= 0. )`
with the nodes ends		`(GROUP_NO=AB  DX= 0. )`

Names of the nodes:

$Point\ A = N1$ $P_1 = N2$

$Point\ B = N10$ $P_2 = N3$

....

$P_8 = N9$

3.2 Characteristics of the grid

Many nodes: 10
Many meshs and types: 9 SEG2

3.3 Sizes tested and results

Parts real and imaginary of the component DX displacement of the point P_4.

Frequency	Reference
5.00	1.0237 E-4 – 8.5187 E-6
5.50	4.5066 E-4 – 7.7914 E-4
6.00	– 9.4101 E-5 – 1.0585 E-5
10.00	8.4143 E-7 – 1.0335 E-6
15.00	1.2656 E-5 – 5.6652 E-6
20.00	2.9784 E-6 – 6.6970 E-6
25.00	– 1.2536 E-6 – 5.2703 E-6
30.00	– 2.0904 E-6 – 5.4821 E-6

35.00	– 4.5447 E-6 – 1.1190 E-6
39.50	– 2.6895 E-6 – 3.0505 E-7

Parts real and imaginary of the component DX speed of the point P_4.

Frequency	**Reference**
5.00	2.6762 E-4 3.2160 E-3
5.50	2.6925 E-2 1.5574 E-2
6.00	3.9904 E-4 – 3.5475 E-3
10.00	6.4937 E-5 5.2869 E-5
15.00	5.3393 E-4 1.1928 E-3
20.00	8.4157 E-4 3.7428 E-4
25.00	8.2786 E-4 – 1.9691 E-4
30.00	1.0333 E-3 – 3.9403 E-4
35.00	2.4608 E-4 – 9.9943 E-4
39.50	7.5709 E-5 – 6.6749 E-4

Parts real and imaginary of the component DX acceleration of the point P_4.

Frequency	**Reference**
5.00	– 1.0103 E-1 8.4076 E-3
5.50	– 5.3819 E-1 9.3047 E-1
6.00	1.3374 E-1 1.5044 E-2
10.00	– 3.3218 E-3 4.0801 E-3
15.00	– 1.1242 E-1 5.0322 E-2
20.00	– 4.7033 E-2 1.0575 E-1
25.00	3.0931 E-2 1.3004 E-1
30.00	7.4273 E-2 1.9478 E-1
35.00	2.1979 E-1 5.4116 E-2
39.50	1.6566 E-1 1.8789 E-2

3.4 Remarks

Contents of the file results:

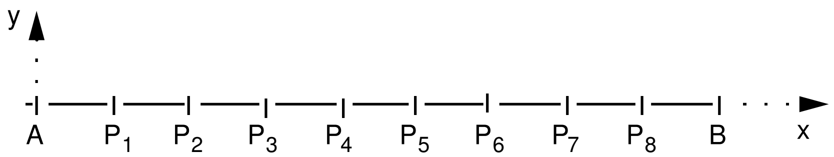
Values of the displacement of the component DX point P_4 for all the frequencies of 5 with $40\,Hz$ by step of 0.5 (Case initial test of VPCS).

Values the speed and the acceleration of the component DX point P_4 for some frequencies of vibration.

4 Modeling B

4.1 Characteristics of modeling

Discrete element of rigidity in translation



Characteristics of the elements

`DISCRETE :`	with nodal masses	`M_T_D_N`
	and matrices of rigidity	`K_T_D_L`
	and matrices of damping	`A_T_D_L`

Limiting conditions:

in all the nodes	`DDL_IMPO`	`(TOUT=' OUI'  DY= 0. , DZ= 0. )`
with the nodes ends		`(GROUP_NO=AB  DX= 0. )`

Names of the nodes:

$Point\ A = N1$ $\quad P_1 = N2$

$Point\ B = N10$ $\quad P_2 = N3$

....

$P_8 = N9$

4.2 Characteristics of the grid

Many nodes: 10
Many meshs and types: 9 SEG2

To test `REST_SPEC_TEMP`, one will compare several approaches (on modal and physical basis) by testing the component DX at the point P_4 displacement, speed and acceleration.
Two methods thus are tested:

- calculation on modal basis, then afterwards `REST_SPEC_TEMP`, return on physical basis with `RECU_FONCTION` on `RESU_GENE`,
- calculation on physical basis directly.

Each time the option is tested `TOUT_CHAM=' OUI'` or by calculating the three fields kinematics separately with `NOM_CHAM = 'DEPL'`, `'QUICKLY'` or `'ACCE'`.

By these various ways one must find the same results because the modal base is complete (it is not problematic because one has a small number of physical degrees of freedom).
Rather than to test only into cubes urgent individuals, the comparisons (on physical basis) are done by analyzing the sum over every moment of the absolute values of the maximum (with each step) of the differences between the temporal solutions (obtained by FFT reverses with `REST_SPEC_TEMP`). This standard must each time be strictly worthless.

Being given the number of additional operations related to the tests of `REST_SPEC_TEMP`, time CPU of modeling B is clearly increased, compared to other modelings which do not comprise these tests.

4.3 Sizes tested and results

Parts real and imaginary of the component DX displacement of the point P_4.

Frequency	Reference
5.00	1.0237 E-4 – 8.5187 E-6
5.50	4.5066 E-4 – 7.7914 E-4
6.00	– 9.4101 E-5 – 1.0585 E-5
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30.00	– 2.0904 E-6 – 5.4821 E-6
35.00	– 4.5447 E-6 – 1.1190 E-6
39.50	– 2.6895 E-6 – 3.0505 E-7

For the tests on `REST_SPEC_TEMP`, all the standards on the maximum values of the differences between the solutions calculated for the fields of displacement, speed and acceleration are strictly worthless: there are thus the same results on modal or physical base and whatever the mode of use of `REST_SPEC_TEMP`.

4.4 Remarks

Contents of the file results:

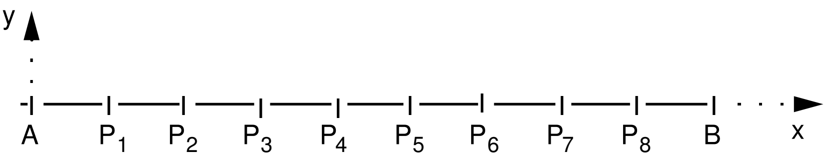
Values of the displacement of the component DX point P_4 for all the frequencies of 5 with $40\,Hz$ by step of 0.5 (Case initial test of VPCS).

Values the speed and the acceleration of the component DX point P_4 for some frequencies of vibration.

5 Modeling C

5.1 Characteristics of modeling

Discrete element of rigidity in translation



Characteristics of the elements

`DISCRETE :`	with nodal masses	`M_T_D_N`
	and matrices of rigidity	`K_T_D_L`
	and matrices of damping	`A_T_D_L`

Limiting conditions:

in all the nodes	`DDL_IMPO`	`(TOUT=' OUI'   DY= 0. , DZ= 0. )`
with the nodes ends		`(GROUP_NO=AB  DX= 0. )`

Names of the nodes:

$Point\ A = N1$ $P_1 = N2$

$Point\ B = N10$ $P_2 = N3$

....

$P_8 = N9$

5.2 Characteristics of the grid

Many nodes: 10
Many meshs and types: 9 SEG2

5.3 Sizes tested and results

Eigen frequencies of the structure for the sequence numbers from 1 to 5.

Sequence number	Reference
1	5.5271
2	10.8868
3	15.9155
4	20.4606
5	24,384

Damping reduce structure for the sequence numbers from 1 to 5.

Sequence number	Reference
1	0.00868241
2	0.017101
3	0,025
4	0.0321394
5	0.0383022

6 Modeling D

6.1 Characteristics of modeling

This modeling is identical to modeling A, the only difference is on the level of the solvor employed: one uses here MUMPS.

6.2 Characteristics of the grid

Many nodes: 10
Many meshs and types: 9 SEG2

6.3 Sizes tested and results

As for modeling A, one tests on the parts real and imaginary component DX displacement, speed and acceleration of the point P_4. The results are equal to those obtained with modeling A, until less the eleventh decimal included.

7 Summary of the results

The got results are excellent, which is normal for a direct integration.