ZZZZ294 – Validation of the position of the under-points of the plates 3D

Summary:
This test validates the calculation of the position of the under-points of integration in the total reference mark for modelings DKT, DST, COQUE_3D and GRILLE_EXCENTRE. An elementary mechanical calculation is carried out in order to allow the creation of a table with CREA_TABLE starting from the result. Only the coordinates of some under-points are tested in the table.

1 Problem of reference

1.1 Geometry

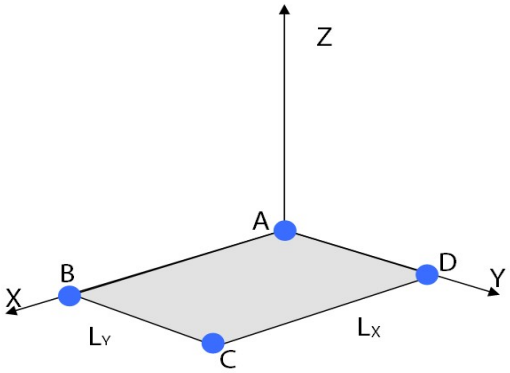


Plate 3D:

Length X : $L_X = 2.0\,m$

Length Y : $L_Y = 1.0\,m$

Thickness: $e = 0.5\,m$

Local coordinates of the points A, B, C and D

$X_A = 0.0 ; Y_A = 0.0 ; Z_A = 0.0$

$X_B = 2.0 ; Y_B = 0.0 ; Z_B = 0.0$

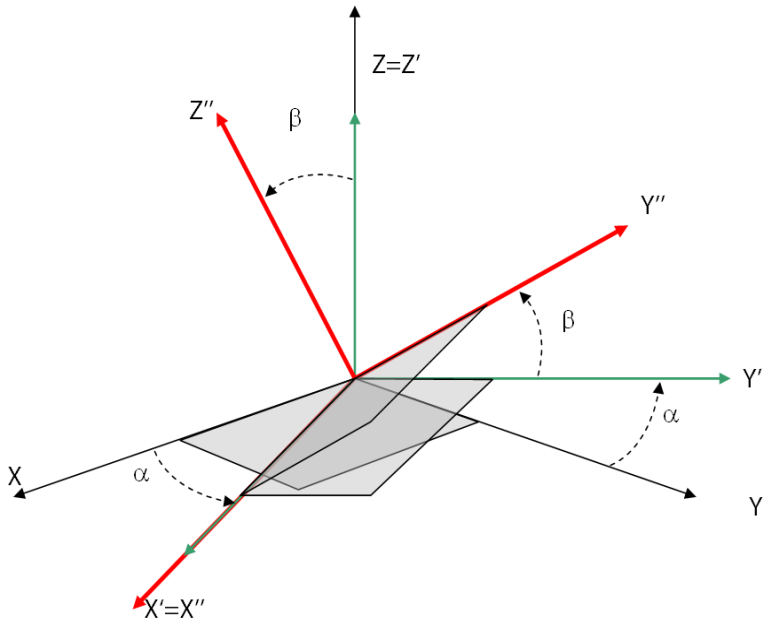
$X_C = 2.0 ; Y_C = 1.0 ; Z_C = 0.0$

$X_D = 0.0 ; Y_B = 1.0 ; Z_B = 0.0$

The orientation of the plate in the total reference mark is obtained by two rotations:

$\alpha = 30°$ around Z

$\beta = 60°$ around the new axis X'



$X ; Y ; Z$: total axes

$X'' ; Y'' ; Z''$: local axes in final position

1.2 Properties of materials

Concrete:

Young modulus: $E=3.7272^{10}\,Pa$

Poisson's ratio: $\nu=0.0$

1.3 Boundary conditions and loadings

On the points A and B one blocks displacements according to X, Y, Z and rotations around the axes X, Y, Z :

$$D_X^A=0.0\,;\ D_Y^A=0.0\,;\ D_Z^A=0.0\,;\ DR_X^A=0.0\,;\ DR_Y^A=0.0\,;\ DR_Z^A=0.0$$

$$D_X^B=0.0\,;\ D_Y^B=0.0\,;\ D_Z^B=0.0\,;\ DR_X^B=0.0\,;\ DR_Y^B=0.0\,;\ DR_Z^B=0.0$$

On the points C and D one applies a loading according to Z :

$$F_Z=-100.0\,N$$

2 Reference solution

2.1 Method of calculating

One calculates the position of the nodes, points of integration and under-points of integration from their cordonnées in the local axes in final position of the plate, and the matrices of passage between the local axes and the total axes

Rotation around the axis Z is made starting from the following matrix:

$$Tz(\alpha)=\begin{bmatrix}\cos(\alpha) & -\sin(\alpha) & 0\\ \sin(\alpha) & \cos(\alpha) & 0\\ 0 & 0 & 1\end{bmatrix}$$

Rotation around the axis X' is made starting from the following matrix:

$$Tx'(\beta)=\begin{bmatrix}1 & 0 & 0\\ 0 & \cos(\beta) & -\sin(\beta)\\ 0 & \sin(\beta) & \cos(\beta)\end{bmatrix}$$

For any point of initial coordinates (X,Y,Z) one can calculate his coordinates expressed in the total reference mark (X',Y',Z') after rotations with the following transformation:

$$\begin{bmatrix}X'\\ Y'\\ Z'\end{bmatrix}=[Tz(\alpha)][Tx'(\beta)]\begin{bmatrix}X\\ Y\\ Z\end{bmatrix}$$

2.2 Sizes and results of reference

One calculates the position of the under-points of integration in the total reference mark knowing his position expressed in the local axes.

Here one a: $Tx'(\beta)=\begin{bmatrix}1 & 0 & 0\\ 0 & 0.5 & -0.866\\ 0 & 0.866 & 0.5\end{bmatrix}$ and $Tz(\alpha)=\begin{bmatrix}0.866 & -0.5 & 0\\ 0.5 & 0.866 & 0\\ 0 & 0 & 1\end{bmatrix}$

For an element QUA4 of plate length $L_X=2.0\,m$ and of width $L_Y=1.0\,m$, the positions in the plan of the point of integration are, for modelings A and B (DKT and DST) who have 4 points of integration (see R3.01.01):

Not	x	y
1	0.42264973081037416	0.21132486540518708
2	1.5773502691896257	0.21132486540518708
3	1.5773502691896257	0.78867513459481287
4	0.42264973081037416	0.78867513459481287

And for modeling C (COQUE_3D) who has 9 points of intégration (see R3.01.01):

Not	x	y
1	0.22540333075851704	0.11270166537925852
2	1.774596669241483	0.11270166537925852
3	1.774596669241483	0.88729833462074148
4	0.22540333075851704	0.88729833462074148
5	1	0.11270166537925852
6	1.774596669241483	0.5
7	1	0.88729833462 074148
8	0.22540333075851704	0.5
9	1	0.5

The thickness $EP=0.5\,m$, is discretized in 4 layers, which makes 12 under-points of which heights compared to the average plan (except case modeling D GRILLE_EXCENTREE) are:

Under-point	z	Under-point	z
1	-0,250	7	0,000
2	-0.1875	8	0.0625
3	-0,125	9	0,125
4	-0,125	10	0,125
5	-0.0625	11	0.1875
6	0,000	12	0,250

In the case of modeling D (GRILLE_EXCENTRE), with a offsetting of 0.05, the position of the points and under-point in the plan of the points of integration are:

Not	Under-point	x	y	z
1	1	0.42264973081037416	0.21132486540518708	0.05
2	1	1.5773502691896257	0.21132486540518708	0.05
3	1	1.5773502691896257	0.78867513459481287	0.05
4	1	0.42264973081037416	0.78867513459481287	0.05

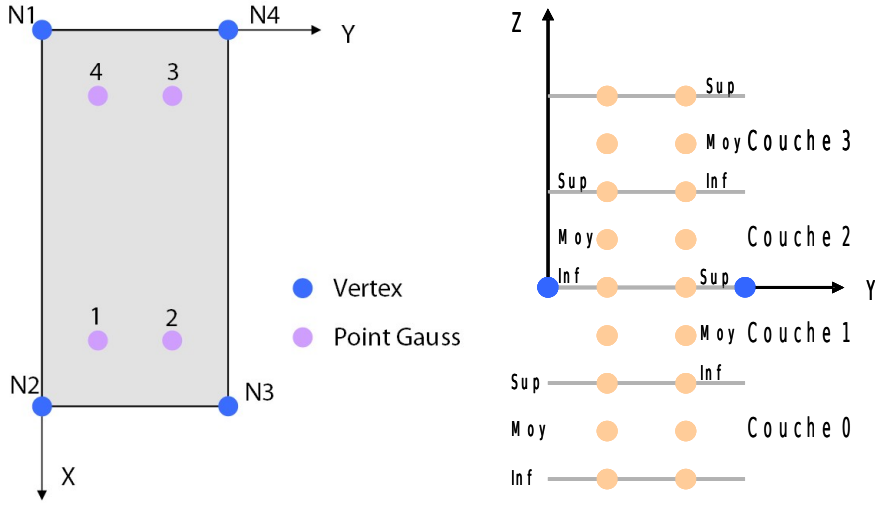
2.3 Uncertainties on the solution

No, exact solution.

3 Modeling A

3.1 Characteristics of the grid

The grid is composed of a standard mesh QUAD4 with four nodes ($N1$, $N2$, $N3$ and $N4$).



For each mesh, there are 4 points of integration

- The group of nodes ENC is composed of the nodes $N1$ and $N2$
- The group of nodes CHA is compound due nodes $N3$ and $N4$

3.2 Characteristics of modeling

Modeling:

```
MO=AFFE_MODELE (MAILLAGE=MA,
    AFFE=_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
       MODELISATION=' DKT',),
)
```

Boundary conditions:

```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
    DDL_IMPO=_F (GROUP_NO=' ENC',
       DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

Mechanical loading:

```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
    FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 0, FY = 0, FZ = -100.),
)
```

Assignment of the characteristics of the elements:

```
PLAQUE=AFFE_CARA_ELEM (MODELE=MO,
    COQUE=_F (GROUP_MA= ('PLA'), THICK = 0.5, COQUE_NCOU = 4,),
)
```

3.3 Values tested and results

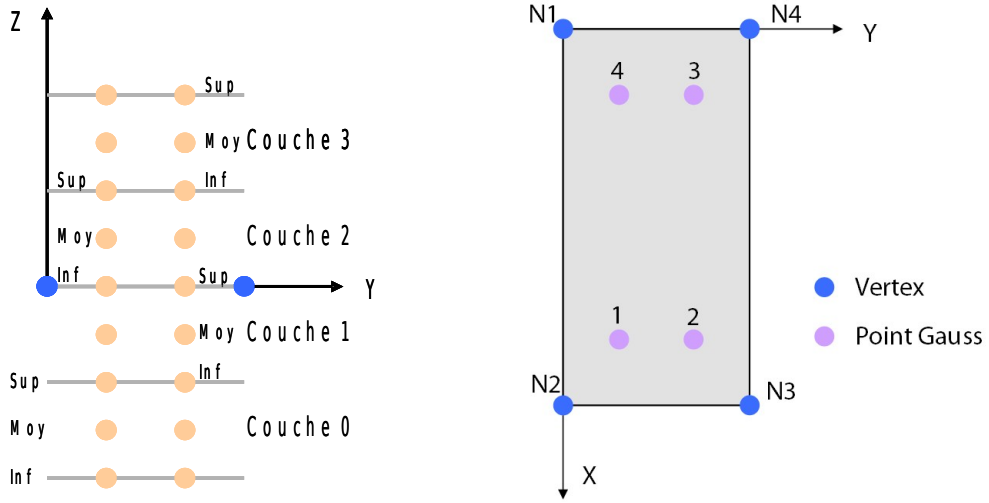
Coordinate mesh QUA1	Not of integration	under-point	Reference
COOR_X	1	1	0.204941012
COOR_X	1	2	0.232004306
COOR_X	1	3	0.2590676
COOR_X	1	4	0.2590676
COOR_X	1	5	0.286130894
COOR_X	1	6	0.313194187
COOR_X	1	7	0.313194187
COOR_X	1	8	0.340257481
COOR_X	1	9	0.367320775
COOR_X	1	10	0.367320775
COOR_X	1	11	0.394384069
COOR_X	1	12	0.421447363
COOR_Y	1	1	0.490331216
COOR_Y	1	2	0.443456216
COOR_Y	1	3	0.396581216
COOR_Y	1	4	0.396581216
COOR_Y	1	5	0.349706216
COOR_Y	1	6	0.302831216
COOR_Y	1	7	0.302831216
COOR_Y	1	8	0.255956216
COOR_Y	1	9	0.209081216
COOR_Y	1	10	0.209081216
COOR_Y	1	11	0.162206216
COOR_X	1	12	0.115331216
COOR_Z	1	1	0.058012702
COOR_Z	1	2	0.089262702
COOR_Z	1	3	0.120512702
COOR_Z	1	4	0.120512702
COOR_Z	1	5	0.151762702
COOR_z	1	6	0.183012702
COOR_Z	1	7	0.183012702
COOR_Z	1	8	0.214262702
COOR_z	1	9	0.245512702
COOR_Z	1	10	0.245512702
COOR_Z	1	11	0.276762702
COOR_Z	1	12	0.308012702

The tolerance is of $1.0\mathrm{E}-03$ for all the tests.

4 Modeling B

4.1 Characteristics of the grid

Grid: the grid is made up of a standard mesh QUAD4 aveC four nodes ($N1$, $N2$, $N3$ and $N4$).



For each mesh, there are 4 points of integration:

- The group of nodes ENC is composed of the nodes $N1$ and $N2$
- The group of nodes CHA is compound due nodes $N3$ and $N4$

4.2 Characteristics of modeling

Modeling:

```
MO=AFFE_MODELE (MAILLAGE=MA,
    AFFE=_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
        MODELISATION=' DST',),)
```

Boundary conditions:

```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
    DDL_IMPO=_F (GROUP_NO=' ENC',
        DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

Mechanical loading:

```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
    FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 0, FY = 0, FZ = -100.),
)
```

Assignment of the characteristics of the elements:

```
PLAQUE=AFFE_CARA_ELEM (MODELE=MO,
    COQUE=_F (GROUP_MA= ('PLA'), EPAIS= 0.5, COQUE_NCOU = 4,),
)
```

4.3 Values tested and results

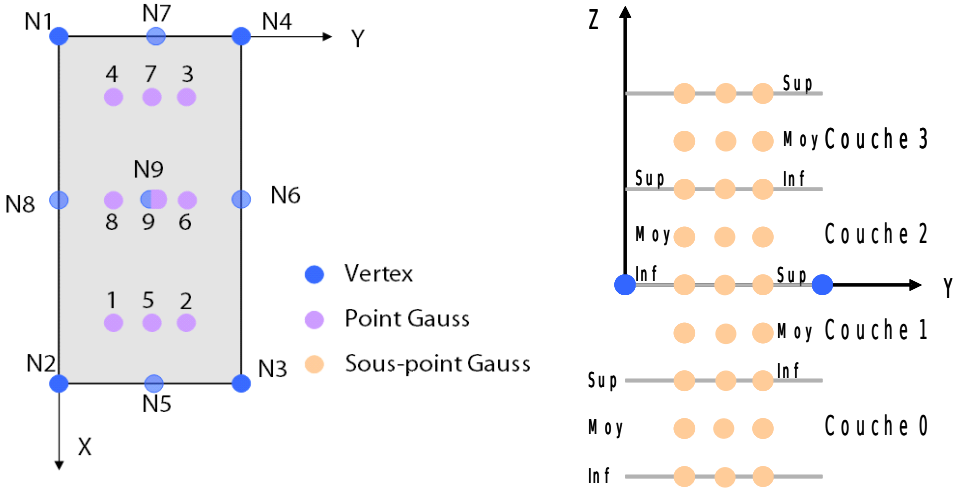
Coordinate mesh QUA1	Not of integration	under-point	Reference
COOR_X	1	1	0.204941012
COOR_X	1	2	0.232004306
COOR_X	1	3	0.2590676
COOR_X	1	4	0.2590676
COOR_X	1	5	0.286130894
COOR_X	1	6	0.313194187
COOR_X	1	7	0.313194187
COOR_X	1	8	0.340257481
COOR_X	1	9	0.367320775
COOR_X	1	10	0.367320775
COOR_X	1	11	0.394384069
COOR_X	1	12	0.421447363
COOR_Y	1	1	0.490331216
COOR_Y	1	2	0.443456216
COOR_Y	1	3	0.396581216
COOR_Y	1	4	0.396581216
COOR_Y	1	5	0.349706216
COOR_Y	1	6	0.302831216
COOR_Y	1	7	0.302831216
COOR_Y	1	8	0.255956216
COOR_Y	1	9	0.209081216
COOR_Y	1	10	0.209081216
COOR_Y	1	11	0.162206216
COOR_X	1	12	0.115331216
COOR_Z	1	1	0.058012702
COOR_Z	1	2	0.089262702
COOR_Z	1	3	0.120512702
COOR_Z	1	4	0.120512702
COOR_Z	1	5	0.151762702
COOR_z	1	6	0.183012702
COOR_Z	1	7	0.183012702
COOR_Z	1	8	0.214262702
COOR_z	1	9	0.245512702
COOR_Z	1	10	0.245512702
COOR_Z	1	11	0.276762702
COOR_Z	1	12	0.308012702

The tolerance is of $1.0\mathrm{E}-03$ for all the tests.

5 Modeling C

5.1 Characteristics of the grid

Grid: the grid is composed of a standard mesh QUAD9 withuf nodes ($N1$, $N2$, $N3$, $N4$, $N5$, $N6$, $N7$, $N8$, $N9$).



For each mesh, there are 9 points of integration.

- The group of nodes ENC is composed of the nodes $N1$ and $N2$
- The group of nodes CHA is composed of the nodes $N3$ and $N4$

5.2 Characteristics of modeling

Modeling:

```
MO=AFFE_MODELE (MAILLAGE=MA,
    AFFE=_F (TOUT=' OUI', PHENOMENE=' MECANIQUE',
       MODELISATION=' COQUE_3D',),)
```

Boundary conditions:

```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
    DDL_IMPO=_F (GROUP_NO=' ENC',
       DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

Mechanical loading:

```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
    FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 0, FY = 0, FZ = -100.),
)
```

Assignment of the characteristics of the elements:

```
PLAQUE=AFFE_CARA_ELEM (MODELE=MO,
    COQUE=_F (GROUP_MA= ('PLA'), THICK = 0.5, COQUE_NCOU = 4,),
)
```

5.3 Values tested and results

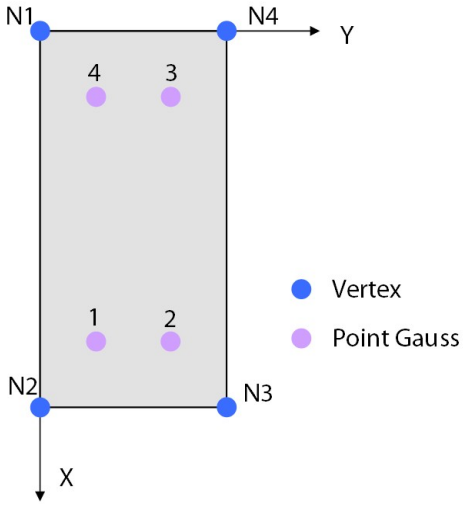
Coordinate mesh QUA1	Not of integration	under-point	Reference
COOR_X	1	1	0.058776419
COOR_X	1	2	0.085839713
COOR_X	1	3	0.112903006
COOR_X	1	4	0.112903006
COOR_X	1	5	0.139966300
COOR_X	1	6	0.167029594
COOR_X	1	7	0.167029594
COOR_X	1	8	0.194092888
COOR_X	1	9	0.221156182
COOR_X	1	10	0.221156182
COOR_X	1	11	0.248219476
COOR_X	1	12	0.275282770
COOR_Y	1	1	0.349002918
COOR_Y	1	2	0.302127918
COOR_Y	1	3	0.2552529180
COOR_Y	1	4	0.2552529180
COOR_Y	1	5	0.2083779180
COOR_Y	1	6	0.1615029180
COOR_Y	1	7	0.1615029180
COOR_Y	1	8	0.1146279180
COOR_Y	1	9	0.0677529180
COOR_Y	1	10	0.0677529180
COOR_Y	1	11	0.0208779180
COOR_Y	1	12	-0.025997082
COOR_Z	1	1	-0.027397495
COOR_Z	1	2	3.852505E-03
COOR_Z	1	3	0.0351025050
COOR_Z	1	4	0.0351025050
COOR_Z	1	5	0.0663525050
COOR_z	1	6	0.0976025050
COOR_Z	1	7	0.0976025050
COOR_Z	1	8	0.1288525050
COOR_z	1	9	0.1601025050
COOR_Z	1	10	0.1601025050
COOR_Z	1	11	0.1913525050
COOR_Z	1	12	0.2226025050
COOR_Z	9	1	0.3080127020
COOR_Z	9	2	0.3392627020
COOR_Z	9	3	0.3705127020
COOR_Z	9	4	0.3705127020
COOR_Z	9	5	0.4017627020
COOR_z	9	6	0.4330127020
COOR_Z	9	7	0.4330127020
COOR_Z	9	8	0.4642627020
COOR_z	9	9	0.4955127020
COOR_Z	9	10	0.4955127020
COOR_Z	9	11	0.5267627020
COOR_Z	9	12	0.5580127020

The tolerance is of $1.0\mathrm{E}-03$ for all the tests.

6 Modeling D

6.1 Characteristics of the grid

Grid: the grid is composed of two superimposed meshs of type QUAD4 with four nodes ($N1$, $N2$, $N3$, $N4$). Dbe meshs is intended to model an element DKT and the other is intended to model an element GRILLE_EXCENTRE.



The mesh of offset grid has four points of integration and only one under-point.

- The group of nodes ENC is composed of the nodes $N1$ and $N2$
- The group of nodes CHA is compound due nodes $N3$ and $N4$

6.2 Characteristics of modeling

Modeling:

```
MO=AFFE_MODELE (MAILLAGE=MA,
    AFFE= (
        _F (GROUP_MA=' PLA', PHENOMENE=' MECANIQUE', MODELISATION=' DKT'),
        _F (GROUP_MA=' GRI', PHENOMENE=' MECANIQUE',
            MODELISATION=' GRILLE_EXCENTRE',))
)
```

Boundary conditions:

```
BLOCAGE=AFFE_CHAR_MECA (MODELE=MO,
    DDL_IMPO=_F (GROUP_NO=' ENC',
    DX=0.0, DY=0.0, DZ=0.0, DRX=0.0, DRY=0.0, DRZ=0.0,),
)
```

Mechanical loading:

```
CHARGE=AFFE_CHAR_MECA (MODELE=MO,
    FORCE_NODALE=_F (GROUP_NO=' CHA', FX = 0, FY = 0, FZ = -100.),
)
```

Assignment of the characteristics of the elements:

```
PLAQUE=AFFE_CARA_ELEM (MODELE=MO,
    COQUE=_F (GROUP_MA= ('PLA'), EPAIS=0.05,),
    GRILLE=_F (GROUP_MA= ('GRI'), SECTION = 0.01, OFFSETTING = 0.05,
        ANGL_REP= (10.10),),
)
```

6.3 Values tested and results

Coordinate mesh QUA2	Not of integration	under-point	Reference
COOR_X	1	1	0.334844823
COOR_Y	1	1	0.265331216
COOR_Z	1	1	0.208012702

The tolerance is of $1.0E-03$ for all the tests.

7 Summary of the results

The purpose of this test is principal to check if the positions of under points of integration of modeling DKT, DST, COQUE_3D and GRILLE_EXCENTRE are well calculated.

For this modeling, the maximum error found is of $3.0\mathrm{E}-07\%$.