Reduction of model per dynamic condensation and static under-structuring

Summary:

This document describes a mode of use of *Code_Aster* to allow transitory calculation by the operator of transitory dynamics nonlinear DYNA_NON_LINE on a scale model strictly with nonlinear under-fields, linear under-fields being taken into account once and for all by dynamic condensation by means of static macronutrients on the interfaces.

The method of condensation is exerted on all the degrees of freedom of interface commun runs to the condensed under-fields and the under-fields calculated by nonlinear analysis in finite elements. A complete general method requires the calculation of all the constrained static modes of interface, associated each one with an answer to a unit displacement imposed for each degree of freedom of interface. An alternative method makes it possible to use a reduced base of modes of interface of lower number but representative of displacements of all the degrees of freedom of interface, in the frequency band discounted.

1 Introduction

The objective of the dynamic condensation of model per static under-structuring is to reduce the full number of degrees of freedom of a model made up of several under-fields by condensing the resolution of the under-fields of linear behavior on the degrees of freedom of their interfaces with under-fields of nonlinear behavior, treated in finite elements.
The idea is then to represent each one of these under-fields by macronutrient of a static type to be able to use it in a "mixed" scale model also including finite elements for the parts non-condensed of nonlinear behavior in order to carry out nonlinear analyses by means of the nonlinear operators of calculation `STAT_NON_LINE` [U4.51.03] or `DYNA_NON_LINE` [U4.53.01].

In the condensation of each linear under-field, one wants to also integrate the contribution of inertial forces, therefore the dynamic part of its behavior obtained by analysE modal, and for this reason, one will thus use dynamic macronutrients like the static macronutrients necessary in static under-structuring. This approach is known in the literature under the term: "System Are equivalent Reduction-Expansion Process". Into linear, it preserves the own pulsations of the initial system in the discounted frequency band.

The principle of this condensation and its application is described in the documents [bib1, bib2, bib5].

2 Classical problem of condensation and principle of the method

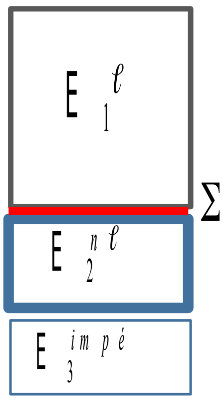

Figure 2.1: Under-fields of the classical problem of condensation.

On figure 2.1 above are schematized the possible under-fields which one can meet within the framework of a classical problem of condensation per static under-structuring:

- under-fields of the type E_1^l : under-fields of structure of constant linear behavior condensed and modelled by dynamic macronutrient,
- under-fields of the type E_2^{nl} : under-fields of structure of behavior potentially nonlinear non-condensed and modelled by finite elements,
- under-fields of the type $E_3^{impé}$: under-field of ground represented by a macronutrient resulting from the conversion of a matrix complexes impedance of ground.

The basic principle of condensation by macronutrient consists with partitionner for each under-field of the type E_1^l degrees of freedom j their noted interfaces Σ with the under-fields of the type E_2^{nl} like their other degrees of freedom i said "internal".

Matrices of rigidity $[\mathbf{K}]$ and of mass $[\mathbf{M}]$ break up thus on these degrees of freedom:

$$[\mathbf{K}]=\begin{bmatrix} K_{ii} & K_{ij} \\ K_{ji} & K_{jj} \end{bmatrix} \qquad [\mathbf{M}]=\begin{bmatrix} M_{ii} & M_{ij} \\ M_{ji} & M_{jj} \end{bmatrix} \qquad \begin{matrix} i & \text{ddl internes} \\ j & \text{ddl d'interfaces} \end{matrix} \tag{2.1}$$

The static principle of under-structuring returns for the matrix $[\mathbf{K}]$ for example to condense it only on the degrees of freedom j interfaces Σ by correcting the initial term K_{jj} of a complementary term called “complement of Schur” according to the principle of the condensation known as of Guyan [bib3]:

$$\bar{K}_{jj}=K_{jj}-K_{ji}\left(K_{ii}\right)^{-1}K_{ij} \;=\; \mathbf{\Psi}^T.[\mathbf{K}].\mathbf{\Psi} \tag{2.2}$$

The calculation of the new term $\bar{K}_{jj}$ cost with a projection of the matrix $[\mathbf{K}]$ on a basis of static modes $\mathbf{\Psi}$ of “constrained” type obtained by static answer to the unit displacements imposed in each degree of freedom j interfaces Σ :

$$\mathbf{\Psi}=\begin{bmatrix} [\Psi_{ij}] \\ \mathbf{Id} \end{bmatrix}=\begin{bmatrix} [-K_{ii}^{-1}K_{ij}] \\ \mathbf{Id} \end{bmatrix} \tag{2.3}$$

Condensation on the degrees of freedom j interfaces Σ matrix of mass $[\mathbf{M}]$ and of the loads of the under-fields of the type E_1^l will be done in a similar way by the matric product of the matrix and the vectors of loading on the basis of static modes $\mathbf{\Psi}$.

It is noted whereas the terms $\mathbf{\Psi}^T.[\mathbf{K}].\mathbf{\Psi}$ and $\mathbf{\Psi}^T.[\mathbf{M}].\mathbf{\Psi}$ can be directly obtained for each under-field of the type E_1^l by means of the operator of calculation of a dynamic macronutrient `MACR_ELEM_DYNA` [U4.65.01] definite on the interface Σ and that terms of loads, applied to the under-field of the type E_1^l , are obtained by projection $\mathbf{\Psi}^T.\mathbf{F}$ by means of the operator `PROJ_VECT_BASE` [U4.63.13].

3 Methods of dynamic condensation per complete or reduced base of movements on the interfaces

One extends the preceding approach by including the condensation of the dynamic behavior of the under-fields of the type E_1^l using a reduced modal representation of clean modes blocked on the interface Σ , selected to be representative of dynamics in a certain frequency band.
According to the way of calculating of the modes of representation of the movements of the interfaces, two methods of condensation are proposed.

3.1 The method of calculating by base supplements movements with the interface

The movement $\mathbf{U}$ of an unspecified point of a under-field of the type E_1^l express yourself starting from the decomposition on the static modes $\mathbf{\Psi}$ and on a complement brought by the dynamic part of the movement expressed by clean modes $\mathbf{\Phi}$, blocked on the interface Σ :

$$\mathbf{U}=\sum \Psi\, q_{\Sigma}+\Phi\, q \tag{3.1.1}$$

With the method by complete base, on the interface Σ, clean modes $\mathbf{\Phi}$ have a worthless contribution and the static modes $\mathbf{\Psi}$ have an unit value and thus displacement $\mathbf{U}_\Sigma$ of a point of the interface expresses itself:

$$\mathbf{U}_\Sigma=\mathbf{X}_\Sigma=q_\Sigma \quad (3.1.2)$$

In this case, physical degrees of freedom $\mathbf{X}_\Sigma$ interface Σ merge with the coordinates generalized q_Σ associated with the static modes $\mathbf{\Psi}$.

With regard to the generalized coordinates q associated with the clean modes $\mathbf{\Phi}$ under-fields of the type E_1^l who are worthless on the interface, their resolution will be uncoupled from that of the generalized coordinates q_Σ. In the assembly of the matrices of the problem condensed, the terms of projection on the clean modes $\mathbf{\Phi}^T.[\mathbf{K}].\mathbf{\Phi}$ and $\mathbf{\Phi}^T.[\mathbf{M}].\mathbf{\Phi}$ will be juxtaposed. To be able “to mix” physical degrees of freedom with generalized coordinates, one uses the artifice transparency for the user to make pass these last for additional physical degrees of freedom taken among the internal degrees of freedom to a under-field of the type E_1^l.
This alias is active in the model where fields of the type E_1^l are condensed by macronutrients, but to have the true displacement of the internal degrees of freedom to these fields, it is obtained in postprocessing after resolution of the total problem assembled by means of the operator `REST_COND_TRAN` [U4.63.33] by applying the initial relation:

$$\mathbf{U}=\sum \Psi q_\Sigma+\Phi q \quad (3.1.3)$$

It is necessary to retain a number of clean modes $\mathbf{\Phi}$ exact multiple amongst components maximum of the condensed field (3 for the finite elements mechanical solid masses, 6 for the plates and hulls,…) in order to make correspond exactly generalized coordinates q with components of Nœuds additional of interface.

On the level of the assembly of the ddls of a under-field of the type E_1^l, the initial storage represented on the figure 3.1-a understands terms at the same time on internal degrees of freedom and the degrees of freedom of interface with a profile of noncomplete filling.

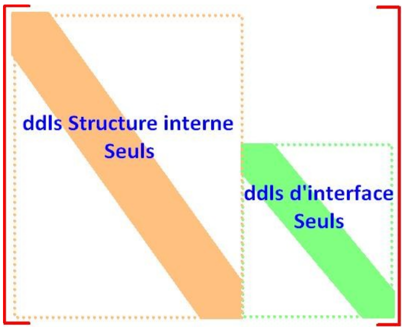

Figure 3.1-a: Initial assembly of the ddls of the E1 under-fields before condensation.

After condensation by the complete method, most internal degrees of freedom disappear, but an any small portion is devoted to represent them generalized coordinates associated with the clean modes and the terms of the static macronutrient are added to the degrees of freedom of interface with a profile of complete filling this time. Resulting storage is represented on the figure 3.1-b below.

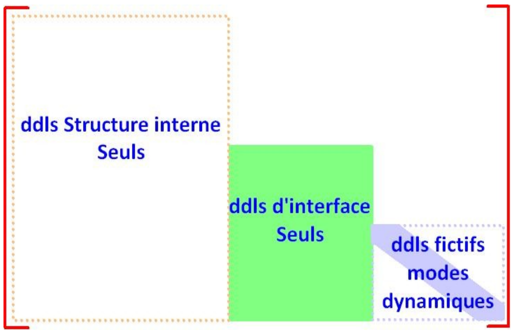

Figure 3.1-b: Assembly of the ddls of the under-fields E_1^l with complete method.

The assembly of the terms of the static macronutrient is carried out by simple summation on the degrees of freedom of interface and in general does not require an additional connection on the interface when one takes into account all the constrained static modes of the interface.
It is not thus in the typical case where one decides to make a condensation on only one node of interface where, there, it is necessary to add a solid relation of connection on the degrees of freedom of interface.

In the case of the presence of a under-field of the type $E_3^{impé}$, it postprocessing by `REST_COND_TRAN` is useless because displacement does not have a component on dynamic modes and displacement with the interface is definitively obtained as of the resolution of the total problem.

3.2 Method of calculating by modal reduction with the interface

With the method by modal reduction, the movement $\mathbf{U}$ of an unspecified point of a under-field of the type E_1^l always express yourself starting from the decomposition on the static modes $\mathbf{\Psi}$ and on a complement brought by the dynamic part of the movement expressed by clean modes $\mathbf{\Phi}$:

$$\mathbf{U}=\sum \Psi\, q_{\Sigma}+\Phi\, q \tag{3.2.1}$$

But this time, on the interface Σ , clean modes $\mathbf{\Phi}$ have a nonworthless contribution because the static modes $\mathbf{\Psi}$ chosen do not have an unit value on each node and are of number generally much lower than the number of physical degrees of freedom $\mathbf{X}_{\Sigma}$ interface Σ who, in this case, do not merge with the coordinates generalized q_{Σ} associated with the static modes $\mathbf{\Psi}$. It is thus necessary to distinguish in the assembly and the resolution the terms associated with the degrees of freedom physical and those associated with the generalized coordinates. In the same way that for the coordinates generalized q associated with the clean modes Φ , one uses the artifice transparency for the user to make pass the generalized coordinates q_{Σ} associated with the static modes $\mathbf{\Psi}$ for additional physical degrees of freedom also taken among internal degrees of freedom to a under-field of the type E_1^l .
But this time, the resolution of the physical degrees of freedom $\mathbf{X}_{\Sigma}$ interface Σ will not be uncoupled from that of the generalized coordinates q_{Σ} . In the assembly of the matrices of the problem condensed, the terms of projection on the clean modes $\mathbf{\Phi}^T.[\mathbf{K}].\mathbf{\Phi}$ and $\mathbf{\Phi}^T.[\mathbf{M}].\mathbf{\Phi}$ will be juxtaposed. But it will then be necessary to add a connection and thus degrees of freedom of Lagrange between the physical degrees of freedom $\mathbf{X}_{\Sigma}$ interface Σ and generalized coordinates q_{Σ} . This connection is expressed by the relation:

$$\mathbf{X}_{\Sigma}=\sum \Psi\, q_{\Sigma} \tag{3.2.2}$$

This relation was expressed by the keyword répétable `LIAISON_INTERF` of the operator `AFFE_CHAR_MECA` [U4.44.01], which makes it possible to connect each under-field of the type E_1^l with under field of the type E_2^{nl} (cf figure 2.1) by generating a typical case of `LIAISON_DDL` with the coefficients of the preceding linear relation.

Then, to have the true displacement of the internal degrees of freedom to the under-fields of the type E_1^l, one proceeds in the same way in postprocessing after resolution of the total problem "mixed" by means of the operator `REST_COND_TRAN` [U4.63.33] by applying the initial relation:

$$\mathbf{U}=\sum \Psi\, q_\Sigma+\Phi q \tag{3.2.3}$$

Notice :

There too, it is necessary to retain a number of clean modes $\mathbf{\Phi}$ and a number of static modes $\mathbf{\Psi}$ both exact multiples amongst components maximum of the condensed field (3 for the finite elements mechanical solid masses, 6 for the plates and hulls,…) in order to make correspond exactly coordinates generalized to components of additional nodes of interface .

After condensation by the method of modal reduction, most internal degrees of freedom disappear, but an any small portion is devoted to represent them generalized coordinates associated with the clean modes, another small portion is devoted to the generalized coordinates associated with the clean modes of interface. This time, the terms of the static macronutrient are not added to the degrees of freedom of interface which preserve a profile of noncomplete filling then. But one obtains a quadrangular block then of degrees of freedom of Lagrange enters the physical degrees of freedom $\mathbf{X}_\Sigma$ interface Σ and generalized coordinates q_Σ.
Resulting storage is represented on figure 3.2 below.

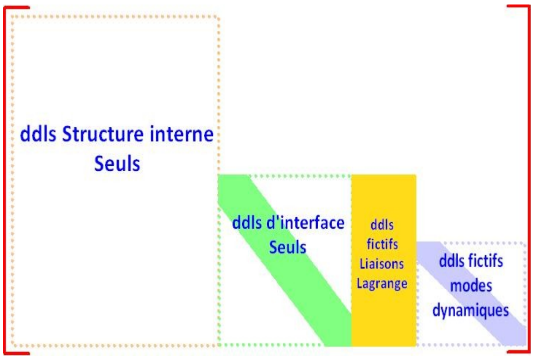

Figure 3.2: Storage of the ddls of the under-fields E_1^l with method of modal reduction.

It will be noticed that there is no choice of calculation imposed for the base of reduction of static modes of interface $\mathbf{\Psi}$. A possible choice will be that used in interaction ground-structure [bib4] by considering the restriction on the interface Σ of a base of clean modes of the under-field of the type E_1^l on carpet of springs representative of the rigidity of the adjacent under-fields of type E_2^{nl} or $E_3^{impé}$, cf [U2.06.07, § 11.5].

3.3 Validation of the use of the methods of dynamic condensation

A good approach of validation of the dynamic model of condensation – before carrying out the nonlinear analysis – consists of the comparison of the modal analysis on the first modes between the complete initial model nonsmall-scale, containing all the under-fields, and the "mixed" scale model final containing the under-fields condensed in the form of dynamic macronutrients as well as the model of under-fields of the type E_2^{nl} treaties into linear.

3.4 Respective interests of use of the methods of dynamic condensation

The method of condensation dynamic allows in general, compared to the model non-condensed, to reduce the size of the matric systems, the full number of degrees of freedom and to obtain profits in term of memory and performances for factorization and the resolution.
For the complete method, because of storage full with the terms of the modes of interface, this method present of the interest in the operations of factorization especially if the number of degrees of freedom of interface hardly exceeds $1/10$ full number of degrees of freedom before condensation.
On the other hand, this method requires the calculation of the modes of interface for all the degrees of freedom and can be crippling in memory for this calculation if the full number of degrees of freedom before condensation is higher than 100000 and if the number of modes of interface is about 10000.

The method of condensation with modal reduction is then of an interest one reduces the number of modes of interface to less $1/10$ amongst degrees of freedom of interface, that is to say less than 1000.
While knowing that in terms of performances:

- one adds equations related to the degrees of freedom of Lagrange relating to the relations between degrees of freedom and reduced modes of interface, which is likely to penalize the resolution,
- one obtains a profit on the factorization of the system because its size will decrease as on the restitution on the basis of each physical under-field in the report amongst degrees of freedom on the number of reduced modes of interface.

The complete method seems powerful and sufficient for a full number of degrees of freedom before condensation ranging between approximately 5000 and 50000. The method with modal reduction becomes interesting beyond 50000 degrees of freedom before condensation.

4 Synthesis of the calculation of the “mixed” scale model with dynamic condensation

Calculation with dynamic condensation requires to place by the following stages of sequence of the orders of *Code_Aster:*

- Initial classical instructions: definition of the complete grid in finite elements of all the structure and the groups of meshs, in particular groups of meshs and nodes of the interfaces Σ, and of the under-field of the type E_2^{nl}, definition of materials, assignment of materials to the groups of meshs.

And for each under-field of the type E_1^l who will be condensed:

- Definition of the model and assignment of the finite elements with the model, assignment of materials to the elements, assignment of the elementary characteristics,
- Calculation of the clean modes on basis blocked on the interface Σ by the order `CALC_MODES` [U4.52.02], with the boundary conditions of origin of it under-field,
- Definition of the dynamic interface of type `CRAIGB` on the group of nodes of the interface Σ by the order `DEFI_INTERF_DYNA` [U4.64.01],
- Calculation of the static modes of the dynamic interface previously definite with `MODE_STATIQUE` [U4.52.14] in the case choice of a complete base or by `CALC_MODES` in the case of the choice of a reduced base, with the suitable conditions,
- Definition of a modal base by the order `DEFI_BASE_MODALE` [U4.64.02] with option `RITZ` by integrating the static modes of the dynamic interface previously definite to supplement the base of the clean modes on the interface (keywords `MODE_INTF` and `INTERF_DYNA` to indicate the dynamic interface) ,
- Calculation of the assembled vectors of loading (imposed forces, displacements…) under-field of the type E_1^l and projection of those on the basis supplements `RITZ`, with the classification of the degrees of associated freedom,

- Assembly of the dynamic macronutrient of under-field of type E_1^l on the interface Σ by the order MACR_ELEM_DYNA [U4.65.01], starting from the base of RITZ previously defined by integrating there the vectors of projected load, or a dynamic macronutrient of under-field of type $E_3^{impé}$ by converting an impedance of ground on the interface Σ, cf the order LIRE_IMPE_MISS [U7.02.32] and the keyword MATR_IMPE order MACR_ELEM_DYNA [U4.65.01].

Then, for the dynamic analysis of the “mixed” scale model with the nonlinear under-fields:

- Definition and assembly with the initial grid by DEFI_MAILLAGE [U4.23.01] and ASSE_MAILLAGE [U4.23.03] super-meshs corresponding each one to a dynamic macronutrient of the under-field of type E_1^l to create a new reduced grid “mixed”. The nodes of these super-meshs will carry the degrees of freedom of the condensed under-field,
- Definition of a “mixed” scale model affected to the new reduced grid “mixed” integrating the under-field non-condensed in finite elements of type E_2^{nl} as well as the assignments of the super-meshs by the keyword AFFE_SOUS_STRUC of the operator AFFE_MODELE [U4.41.01],
- Definition of the specific charges of the model by the keywords LIAISON_SOLIDE or LIAISON_INTERF of the operator AFFE_CHAR_MECA [U4.44.01],
- Call to the nonlinear operator of calculation STAT_NON_LINE [U4.51.03] or DYNA_NON_LINE [U4.53.01] on the “mixed” scale model by including the field material defined on the under-field of type E_2^{nl} affected with the reduced grid “mixed”, preceding specific charges by the keyword EXCIT as well as the vectors of loading defined on the under-fields which were condensed and projected on the macronutrients, via the keyword SOUS_STRUC,
- Lastly, one can validate the “mixed” scale model final, under entirely linear assumption, containing the under-fields condensed in the form of macronutrients by comparison between Eigen frequencies and the modal analysis on the first modes of the complete initial model nonsmall-scale, containing all the under-fields, by using the keyword MODE_VIBR of the operator DYNA_NON_LINE.

Then for each under-field of the type E_1^l out those of an impedance of ground:

- Restitution in physical base on the model of the under-field on which the macronutrient of the under-field by the operator was built REST_COND_TRAN [U4.63.33].

Command files miss06b.comm and sdnv107b.comm illustrate this kind of calculation.

5 Bibliography

[1] O. NICOLAS: Specification of the development of the translation MACR_ELEM_STAT / MACR_ELEM_DYNA. Report EDF/AMA-05.149.

[2] G. DEVESA: Application of methods of modal condensation in the operators of nonlinear dynamic calculation used for the study of benchmark EUROPLEXUS – *Code_Aster* into explicit. Report EDF/AMA-07-165.

[3] Document Aster [U2.07.02]: Note of use of the static under-structuring.

[4] Document Aster [U2.06.07]: Interaction ground-structure in seismic analysis with the interface *Code_Aster* - ProMISS3D.

[5] Document Aster [R4.06.02]: Modal calculation by classical dynamic under-structuring.