Realization of calculations of damage into quasi-static (brittle fracture)

Summary:

The purpose of this document is to advise the user of *Code_Aster* for the realization of simulation using models of damage and in particular of the techniques to be implemented to have a solution of good quality and a robust way.

Contents

1 Introduction and warning

To study the degradation of a structure, several tools are available and in particular (by level of complexity and time calculation crescents):

- breaking process with criteria of starting based on criteria in constraint or the rate of refund of energy;
- models of cohesive zones which make it possible to study the starting and the propagation of cracks when one knows a priori the potential way of cracking;
- the models of damage which make it possible to model the degradation of material in a diffuse or localised way.

Guides of implementation in *Code_Aster* exist for the first 2 families of method. They is the documents [U2.05.01]: “Scope of application of the operators of breaking process and advices of use” and [U2.05.07]: “Note of use of the cohesive elements”. This document aims at supplementing the offer of the user guides while concentrating on the implementation of calculations of damage into quasi-static.

However, it should be specified that, compared to the other techniques, calculations of damage are most complicated to implement, most expensive in computing times but as those which are controlled as much from the point of view of the robustness as from the point of view of the quality of the result, in particular because:

- the problems of damage do not have in general a single solution at the level of the structure;
- the solutions obtained with the local models of damage strongly depend on the grid;
- one cannot propose means yet of identifying the lengths characteristic of materials, and these lengths are not transposable of a method of regularization to the other;
- the management of broken meshs often pose problem in particular with the models regularized. According to modelings, one can observe nonphysical widenings of the bands damaged rather than the appearance of multi-cracking or very strong difficulties of convergence.

It is thus advisable always to have a critical glance on the got results.

2 Choice of the model of damage

To represent the degradation of a material even its rupture, one of the possible methods (in particular when the mode of ruin is not known) is to use a “lenitive” law of behavior, i.e. such as, once passed a threshold (in constraint or deformation), the constraint decreases when the deformation increases. This kind of behavior can be obtained:

- by introducing a variable of damage D understood enters 0 and 1 (= mechanical of the damage as introduced by Kachanov or Lemaitre): laws ENDO_FRAGILE, ENDO_ISOT_BETON, ENDO_ORTH_BETON, MAZARS in *Code_Aster;*
- by using models of plasticity with a negative “work hardening” such as for example the laws BETON_DOUBLE_DP, DRUCK_PRAGER, but also various laws of ground available in *Code_Aster* ;
- by introducing a variable such as the porosity coupled to plasticity for the ductile rupture: law of ROUSSELIER in *Code_Aster*.

Whatever the method employed, all these lenitive laws “local” lead to the same difficulty: it arrives one moment when the problem becomes badly posed and the solution obtained becomes strongly dependent on the network. Indeed, the damage is located in a band having for thickness only one element (from where an energy of cracking which tends towards zero when the grid is refined) and there is dependence of the “way” of cracking to the topology of the grid. Relevance of these calculations (although still enough spread in Littéerasure) is thus very contestable, and the fact of making depend energy dissipated on the size of mesh brings only one solution very partial.

To circumvent this problem, it is usually allowed that it is necessary to introduce into the problem to solve a characteristic length, which will control the thickness of damaged zone the independently of the grid and will again make it possible to have convergence of the solution when the grid is refined.
Several methods of regularization were developed these last years which have each one their advantages and their limits. In *Code_Aster*, the methods available are based all on a concept of gradient (in opposition to methods of the integral type, largely widespread in the literature also).

One distinguishes the methods which introduce:

- the gradient of the variable of damage (model `GRAD_VARI`, confer documentation [R5.04.01]);
- a regularized deformation calculated starting from the deformation and of its Laplacian (model `GRAD_EPSI`, cf Doc. [R5.04.02]);
- modelings second gradient and second gradient of dilation which introduce an energy depending completely, or partly, of the components of the gradient of the deformation (models `2DG` and `DIL`, cf Doc. [R5.04.03]);
- modelings with gradient of gonflemeNT (model `INCO_UPG`, cf Doc. [R3.06.08]).

In all the cases, the principle is to penalize from an energy point of view the localization of the damage.
The table below recapitulates for the various laws of behavior, which regularized modeling is available (and valid) in *Code_Aster*.

Law of behavior	**Modeling**
ENDO_FRAGILE	GRAD_VARI/ 2DG
ENDO_ISOT_BETON	GRAD_EPSI/GRAD_VARI/ 2DG
MAZARS	GRAD_EPSI/ 2DG
ENDO_ORTH_BETON	GRAD_EPSI/ 2DG
DRUCK_PRAGER	2DG/DIL
ROUSSELIER	INCO_UPG

Table 1: correspondence law of damage/not-local modeling

Note:

1) There does not exist not-local version of the model `BETON_DOUBLE_DP`. The established version includes however a regularization of the Hillerborg type to avoid the problem of the energy which tends towards zero when the size of the elements tends towards zero.
2) All the laws of behavior can be used with modelings `2DG` and `DIL`. However, modeling 2DG for the moment was used only with the laws of grounds and modeling `DIL`, has direction only to regularize the "voluminal damage": it thus is well adapted to treat the case of dilating materials and thus grounds. In addition to `DRUCK_PRAGER`, it is thus possible to use the laws `CAM_CLAY` (which is a typical case of the law of Hujeux) and `HUJEUX` (cf thesis of Alexandre Foucault and CR-AMA-09-154), `VISC_DRUCK_PRAG` (see note H-T64-2009-03498) and `LETK, BARCELONA, CJS, HOEK-BROWN`; but the experience feedback with these laws is still weak.
3) Modeling `INCO_UPG` for the moment is badly controlled, and must be the object of complementary studies and development. His use is disadvised.
4) Cost CPU of regularized modelings is important, on the one hand because they require to net rather finely (see §3) and of other because they introduce additional degrees of freedom returning the matrices to be reversed much larger and less hollow.

3 Grid

For a mechanical calculation with local damage, one can use linear or quadratic grids indifferently. On the other hand, it is advised to have rather homogeneous sizes of meshs so that dissipation is to it also (recall: dissipation locally is dependent in keeping with the mesh). It is also advised to have carried out the identification of the parameters post-peak on tests using about the same sizes of mesh.

Most models not-rooms are based on quadratic grids. The following table recapitulates the elements available for each modeling.

	2D	AXI	3D
GRAD_EPSI	TRIA6 TRIA3 QUAD8	-	TETRA10 TETRA4 HEXA20 PENTA15 PYRAM13
GRAD_VARI	TRIA6 QUAD8	TRIA6 QUAD8	TETRA10 HEXA20 PENTA15 PYRAM13
2DG	TRIA7 QUAD9	-	-
DIL	TRIA7 QUAD9 TRIA6 QUAD8	-	-*
INCO_UPG	TRIA6 QUAD8	TRIA6 QUAD8	TETRA10 HEXA20

Table 2: meshs available according to modelings

Concerning the smoothness of the grid, one recalls that so that the regularization of the problem is effective, at least three meshs are needed even ten meshs (for modeling GRAD_VARI) for a band of damage. Thus, if the size of the damaged zone makes $1\,cm$, it is necessary that in this zone, the size of the meshs are lower than $3\,mm$ even $1\,mm$ for GRAD_VARI.

4 The resolution of the problem with STAT_NON_LINE

The goal of this chapter is to give advices on the manner of solving the lenitive problems with STAT_NON_LINE. One proposes here a certain gradation of the tools according to their originality, of their effectiveness, their difficulties of implementation,… allowing a new user to have his own experience. According to the law of behavior used, modeling, structure, it could be more judicious to directly use one or the other of the methods (in particular piloting or mixed linear research). However, it is possible also to increase the convergence criteria slightly (RESI_GLOBA_RELA $=10^{-6}$ by default) in order to reduce the computing time and to help with convergence for strongly damaged structures. One should not exceed 10^{-4} .

4.1 Simple newton-Raphson (METHODE=' NEWTON')

It is the method simplest to use and which guarantees the quality of the solution obtained, in condition of course of using sufficiently small convergence criteria. However:

* The version 3D will be available in version 10

(i) the problems being lenitive, it is recommended to work in displacement imposed rather than in imposed effort. If not, as soon as the total effort to apply so much to decrease, it is possible no more no to converge.
(ii) In the case of fragile material, one often observes an instability of the solution, i.e. a brutal rupture of the material (snap-back in the total answer force-displacement). In this case, there is little chance that Newton alone crosses this instability (except sometimes in the case of small instabilities or of the passage in a completely broken state), and it is advisable to rock towards mixed linear research or especially piloting.

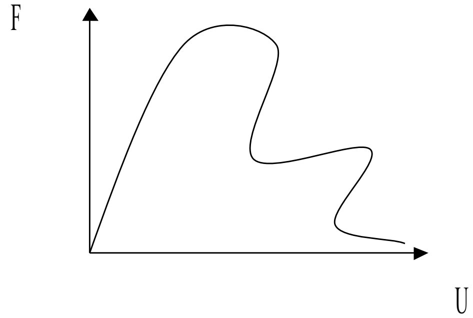

Figure 1: unstable example of answer

One recommends in general, to activate the automatic step division of time (several levels, to see the order DEFI_LIST_INST) and to use the tangent matrix (reactualized for the standard laws ENDO_ **, inevitably not reactualized for Mazars) but to if required allow the swing towards the secant matrix when one redécoupé much the step of time (keyword PAS_MINI_ELAS). Once the structure is strongly damaged, the secant matrix is sometimes more effective. Let us announce that for the laws of grounds, it is useless to reactualize the secant matrix, because one uses the elastic matrix (REAC_ITER_ELAS=0).
With the tangent matrix, one can authorize between 10 and 30 iterations of Newton according to the law of behavior used (ITER_GLOB_MAXI). On the other hand, with the secant matrix, convergence is much slower and it is necessary to authorize much more iterations (ITER_GLOB_ELAS≥100).

4.2 Linear newton-Raphson + research

In certain cases, classical linear research (keyword RECH_LINEAIRE, METHODE=' CORDE') can help with convergence. But method MIXED is definitely more robust.

4.3 Linear newton-Raphson + research mixed method

Linear research mixed method (keyword RECH_LINEAIRE, METHODE=' MIXTE') is able to go to seek solutions further that the basin from attraction of Newton and will thus allow to cross snap-back. To avoid going to seek too distant solutions (the problems of damage do not admit a single solution), one advises:

- to use small steps of time,
- not to activate the recutting of the step of time (or very little), because in the presence of a great instability the size of the step of time imports little
- to authorize a large number of iterations of Newton (ITER_GLOB_MAXI=1000),
- to use the reactualized tangent matrix (REAC_ITER=1),
- to authorize a large number of iterations of linear research to leave with the method opportunity of finding the solution in the direction of descent (ITER_LINE_MAXI=50)

It is in general the most robust method, but it can prove relatively expensive, since many iterations are authorized. On the other hand, it is not in general any more very effective when the structure is completely broken. In this case, she tends to do many iterations and can sometimes artificially increase the size of the band of localization.

In the event of large snap-back, corresponding to a brutal of test-tube, identifiable rupture on curved force-displacement by a brutal reduction in the effort applied, it is advised to use the piloting, which will seek a solution continues in order to make sure of the solution of quality.

4.4 Piloting

The piloting or method of continuation is a method which makes it possible to follow the answer of a structure in the event of instability (keyword PILOTING, confer documentation [R5.03.80]). With this intention, the intensity of the loading becomes a new unknown factor of the problem. Consequently, this method is not applicable when the problem depends explicitly on time (damage coupled with creep, thermal loading, etc).

Two modes of piloting adapted to the problems of damage exist in Code_Aster. It is on the one hand piloting by increment about deformation (TYPE=' DEFORMATION'), and in addition, of piloting by elastic prediction (TYPE=' PRED_ELAS'). Piloting by elastic prediction is in general more performing, but it is not generic (contrary to piloting by deformation); it is available for the laws ENDO_FRAGILE, ENDO_ISOT_BETON and ENDO_ORTH_BETON.

4.4.1 Preliminary valid for the 2 types of pilotings

When you decide to control a loading, you do not control any more his evolution in the course of time. Indeed, the intensity of the loading becomes an unknown factor of the problem: it can increase or decrease during increments. You can nevertheless fix the speed with which it will evolve, thanks to the increments of time and the parameter COEF_MULT (see further).
You can control only one loading or then several loadings which are proportional and thus defined in the same occurrence of AFFE_CHAR_MECA and multiplied by the same multiplying function. (e.g. simultaneous traction + shearing). Thus, if the efforts are broken up into two terms: fixed efforts F_{fixe} and controlled efforts F_{pilo}, the loading will be worth $F = F_{fixe} + \eta F_{pilo}$ where η will be the intensity of the controlled effort.
Let us admit that the loading is composed of the boundary conditions and a tractive effort.

```
EXAM NERVES = AFFE_CHAR_MECA (DDL_IMPO= (_F (GROUP_MA=' HAUT', DY = 0.0002)
FUNCTION  = DEFI_FONCTION (VALE = 0. , 0. , 1. , 1.)
```

If EXAM NERVES is the loading to be controlled, in STAT_NON_LINE, it is enough to replace:
```
EXCIT=_F (CHARGE= EXAM NERVES, FONC_MULT=FONCTION)
```
by
```
EXCIT=_F (CHARGE= EXAM NERVES, TYPE_CHARGE=' FIXE_PILO').
```

During step of time, you can follow the intensity of the loading thanks to the value ETA_PILOTAGE indicated in the tables of convergence of STAT_NON_LINE (see example below). In our case, with the end of the step of time considered, displacement will be worth:

$$dy = 0{,}0002 \times \text{eta\_pilotage} = 0{,}0002 \times 8{,}66245 \times 10^{-3}$$

```
--------------------------------------------------------------------------------
|  ITERATIONS    |     RESIDUE      |     RESIDUE      |   PARAMETER     |     OPTION     |
|    NEWTON      |     RELATIVE     |     ABSOLUTE      |     PILOTING     |   ASSEMBLY   |
|                | RESI_GLOB_RELA | RESI_GLOB_MAXI |  ETA_PILOTAGE  |                |
--------------------------------------------------------------------------------
|     0        X | 6.19721E-02  X | 1.61697E+02    | 9.58385E-03    |TANGENT        |
|     1        X | 1.35762E-04  X | 3.20143E-01    | 8.66483E-03    |TANGENT        |
|     2          | 2.15218E-07    | 5.07372E-04    | 8.66245E-03    |TANGENT        |
--------------------------------------------------------------------------------
```

You can control the terminals of the loading while fixing in the keyword PILOTING, parameters ETA_PILO_MAX and ETA_PILO_MIN. When these maximum or minimal values are reached, calculation will stop, whatever the moment. ETA_PILO_MIN is especially useful when one works with effort forced to stop calculation when the effort becomes very small (because the structure is broken) and ETA_PILO_MAX when one works with imposed displacement, when desired displacement was reached.

One can also inform the values ETA_PILO_R_MIN and ETA_PILO_R_MAX who fixes the physical values for ETA. ETA_PILO_R_MIN is in particular essential for ENDO_FRAGILE who presents same behaviour in traction and compression under penalty of jumping of one solution to the other.
Lastly, it can be interesting to specify it GROUP_MA on which to carry out piloting. Indeed, if the damage is located, that can make it possible to limit the computing times. On the other hand, it is essential if there are several laws of behavior in the model, of which certain not "controllable" (for standard piloting PRED_ELAS).

In general, calculation with piloting asks for more step of time than a direct calculation, because the evolution of the damage is limited. If calculation stopped because it reached the end of the list of moment, and not the loading, it is enough "to prolong" the initial list of moment at the time of CONTINUATION calculation.

4.4.2 The Councils for the implementation of piloting PRED_ELAS.

For the laws ENDO_FRAGILE and ENDO_ISOT_BETON, the coefficient COEF_MULT is connected to the step of time and the required variation of damage minimal:

$$\text{COEF\_MULT}=\frac{\Delta t}{\Delta D}$$

Thus, if you wish that there be at least a point which sees its damage growing of 20%, and which your steps of time are worth $\Delta t=1$, you must define $\text{COEF\_MULT}=\frac{1.}{0.2}=5$. Thus more COEF_MULT is small, more the damage progresses quickly.

For the other laws, the coefficient COEF_MULT ensure that, at least a point of Gauss left the threshold of elasticity linearized f_{pred_elas} of a quantity $\frac{\Delta t}{\text{COEF\_MULT}}$; one thus has $\text{COEF\_MULT}=\frac{\Delta t}{\underset{\text{pts de Gauss}}{MAX}(f_{\text{pred\_elas}})}$. In this case, the value of the parameter required is not obvious and depends on the laws. One advises to put 1 to start. According to the progression of the damage observed, one will be able to accelerate the phenomenon while decreasing COEF_MULT or to slow down it while increasing COEF_MULT. One can also prefer in an equivalent way to modify the list of the steps time.

One in general advises to choose SELECTION=' RESIDU',

When the loading is controlled, one should not hesitate to authorize much of recutting (by using the order DEFI_LIST_INST) because certain passages are delicate.

4.4.3 The Councils for the implementation of piloting DEFORMATION.

For piloting in deformation, it is obligatory to make at least an increment of load without piloting. One in general advises to rock with piloting only when one does not converge any more with the other methods.
The idea here is to ensure that at least a point of the structure sees its deformation progressing in a monotonous way:

$$\text{COEF\_MULT}\times\underset{\text{pts de Gauss}}{MAX}\left(\frac{\varepsilon^-}{\|\varepsilon^-\|}.\Delta\varepsilon\right)=\Delta t\ .$$

The value of COEF_MULT is not always easy to gauge, but it is in general important values since inversely proportional to the increment of deformation (example for steps of $0,1\,s$ and an increase in deformation of 10^{-6} , $\text{COEF\_MULT}=\frac{0,1}{10^{-6}}=100\,000$). After a first test, one will if need be adjust the value of this coefficient by increasing the value to charge less quickly, and by decreasing it to damage more quickly. One can also prefer to act on the list of the steps of time.

4.5 Dynamics

As a last resort, one can also try to launch out in a dynamic calculation. In certain cases, that can bring solutions but is to be used with many precautions. A U2 documentation was especially written to gather the advices, it acts of Doc. [U2.04.07], which it is essential to consult before launching out in a simulation in dynamics (to solve a quasi-static problem).

Note:
Since version 10 of Code_Aster, two methods will be also available for the user: the automatic management of the step of time (which can appear an interesting help) and method IMPLEX for the laws of behavior `ENDO_FRAGILE` and `ENDO_ISOT_BETON` but only in local version.