Note of use of the operators of breaking process for the classical approach (non-linear elasticity)

Summary:

This document introduces the operators of breaking process available in *Code_Aster* within the framework of the classical approach. The classical approach is limited to non-linear elasticity. Advices of use are given.

It is advised to have taken knowledge of the methodological guide general in breaking process, which counts the various approaches available [U2.05.00].

Contents

1 General information

The classical approach is the historical approach for the breaking process. It was initially developed within the framework of the linear elasticity, then extended to non-linear elasticity.

1.1 Linear elastic mechanics of the rupture into quasi-static

This paragraph points out the parameters characteristic in linear elastic mechanics of the rupture.

1.1.1 Factors of intensity of the constraints

The factors of intensity of the constraints characterize the singularity of the constraints at a peak of crack. Their general expression is form: $K=\lim_{r\to 0}\sigma(r)\sqrt{r}$. Three factors of intensity of the constraints are defined, associated with the three modes of opening of the crack.
In linear elasticity, the factors of intensity of the constraints make it possible to break up the field of displacement $\mathbf{u}$ in a singular part and a regular part [1][6] :

$$u=u_R+K_I u_S^I+K_{II}u_S^{II}+K_{III}u_S^{III}.$$

1.1.2 Rate of refund of energy

One considers a fissured elastic solid occupying the field Ω. Are:

$\mathbf{u}$ the field of displacement,
T the field of temperature,
$\mathbf{f}$ the field of voluminal forces applied to Ω,
$\mathbf{g}$ the field of surface forces applied to a part S of $\partial\Omega$,
$\mathbf{U}$ the field of displacements imposed on a part S_d of $\partial\Omega$.
$\boldsymbol{\sigma}$ the tensor of the constraints,
$\boldsymbol{\varepsilon}$ the tensor of the deformations,
$\boldsymbol{\varepsilon}^{th}$ the tensor of the deformations of thermal origin,
$\psi(\boldsymbol{\varepsilon}, T)$ density of free energy.

The rate of refund of energy G corresponds to the energy approach of the rupture of Griffith [5]. It is defined by the opposite of the derivative of the potential energy in balance $W(\mathbf{u})$ compared to the field Ω :

$$G=\frac{-\partial W(\mathbf{u})}{\partial\Omega}$$

with: $W(\mathbf{u})=\int_\Omega \psi(\boldsymbol{\varepsilon}(\mathbf{u}), T)\,d\Omega-\int_\Omega \mathbf{f}\,\mathbf{u}\,d\Omega-\int_S \mathbf{g}\,\mathbf{u}\,d\Gamma$

It is pointed out that the rate of refund of energy is equivalent to the integral of Rice in linear elasticity [4].
In plane linear elasticity, the coefficients D'intensity of constraints are connected to the rate of refund of energy by the formula of Irwin:

$G=\frac{1-\nu^2}{E}(K_I^2+K_{II}^2)$ in plane deformations

$G=\frac{1}{E}(K_I^2+K_{II}^2)$ in plane constraints

$$G=\frac{1-\nu^2}{E}(K_I^2+K_{II}^2)+\frac{K_{III}^2}{2\mu} \text{ with } \mu=\frac{E}{2(1+\nu)} \text{ , in 3D}$$

1.2 Extension to non-linear elasticity

The preceding definitions are rigorous only in linear thermoelasticity but extensions are possible with the nonlinear problems. In particular, it is possible to define and calculate the rate of refund of energy in nonlinear elasticity, provided that the loading remains radial and monotonous.

The application of the comprehensive approach apart from its field of validity led to nonsatisfactory results: problems of “transferability” of test-tubes with structures (effect small defect [3]), bad taking into account of the history of the loading (effect of hot preloading [7]),…

2 Tally of use of the features of breaking process in *Code_Aster*

2.1 Recall of the methods of calculating available

2.1.1 Method theta

Calculation of the rate of refund of energy

The difficulty of the calculation of the rate of refund of energy comes from derivation compared to the field of an integral depending on this same field. A rigorous method is the method theta, which is a Lagrangian method of derivation of the potential energy [8][9][10]. It consists in introducing a field θ and to consider transformations $F^{\eta}: \mathbf{M} \to \mathbf{M} + \eta\theta(\mathbf{M})$ area of reference Ω in a field Ω_{η} who correspond to propagations of the crack. These transformations should not modify the edges of the field except the bottom of crack.

This method is detailed in [R7.02.01] and [R7.02.04]. The use of the method, developed in the operator CALC_G of *Code_Aster*, is described with the §4.1.

With the method theta, the rate of refund of energy G is solution of the variational equation:

$$\int_{\Gamma_0} G(s)\theta(s)\cdot\mathbf{m}(s)\,ds = G(\theta),\ \forall\,\theta\in\Theta$$

where $\mathbf{m}$ is the unit normal at the bottom of crack Γ_0 located in the tangent plan at $\partial\Omega$ and returning in Ω, and where $G(\theta)$ is defined by the opposite of the derivative of the potential energy $W(\mathbf{u}(\eta))$ with balance compared to the initial evolution of the bottom of crack η :

$$G(\theta) = \left.\frac{-dW(\mathbf{u}(\eta))}{d\eta}\right|_{\eta=0}$$

One notes Θ the whole of the fields θ acceptable (see §4.1.1).

For a linear or non-linear thermoelastic problem the expression of $G(\theta)$ is:

$$\begin{aligned}
G(\theta) = & \int_{\Omega}\left[\sigma(\boldsymbol{u}):(\nabla\boldsymbol{u}.\nabla\theta) - \Psi(\varepsilon(\boldsymbol{u}))\,\mathrm{div}\,\theta\right]d\Omega && \leftarrow \textit{terme classique}\\
& -\int_{\Omega}\frac{\partial\Psi}{\partial T}(\nabla T.\theta)\,d\Omega && \leftarrow \textit{terme dû à la thermique}\\
& +\int_{\Omega}\left[(\nabla\boldsymbol{f}.\theta)\boldsymbol{u} + \boldsymbol{f}.\boldsymbol{u}\,\mathrm{div}\,\theta\right]d\Omega && \leftarrow \textit{terme dû aux forces volumiques } \boldsymbol{f} \textit{ sur } \Omega\\
& +\int_{\Gamma_F}\left[(\nabla\boldsymbol{F}.\theta)\boldsymbol{u} + \boldsymbol{F}.\boldsymbol{u}\left(\mathrm{div}\,\theta - \boldsymbol{n}.\frac{\partial\theta}{\partial\boldsymbol{n}}\right)\right]d\Gamma && \leftarrow \textit{terme dû aux forces surfaciques } \boldsymbol{F} \textit{ sur } \Gamma_F
\end{aligned}$$

If one places oneself on the assumption of great displacements (but always in nonlinear elasticity in small deformations), the term should be replaced

$\int_{\Omega}\sigma_{ij}u_{i,p}\theta_{p,j}\,d\Omega$ by $\int_{\Omega}F_{ik}S_{kj}u_{i,p}\theta_{p,j}\,d\Omega$

with $\mathbf{S}$ the tensor of the constraints of Piola-Lagrange called still second tensor of Piola-Kirchoff, $\mathbf{F}$ the gradient of the transformation which makes pass from the configuration of reference to the current configuration.

If one takes account of the initial deformations ε_{ij}^{0} and of the initial constraints σ_{ij}^{0}, the term should be added:

$$\int_{\Omega}\left[\left(\sigma_{ij}-\frac{1}{2}\overset{\circ}{\sigma}_{ij}\right)\overset{\circ}{\varepsilon}_{ij,k}-\left(\varepsilon_{ij}-\varepsilon_{ij}^{th}-\frac{1}{2}\overset{\circ}{\varepsilon}_{ij}\right)\overset{\circ}{\sigma}_{ij,k}\right]\theta_k\, d\Omega\ .$$

It would seem that this expression does not make it possible to impose at the same time initial strains and initial stresses (even if the fields are in balance). It is thus not possible for the moment to impose at the same time initial strains and initial stresses (see §2.2.5).

For a thermoelastoplastic problem the expression of $G(\theta)$ reserve in *Code_Aster* is:

$$G(\theta)=\int_{\Omega}\sigma_{ij}\,u_{i,k}\theta_{k,j}-\tilde{\Psi}\,\theta_{k,k}-\left(\frac{\partial\tilde{\Psi}}{\partial T}T_{,k}+(R+\sigma_y)\,p_{,k}+\frac{\partial\tilde{\Psi}}{\partial\beta_{ij}}\beta_{ij,k}-\sigma_{ij}\,\varepsilon_{ij,k}^{p}\right)\theta_k\, d\Omega$$

with:

$\tilde{\Psi}$ total mechanical energy,

ε^{p} the tensor of the plastic deformations,

p the variable interns scalar isotropic work hardening (cumulated plastic deformation),

β one or more tensorial or scalar variables of kinematic work hardening,

σ_y initial linear elastic limit,

R the ray of the surface of load for isotropic work hardening.

For a radial and monotonous loading: $\sigma_{ij}\,\varepsilon_{ij,k}^{p}=(R+\sigma_y)\,p_{,k}+\frac{\partial\tilde{\Psi}}{\partial\beta_{ij}}\beta_{ij,k}$ and one finds the expression of $G(\theta)$ in nonlinear thermoelasticity [R7.02.03].

Calculation of the factors of intensity of the constraints

In linear thermoelasticity, one can associate with G a symmetrical bilinear form $g(\mathbf{u},\mathbf{v})$ by the formula of polarization. One can then show that this bilinear form defines a scalar product for which singular functions u_S are orthogonal between them and orthogonal with regular displacement u_R [R7.02.05].

Consequently, one can calculate the factors of intensity of the constraints from $g(\mathbf{u},\mathbf{v})$ by the method theta:

Finally, in a general way:

$$\begin{cases}K_I=E\,g(\boldsymbol{u},\boldsymbol{u_S^I})\\ K_{II}=E\,g(\boldsymbol{u},\boldsymbol{u_S^{II}})\end{cases}$$ in plane constraints

$$\begin{cases}K_I=\dfrac{E}{1-\nu^2}\,g(\boldsymbol{u},\boldsymbol{u_S^I})\\ K_{II}=\dfrac{E}{1-\nu^2}\,g(\boldsymbol{u},\boldsymbol{u_S^{II}})\end{cases}$$ in plane deformations and 3D

and $K_{III}=2\mu.g(\boldsymbol{u},\boldsymbol{u}_S^{III})$ in 3D

This calculation is possible in *Code_Aster* with the option CALC_K_G of the operator CALC_G.

Local calculation – total calculation

In dimension 2, bottom of crack Γ_0 brings back itself to a point. Only one field θ is enough to calculate the rate of refund of energy (option CALC_G) or factors of intensity of the constraints (option CALC_K_G).

In dimension 3 dependence of $G(\theta)$ with respect to the field θ on the bottom of crack is more complex. In *Code_Aster*, one can calculate:

the total rate of refund G correspondent with a uniform progression of the crack (option CALC_G_GLOB);

the rate of refund of energy room $G(s)$ solution of the preceding variational equation (option CALC_G). Fields θ_i necessary for the resolution of the variational equation and the calculation of $G(s)$ are described in the §4.2.

2.1.2 Calculation by extrapolation of the field of displacements

Method of calculating of the factors of intensity of the constraints by extrapolation of displacement, developed in the operator POST_K1_K2_K3, is based on the asymptotic development of the field of displacement in bottom of crack [R7.02.08].

In 2D, in a springy medium, linear, isotropic and homogeneous, the displacement and stress fields known for the modes of opening of the crack (are analytically characterized by $K1$), of slip plan ($K2$) and of slip antiplan ($K3$). In the case general in 3D, one can show that the asymptotic behavior of displacements and constraints is the sum of the solutions correspondents to modes 1 and 2 (in plane deformations) and to mode 3 (antiplan), and of four other particular solutions, but which are more regular than the preceding ones.

In all the cases, the singularity is thus the same one and one can write the following relations in the normal plan at the bottom of crack, in a point M :

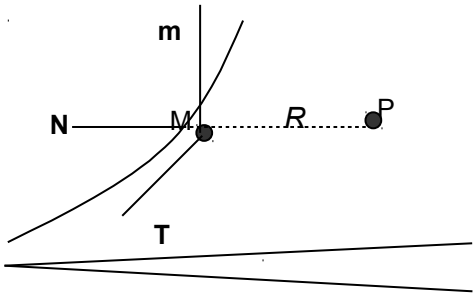
$$K_1(M)=\lim_{r\to 0}\left(\frac{E}{8(1-\nu^2)}[U_m]\sqrt{\frac{2\pi}{r}}\right)$$
$$K_2(M)=\lim_{r\to 0}\left(\frac{E}{8(1-\nu^2)}[U_n]\sqrt{\frac{2\pi}{r}}\right)$$
$$K_3(M)=\lim_{r\to 0}\left(\frac{E}{8(1+\nu)}[U_t]\sqrt{\frac{2\pi}{r}}\right)$$

with:

$\mathbf{t},\mathbf{n}$ in the plan of the crack in M,
$\mathbf{t}$ tangent vector at the bottom of crack in M,
$\mathbf{n}$ normal vector at the bottom of crack in M,
$\mathbf{m}$ normal vector with the plan of the crack in M,
$[U]$ jump of displacement enters the lips of crack:

$$[U_{\mathbf{m}}]=\left(U^{lèvre\ supérieure}-U^{lèvre\ inférieure}\right)\cdot\mathbf{m}$$

$r=\|\mathbf{MP}\|$ where P is a point of the normal plan at the bottom of crack in M, located on one of the lips.



Three methods of extrapolation are available [R7.02.08] and are systematically put in work for the calculation of $K1$, $K2$ and $K3$. Starting from the factors of intensity of the constraints, the formula of Irwin then makes it possible to calculate the rate of refund of energy G .
Ldistance from extrapolation has ABSC_CURV_MAXI ESt the only parameter user. Advices for the choice of this parameter and the interpretation of the results are given in the §4.3.

Note:

- *One can note that the signs of $K2$ and $K3$ depend on the orientation on $\mathbf{t}$ and $\mathbf{n}$. This is not too awkward insofar as the criteria of rupture or tiredness use only the absolute values of $K2$ and $K3$.*
- *The method used here is theoretically less precise than calculation starting from the bilinear form rate of refund of energy and displacements singular [R7.02.01 and R7.02.05] (operator CALC_G). It however makes it possible to easily obtain relatively reliable values of the factors of intensity of the constraints. The comparison of the various methods of calculating is always useful to estimate the precision of the got results.*

Notice on the use of POST_K1_K2_K3 with elements of Barsoum with contact:

> *The contact is generally not taken into account correctly for the nodes with the quarter (except for the formulation contact continues). The calculation of $K1$ on a node top of elements of Barsoum in the presence of contact is thus false. This is not quite serious in practice because when there is contact, it is known that $K1$ must be null.*

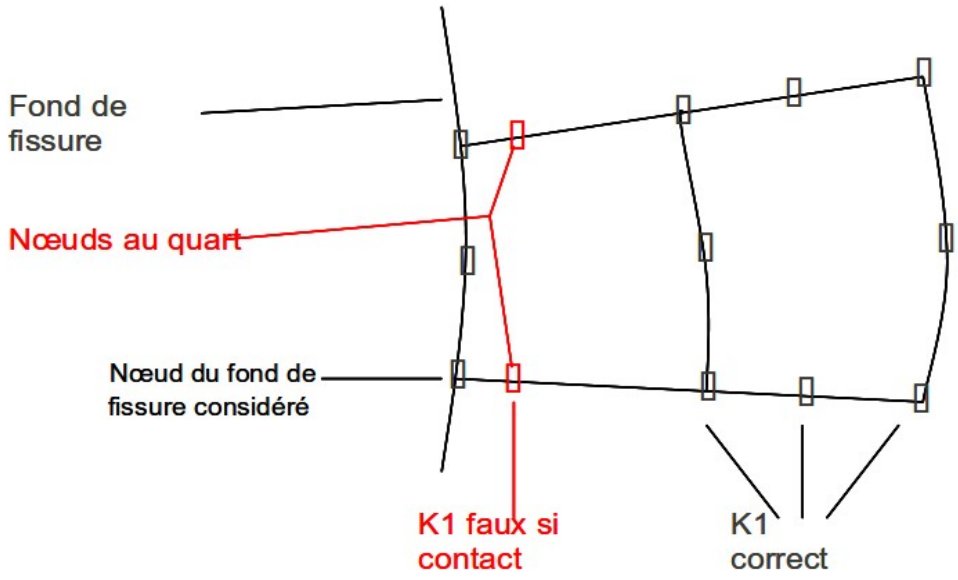


Figure 2.1.2-a : Barsoum and contact

2.2 Field of validity in general

As that will be developed in the following chapter, the crack can be with a grid (classical calculation) or not with a grid (use of method X-FEM). Unless otherwise specified, following information is valid in both cases.

2.2.1 Model

Crack with a grid: operators CALC_G and POST_K1_K2_K3 for all modelings of the continuous mediums 2D and 3D are available: plane strains, plane stresses, 2D axisymmetric and 3D.

These modelings correspond for a two-dimensional medium to triangles to 3 or 6 nodes, quadrangles with 4.8 or 9 nodes and segments with 2 or 3 nodes, for a three-dimensional medium with hexahedrons with 8.20 nodes or 27 nodes, pentahedrons with 6 or 15 nodes, tetrahedrons with 4 or 10 nodes, pyramids with 5 or 13 nodes, faces with 4.8 or 9 nodes.

Crack nonwith a grid (X-FEM): operators CALC_G and POST_K1_K2_K3 for all modelings of the continuous mediums 2D and 3D are available (plane strains, plane stresses, axisymetry and 3D).
All the geometrical types of meshs are available, except the QUAD9 and the HEXA27.
For more specific information relating to X-FEM, one will be able to refer to [U2.05.02].

	D_PLAN	C_PLAN	AXIS	3D
CALC_G	•	•	•	•
CALC_K_G	•	•	•	•
POST_K1_K2_K3	•	•	•	•

Table 2.2.1-1 : Modelings available

2.2.2 Characteristics of material

Calculation rate of refund of energy G by the method theta (operator CALC_G option CALC_G) is valid for an isotropic homogeneous material, or for an isotropic bimatériau (crack with the interface of two isotropic homogeneous materials to the different characteristics).
The calculation of the factors of intensity of the constraints (by the method theta by using the bilinear form associated with G in the operator CALC_G option CALC_K_G , or by extrapolation of the field of displacement with the operator POST_K1_K2_K3) is valid for an isotropic homogeneous material but not for a bimatériau.
Calculation of G or of the factors of intensity of the constraints remain also valid if the properties material (Young, Poisson's ratio modulus, thermal dilation coefficient and possibly elastic and module limit of work hardening) are not homogeneous, but only if the gradient of properties material is orthogonal with the direction of the field theta (direction of propagation).

From a data-processing point of view, the operator CALC_G can be used with a material function (operator DEFI_MATERIAU, keyword ELAS_FO) of a variable of order among all those being able to be affected with a material field (see [U4.43.03]: operator AFFE_MATERIAU, keyword AFFE_VARC). However, the operator CALC_G an alarm emits since the material field is affected with another variable of order that the temperature (TEMP). The operator POST_K1_K2_K3 support only the variables of order TEMP and NEUT1.

	Properties depending on variables of order	Bimatériau (crack with the interface)	Orthotropic material
CALC_G	•	•	-
CALC_K_G	•	-	-
POST_K1_K2_K3	only TEMP and NEUT1	-	-

Table 2.2.2-1 : Characteristics of material

1er case: There is a bimatériau but the point of crack is in only one material, cf Appears 3.1-a. If one is assured that the crown, definite enters the rays inferior R_INF and superior R_SUP, has like support of the elements of same material, calculation is possible whatever the selected option. If not only options CALC_G and CALC_G_GLOB are possible.

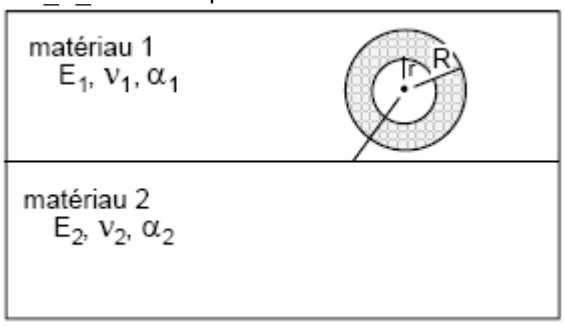


Figure 3.1-a: Bimatériau: 1er case

2Nd case: There is a bimatériau where the point of crack is with the interface, cf Appears 3.1-b. To date, only options of calculation of the rate of refund of energy (options CALC_G_GLOB and CALC_G) are available. The calculation of coefficients of intensity of constraints is not possible in this case.

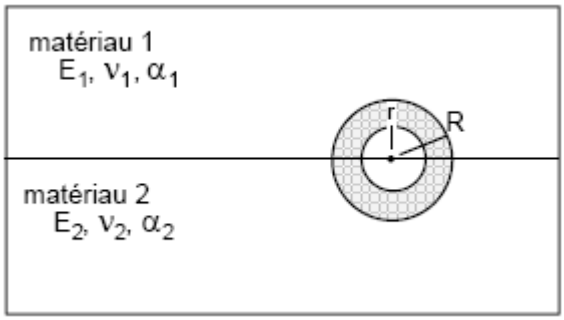


Figure 3.1-b: Bimatériau: 2Nd case

2.2.3 Relation of behavior used in postprocessing of breaking process

Logically, the relation of behavior used during postprocessing in breaking process is that which was useful during mechanical calculation.

It is pointed out that the calculation of the rate of refund of energy is possible in the following cases (for more details, to see it Table 2.2.3-1):

- linear thermoelasticity,
- nonlinear thermoelasticity (hyperelasticity),
- thermoelastoplasticity (criterion of Von Mises with isotropic or kinematic work hardening).

The calculation of the coefficients of intensity of constraints is not as for him possible solely in linear thermoelasticity on the assumption of the small deformations.

In these cases, there is thus no reason to choose a different relation of behaviour between mechanical calculation and postprocessing. In this case, one thus should not **not** to inform COMPORTEMENT under CALC_G. If COMPORTEMENT is not present under CALC_G then, postprocessing will be carried out with the relation of behavior of mechanical calculation.

The keyword factor COMPORTEMENT under CALC_G is useful only in the very particular cases where one wishes to carry out postprocessing with a law of behavior different from that which was used for mechanical calculation. This keyword must be used with an extreme prudence.

RELATION	CALC_G	CALC_K_G	POST_K1_K2_K3

BEHAVIOR	'ELAS'	•	•	•
	'ELAS_VMIS_LINE'	•	-	-
	'ELAS_VMIS_TRAC'	•	-	-
BEHAVIOR	'ELAS'	•	•	•
	'VMIS_ISOT_TRAC'	•	-	-
	'VMIS_ISOT_LINE'	•	-	-
	'VMIS_CINE_LINE'	-***	-	-
DEFORMATION	'SMALL'	•	•	•
	'GROT_GDEP'	• (except X-FEM)	-	-

Table 2.2.3-1 : Compatible laws of behaviour for postprocessing in breaking process

2.2.4 Loading

Certain loadings are associated in the additional terms in the expression of the rate of refund of energy or the factors of intensity of the constraints according to the method theta. The loadings which are currently supported for calculation in breaking process are the following:

- Thermal dilation (transmitted via the variables of order);
- Voluminal forces: FORCE_INTERNE, GRAVITY, ROTATION ;
- Surface forces on the lips of the crack: PRES_REP , FORCE_CONTOUR , FORCE_FACE ;
- Initial deformation (crack with a grid only): PRE_EPSI .

It is not possible to take into account a displacement imposed on the lips of the crack (DDL_IMPO or FACE_IMPO) or a nodal force on those.

The loadings are transmitted for postprocessing with the keyword EXCIT of CALC_G. By default (keyword EXCIT absent), all the loadings of mechanical calculation are used in postprocessing in breaking process. It is thus the advised method. If the keyword EXCIT is present with a part only of the loadings, a message of alarm is emitted.

It is important to note that the only loadings which affect in a calculation of breaking process with the method θ are those applied to the elements inside the crown (enters R_{inf} and $R_{\sup}$ for a linear thermoelastic behavior or not linear [R7.02.01 §3.3], between the bottom of crack and $R_{\sup}$ for a thermoelastoplastic relation [R7.02.07]).

Note:

If one does a calculation in great rotations and great displacements (keyword DEFORMATION = 'GROT_GDEP' under the keyword factor BEHAVIOR) the only supported loadings are died loads, typically an imposed force and not a pressure [R7.02.03 §2.4].

2.2.5 Initial state

For a crack with a grid, it is possible to take account of an initial state (either of the initial constraints, or of the initial deformations) for the calculation of the rate of refund of energy. Two opportunities are given to the user:

- to define initial deformations with the keyword PRE_EPSI in the order AFFE_CHAR_MECA (_F) [U4.44.01] and to recover them under the keyword LOAD in the order CALC_G [U4.82.03] (see an example of installation in the case test sslp102); it is pointed out that recovery is automatic;
- to recover a stress field resulting from a mechanical calculation (evol_noli resulting from the order STAT_NON_LINE [U4.51.03]) with the keyword ETAT_INIT.

It is not possible simultaneously to take into account initial constraints and initial deformations.

- The taking into account of the initial constraint is possible in the option CALC_G only for the moment. It will be possible with CALC_K_G and CALC_GTP soon.

2.2.6 /frottement contact

The calculation of the sizes of breaking process in *Code_Aster* is not **not validates** if there is contact with **friction** between the faces of the crack. Indeed the calculation of the rate of refund of energy does not take into account the dissipative phenomena.
On the other hand if the elements of rubbing contact are beyond the defined crown enters R_{inf} and $R_{\sup}$ calculations of G, $G(s)$, $K1$ and $K2$ are valid.
On the other hand, it is possible for the calculation of G and of $G(s)$ only to take into account conditions of contact without friction to avoid the interpenetration of the lips of the crack.

2.2.7 Factors of intensity of the constraints for a thermomechanical problem

Factors of intensity of the constraints obtained with the option CALC_K_G are calculated while evaluating bilinear form of G with a purely mechanical singular solution (asymptotic solution of Westergaard). If a thermomechanical problem is solved, one then does not take counts some singularity due to the thermal field. An indicator of the error due to this approximation can be obtained while evaluating the difference enters G and G_IRWIN. In practice, one evaluates in any point of the bottom of crack quantity $\frac{|G-G_{irwin}|}{|G|}$, and one makes of it then the arithmetic mean. If this average exceed 50 %, it is estimated whereas one leaves the perimeter of validity of the approach, and a message of alarm is emitted. However, the values of G are right.

For POST_K1_K2_K3, the similar remark is present.

2.3 Validity of the calculation of G into non-linear

The essential problem in the nonlinear situations comes from the difficulty in separating the various energy contributions. It is necessary to consider two very distinct classes of problems:

- that where, in spite of nonthe geometrical linearities or of behavior, one can display a potential for the interior and external actions (nonlinear elasticity or hyperelasticity),
- that where such a potential does not exist (thermo-elastoplasticity).

For the first class, one can extend the criterion of Griffith by using the potential energy to balance, and calculate the rate of refund of energy as in linear thermoelasticity.

For the second class of problem, the essential difficulty comes owing to the fact that dissipation is not only due to the propagation of the crack itself. One cannot distinguish any more which share of restored energy is used for the propagation and which share is directly used by another dissipative phenomenon (plasticity in fact).

2.3.1 Nonlinear thermoelasticity

Not linearity of behavior: the relation of nonlinear elastic behavior is described in [R5.03.20]. It should be noted that the elastoplastic law of Hencky-Von Put (isotropic work hardening) in the case of a radial and monotonous loading is equivalent to the non-linear elastic law. The material hyperelastic has a reversible mechanical behavior, i.e. any cycle of loading does not generate any dissipation. This fact the relation of behavior of material derives from the free potential energy and one can give a direction to the rate of refund of energy within the framework of the energy approach of Griffith.

Not geometrical linearity: Calculations of the rates of refund of energy and factor of intensity of the constraints are theoretically not valid in great deformations (in any case with the relations used in Code_Aster). It however is allowed to the user to carry out these postprocessings starting from a calculation in great deformations, while specifying, in CALC_G, DEFORMATION=PETIT. Load with the user to make sure of the founded good of such a calculation.

We recommend for a calculation of G in great deformations to use an equivalence in opening.

That requires to carry out two mechanical calculations: one in small deformations, the other in the formalism of deformations wished (for example GDEF_LOG). One then recovers for two calculations in postprocessing the openings of defect on each level of desired loading; for calculation in small deformations, one also calculates the rate of refund of energy. For a level of loading given in great deformations noted CHAR1, one determines the opening of the defect; one seeks for which level of loading (probably different and noted CHAR2) the opening in small deformations is the same one; one calculates on this level of loading CHAR2 the rate of refund of energy in small deformations. One applies whereas the rate of refund of energy for calculation in great deformations and the level of loading CHAR1 is equal to that given in small deformations for the level of loading CHAR2.

2.3.2 Thermo-elastoplasticity

The field of validity of the calculation of the rate of classical refund of energy is limited to the linear or non-linear thermoelastic framework. To deal with the elastoplastic problem, two solutions are possible:

- to bring back itself to a non-linear thermoelastic problem with restrictive assumptions,
- to use another formulation, like that of the energy approach.

2.3.2.1 Equivalence enters a nonlinear thermoelastic problem and a thermoelastoplastic problem

The relation of nonlinear elastic behavior gives the opportunity of dealing with the problems of breaking process by approaching the thermoelastoplastic behavior. In the case of one **monotonous radial loading**, it makes it possible to obtain strains and stresses of the structure similar to those which one would obtain if the material presented an isotropic work hardening. The use of the indicators of discharge and loss of radiality makes it possible to make sure of the equivalence of the laws of behavior, cf §3.3.

But the conditions of loadings proportional and monotonous, essential to ensure the coherence of the model with actual material, lead to important restrictions of the field of with the capable problems being dealt by this method (thermal in particular can lead it to local discharges).

2.3.2.2 Another formulation

See methodological guide [U2.05.00]

3 Implementation of a study of breaking process

3.1 Grid

3.1.1 Case of a crack with a grid

Conditions to be respected: there is no condition to respect *a priori* on the type of grid in bottom of crack. However, D E the quality of the grid depends digital quality on the results resulting from mechanical calculation (displacements and constraints) and by consequence from quality from the sizes in breaking process.

The definition of the crack by the operator DEFI_FOND_FISS require relative groups of meshs:

- with the meshs of the bottom of crack (linear meshs in 3D, mesh not in 2D),
- with the meshs of the lips of the crack (grid surface in 3D, linear meshs in 2D).

Remarks and advices:

- Calculations of the sizes of breaking process are valid for linear or quadratic elements, but it is strongly advised to use quadratic elements, in particular in 3D. The calculation of these sizes indeed requires to determine with a good approximation the deformation and stress fields which strongly vary in the vicinity of the bottom of crack. However, to identical number of nodes, the quadratic elements give better results that the linear elements.
- The calculation of the factors of intensity of the constraints with the operator POST_K1_K2_K3 , or with the options 'CALC_K_G' or 'CALC_K_MAX' of the operator CALC_G can be carried out only if the lips of the crack are initially stuck, which corresponds to CONFIG_INIT=' COLLEE' for the operator DEFI_FOND_FISS .
- UN radiant grid in bottom of crack is not obligatory: rays R_{inf} and $R_{\sup}$ are not related to the grid and the crown can be "with horse" on several elements. Nevertheless the practice shows that a radiant grid in bottom of crack gives digital good performances.
- Maillor GIBI comprises a parameterized automatic procedure which makes it possible to conceive grids of blocks fissures in 3D. This procedure was developed by EDF-R&D and was validated to ensure the good quality of the grid. One obtains a grid with the format GIBI which can recognize *Code_Aster* (order PRE_GIBI). The user informs a certain number of geometrical parameters (dimensions of crack, size of block,…) or topological (modeling of the basic torus of crack in crowns, sectors and slices, déraffinement, many elements,…) and the software generates a block fissures, which can then be integrated in another structure. A similar procedure is under development in the platform Salomé and should be available end 2010.
- During the quadratic use of grids, it is **strongly advised** to position the nodes mediums of the quadratic elements concerning the bottom of crack to the quarter of the edges (grid of the type **Barsoum**). Thus dependence in $\sqrt{r}$ field of displacement is represented better and the quality of the results is improved. One can directly introduce of type of elements into an existing quadratic grid by the keyword MODI_MAILLE (option 'NOEUD_QUART') order MODI_MAILLAGE [U4.23.04]. The computing time is not modified, but the profit in term of quality of the results is considerable.

Checking of the quality of the grid: To assess the quality of the grid it is advised to carry out an elastic design and to use the estimators of errors of discretization: estimators of errors of ZHU-ZIENKIEWICZ in elasticity 2D [R4.10.01], the estimator of error by residue [R4.10.02] and it S estimators in Quantity of Interest [R4.10.06], Quantities of Interest available being $K1$, $K2$ and $K3$ resulting from the method n°3 of POST_K1_K2_K3.

These estimators are established in *Code_Aster* in the order CALC_ERREUR [U4.81.06]. They are activated starting from the following options: ERZ1_ELEM for $ZZ1$, ERZ2_ELEM for $ZZ2$ and

ERME_ELEM for the estimator in residue by element. The estimate in Quantity of Interest requires the installation of a dual problem (see for example the case test zzzz257).

Case of a crack nonwith a grid

Conditions to respect: L'use of method X-FEM makes it possible to overcome certain difficulties related to the grid. In particular, a free grid of the healthy structure is enough. However, a sufficiently fine grid remains necessary in the zones to strong gradient (around the bottom of crack for example). It is true qu' in linear elasticity, enrichment by the functions asymptotic to improve the precision of the method: with same size of mesh, elements X-FEM will be thus more precise in bottom of crack than the classical elements. But that only affects very locally. One thus needs a grid relatively refined in bottom of crack.

As comparison a classical grid with a torus enters to n layers of elements in bottom of crack and a free grid X-FEM, one can say that the size of elements X-FEM in bottom of crack must be about that of the elements of the layer $n/2$ torus.

Only the estimator of error by residue is available for elements X-FEM, in 2D only.

Remarks and advices:

- To fix Idées, for a crack length a in infinite medium, the size of the elements in bottom of crack must be enters $a/10$ and $a/20$ to obtain an error on the rate of refund of energy between 1% and 2%.
- In order to obtain a grid refined in bottom of crack, two approaches can be considered:
  1) The introduction of a block crack consists in defining during the creation of the grid, a box including the crack. The grid in this box will be regulated, and the smoothness of the grid must be a parameter of the procedure of grid. The box must be sufficiently broad if the study the propagation of the crack is considered. It is also necessary to create a zone of connection between the regulated box, and the rest of the structure with a grid into free. One can also use the advanced features of the mailleurs as Blsurf+GHS3D who allow to define cards of sizes locally.
  2) The second approach consists in carrying out an adequate grid by successive refinements of an initial free grid considered to be coarse [U2.05.02]. The criterion of refinement is the distance to the bottom of crack (operator RAFF_XFEM [U7.03.51]).

3.2 Linear elastic design

3.2.1 Case of a crack with a grid

The calculation of the various parameters of the breaking process is made only in postprocessing classical mechanical calculation. The implementation of a study is thus the following one:

•Reading of the grid

•Definition of the model, materials, the loadings

•Mechanical calculation with MECA_STATIQUE or STAT_NON_LINE

•Definition of the characteristics of the crack with DEFI_FOND_FISS [U4.82.01].

•Calculation with the operator CALC_G [U4.82.03]:

•option CALC_G or CALC_G_GLOB : calculation of G

•option CALC_K_G : calculation of K and of G

•option CALC_K_MAX, G_MAX, G_MAX_GLOB : maximization of K or of G in the presence of or not signed signed loadings

•Calculation with the operator POST_K1_K2_K3 [U4.82.05]: calculation of K and of G starting from the jumps of displacement on the lips (extracted directly by the operator from the field from total displacement).

3.2.2 Case of a crack nonwith a grid

In the case of a crack nonwith a grid, there is a preliminary stage of definition and enrichment of the model (method X-FEM, cf [U2.05.02] for more details). The implementation of a study is thus the following one:

- Reading of the grid (without crack)
- Definition of the healthy model, materials
- Definition of the crack: DEFI_FISS_XFEM
- Creation of the enriched model: MODI_MODELE_XFEM
- Mechanical calculation with MECA_STATIQUE or STAT_NON_LINE
- Calculation with the operator CALC_G [U4.82.03]:
  - option CALC_G or CALC_G_GLOB : calculation of G
  - option CALC_K_G : calculation of K and of G
- Calculation with the operator POST_K1_K2_K3 [U4.82.05]: calculation of K and of G starting from the jumps of displacement on the lips
- Creation of the grid of visualization and visualization of the fields results (forced, displacement): POST_MAIL_XFEM and POST_CHAM_XFEM.

3.2.3 Use of the results

The factors of intensity of the constraints and the rates of refund of energy calculated can be used for:

- to evaluate the risk of starting of the defect (comparison with tenacity);
- to calculate the propagation velocity in fatigue of the crack (law of Paris) [U4.82.04] for a possible calculation of propagation (by mending of meshes or with PROPA_FISS for a crack nonwith a grid [U4.82.11]);
- to estimate the direction of junction of the crack requested in mixed mode [2], [8], [U4.82.03] [U4.82.04].

3.3 Non-linear calculation: Indicators of discharge and loss of radiality

These indicators make it possible to locate the local discharges and the loss of radiality (field DERA_ELGA and DERA_ELNO calculé with CALC_CHAMP.

Attention with the interpretation of the indicators of discharge and loss of radiality: the value given to time t_i corresponds to the diagnosis from what occurs between t_i and t_{i+1}. Thus, the computed value with the last step of time does not have a direction. The indicator of discharge is negative to indicate a local discharge, and the indicator of radiality is worth 0 for a radial way.

Note:

> *The interpretation of the indicator of loss of radiality is not easy. One cannot in particular define threshold from which calculation is not valid any more.*
> *An alternative solution can consist in comparing in postprocessing of an elastoplastic calculation it G nonlinear rubber band with or without recalculation of the constraints (keyword CALCUL_CONTRAINTE of CALC_G).*
> *If one remains well in the field of validity of the calculation of G (radial and monotonous loading), then the results with or without recalculation of the constraints are identical. As soon as one leaves this field of validity, the variation grows. One can thus check a posteriori that one remains well on the design assumptions of G.*

4 Recommendations of use

4.1 Introduction of the field theta

4.1.1 Conditions to respect

The field θ is a field of vectors, definite on the fissured solid, which represents the transformation of the field during a propagation of crack. This field must check the following conditions:

- the transformation should modify only the position of the bottom of crack and not the edge of the field $\partial\Omega$. The field θ must thus be tangent with $\partial\Omega$ (in particular lips of the crack), i.e while noting $\mathbf{n}$ the normal with $\partial\Omega$: $\theta.\mathbf{n}=0 \quad \text{sur}\,\partial\Omega$.
- The field θ must be locally in the tangent plan with the lips of the crack and in normal 3D with the edge to which it belongs. This corresponds to the direction of propagation of the crack.
- The field θ must also be continuous on Ω .

4.1.2 The Council on the choice of the crowns Rinf and Rsup

In Code_Aster, the choice was made to define the field θ in the following way:

- the direction of the field is colinéaire to the direction of propagation of the crack. In 3D, one takes over the local leadership of the projection of the node considered on the bottom of crack;
- the standard of θ is defined starting from two crowns (or tori in 3D), of ray R_{inf} and R_{sup} . In on this side R_{inf} , the module of the field theta is constant, with beyond it is null and it is linear between the two, cf. Figure 4.1.2-1.

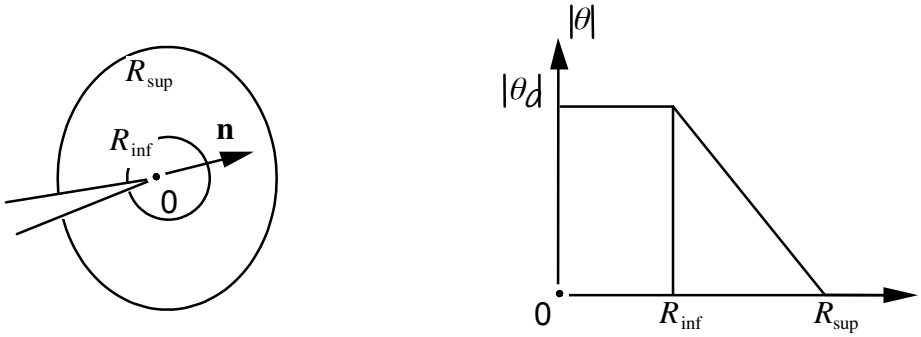


Figure 4.1.2-1: Geometrical definition of the field theta

The construction of the field theta is described precisely in [R7.02.01]. It is established in Lhas order CALC_G.

In 2D and axisymmetric the bottom of crack Γ_0 limits itself to a point. The user defines:

- rays R_{inf} and R_{sup} ,
- the module $|\theta_0|$ in bottom of crack (equal to 1 by default),
- direction of propagation of the crack m .

In 3D the user defines:

- rays $R_{\text{inf}}(s)$ and $R_{\text{sup}}(s)$,
- the topology of the bottom of crack: opened or closed according to if the crack is emerging or not,

- the module $|\theta_0|$ in bottom of crack (only for the calculation of G total if not them P fields θ^i necessary for the resolution of the variational equation and the calculation of $G(s)$ are calculated automatically according to the family of functions of interpolation chosen, to see §4.2).

The directions of the field theta except ends are calculated automatically starting from the lips of the crack, but the user can possibly define them itself by using the keyword DIRE_THETA.

Note:

•The fields of displacement and constraint are singular in bottom of crack; the precision of calculation is thus less good in the vicinity of the bottom. It is noted that the selected shape of the field theta ($\theta.\boldsymbol{m}$ constant between 0 and $R_{\inf}$) precisely allows to cancel the contribution of the classical term of $G(\theta)$ inside the first crown (term in $\int_\Omega \left[\sigma(\boldsymbol{u}):(\nabla \boldsymbol{u}.\nabla\theta)-\Psi(\varepsilon(\boldsymbol{u}))\operatorname{div}\theta\right]d\Omega$).

•To forget only the loadings applied beyond $R_{\sup}$ have a worthless contribution in postprocessings of breaking process. This can be useful if one applies a loading not supported like FORCE_NODALE, DDL_IMPO (in 2D) or FACE_IMPO (in 3D).

The Councils on the choice of the rays $R_{\inf}$ and $R_{\sup}$: the calculation of the sizes of breaking process is theoretically independent of the choice of the crown of integration (in the absence of loading on the lips, voluminal or thermal). Nevertheless it is preferable to comply with some rules:

- never not to take $R_{\inf}$ or too small compared to dimensions of the problem because singular displacements are badly calculated in the vicinity of the bottom of crack (cf notices above);
- the higher ray $R_{\sup}$ can be as large as one wants provided that the crown thus defined either contained in the solid. In 3D, a ray should not be taken $R_{\sup}$ too much large, if not the direction of the field theta (calculated by projection on the bottom of crack) can be vague;
- the choice of the rays $R_{\inf}$ and $R_{\sup}$ is independent of the topology of the grid. However, if it grid is radiant at a peak of crack, it is recommended to take crowns of integration coïncidentes with the crowns of the grid (reduction of the oscillations of G along the bottom of crack in 3D);
- in HTermo-elastoplasticity, one uses a crack as notch. One will make sure that the lower ray $R_{\inf}$ is quite higher than the ray of the notch.
- To take several consecutive crowns to check [R1, R2], [R2, R3], [R3, R4],…

To set the ideas, for a crack length *has* in infinite medium, the size of the elements in bottom of crack must be lower than $a/10$ louseR to obtain a reasonable error (about % on $K1$). For the crowns of integration, one takes then generally $R_{\inf}$ about 1 to 3 times size of the elements in the vicinity of the bottom; and $R_{\sup}$ from 3 to 7 times size of the elements.

An automatic determination of $R_{\inf}$ and $R_{\sup}$ is possible (these keyword are optional). If they are not indicated, they are automatically calculated starting from the maximum h sizes of meshs connected to the nodes of the bottom of crack. It was selected to take $R_{\sup}=4h$ and $R_{\inf}=2h$. If one chooses the value automatically calculated for $R_{\inf}$ and $R_{\sup}$, it is advisable however to make sure that these values (displayed in the file .mess) are coherent with dimensions of the structure.

4.1.3 Problem of the discretization in 3D

In 3D, for a knot slip of the bottom of crack, the direction of propagation is defined as being the average of the normals to the meshs segments of the bottom of crack on its left and its right-hand

side. For the nodes ends, the normal is calculated starting from one only mesh, and can thus be less precise, even distorts.

Crack emerging orthogonally at the edges: L be vectors of direction of propagation in the beginning and at the end of the bottom by taking of account the edges of the structure are automatically calculated. Keywords DTAN_ORIG and D TAN_EXTR (order DEFI_FOND_FISS for the calculation of G on a crack with a grid) which leave ettent to impose the direction of these vectors thus do not need to be specified.

Emerging crack in a nonperpendicular way : at the emerging end of the bottom of crack, the field θ cannot simultaneously be normal with the edge to which it belongs (in the tangent plan of the lips of the crack) and check the condition $\theta.\mathbf{n}=0$ on $\partial\Omega$.
The advised solution is to impose like direction of propagation at the ends (keywords DTAN_ORIG and DTAN_EXTR) the average of the normal at the bottom of crack in this point θ_1 and of the tangent to the edge θ_2.
One can also define the direction of the field θ on all the nodes of the bottom of crack with the keyword DIRE_THETA in Lhas order CALC_G. In the vicinity of the emerging end, one chooses like direction the average of the normal at the bottom of crack in this point θ_1 and of the tangent to the edge θ_2.

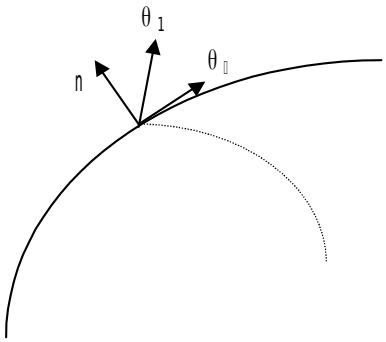


Figure 4.1.3-1: Direction of propagation at the ends of the crack

4.2 Methods of interpolation in 3D

4.2.1 Tally general

The rate of refund of energy room $G(s)$ is solution of the variational equation $\int_{\Gamma_0} G(s)\theta(s).\mathbf{m}(s)\,ds=G(\theta),\quad \forall\theta\in\Theta$.
To solve this equation:
the scalar field is broken up $G(s)$ on a basis which we note $\left(p_j(s)\right)_{1\le j\le N}$. That is to say G_j components of $G(s)$ in this base: $G(s)=\sum_{j=1}^{N} G_j p_j(s)$.
one gives oneself a base of functions tests for θ while choosing P fields θ^i independent for the trace of the field θ on the bottom of crack Γ_0 : $\left(\bar{\theta}^i(s)\right)_{1\le i\le P}$.

G_j are given by solving the linear system with P equations and N unknown factors:

$$\sum_{j=1}^{N} a_{ij} G_j = b_i \quad , \quad i=1,P$$

with $a_{ij} = \sum_{k=1}^{M} \theta_k^i \int_{\Gamma_o} p_j(s) q_k(s) . m(s) ds$

$b_i = G(\theta^i)$

This system has a solution if one chooses P fields θ^i independent such as: $P \geq N$ and if $M \geq N$.
It can comprise more equations than unknown factors, in which case it is solved within the meaning of least squares.

4.2.2 Methods of smoothing of G and Theta

In *Code_Aster*, two families of bases (cf [§2.2] were chosen):

- polynomials of LEGENDRE $\gamma_j(s)$ of degree j ($0=j=7$),
- functions of form of the node k of Γ_0 : $\phi_k(s)$ ($1=k=NNO$ = many nodes of Γ_0) (of degree 1 for the linear elements and of degree 2 for the quadratic elements).

These families of bases and the linear systems to solve are described precisely in the reference material [R7.02.01]. The use of the functions of form is called 'LAGRANGE' in *Code_Aster*, including one alternative is available:

- 'LAGRANGE_NO_NO' : simplified version of the smoothing of LAGRANGE, allowing in certain cases to get more regular results in bottom of crack.

Several options is thus possible according to the base of functions tests for theta and the base of decomposition for G. They are summarized daNS the following table:

		Theta	
		Polynomials of LEGENDRE	**Functions of form**
$G(s)$	**Polynomials of LEGENDRE**	LISSAGE_THETA= 'LEGENDRE' LISSAGE_G = 'LEGENDRE'	LISSAGE_THETA=' LAGRANGE' LISSAGE_G= 'LEGENDRE'
	Functions of form		LISSAGE_THETA = 'LAGRANGE' LISSAGE_G = 'LAGRANGE' or 'LAGRANGE_NO_NO'

Table 4.2.2-1 : Possible combinations for the calculation of G in 3D

4.2.3 Remarks and advices

- **Choice of the method:** it is difficult to give a preference to one or the other method. In theory both give equivalent digital results. Nevertheless the method Theta: Lagrange is a little more expensive in time CPU than the method Theta: Legendre. It is **essential to use several methods** and to compare results, in order to consolidate the validity of the model.
- **Choice of the maximum degree of the polynomials of Legendre** : this choice depends amongst nodes in bottom on crack. If there are a small number of nodes (ten) it is useless to take a degree higher than 3 (it is conceived easily that the results are poor if one tries to find a polynomial of degree 7 passer by 10 points). Beyond of about twenty nodes in bottom of crack one can use degrees going up to 7. The experiment shows that the choice of a degree equal to 5 gives good performances in most case (cf notices below).

- **Case of the closed cracks:** if the bottom of crack is a closed curve, problems of continuity of the solution at the arbitrarily selected point as curvilinear X-coordinate origin prohibit the use of the polynomials of Legendre. If the bottom of crack were declared “closed” in DEFI_FOND_FISS , one must use the functions of form (Lagrange) to describe the functions G and θ .
- **Problem of nonthe respect of symmetry:** if one models only half of the solid compared to the crack, one must have a curve in theory $G(s)$ whose slope of the tangent is worthless with the interface of symmetry. This is not respected by the two methods. Values of $G(s)$ obtained at the ends of the bottom of crack must always be interpreted with prudence, especially if the crack is emerging in a nonperpendicular way (see §4.1.3).
- **Problem of the oscillations with Lagrange:** oscillations can appear with the Lagrange method, in particular if the grid comprises quadratic elements. These oscillations are related to a radial profile of the field theta which is different on the nodes top and the nodes mediums. A smoothing of the type 'LAGRANGE_NO_NO' allows to limit these oscillations. In addition, if the grid is radiant in bottom of crack (crack with a grid), it is pointed out that it is recommended to define crowns R_INF and R_SUP coinciding with the borders of the elements.
- **Case of the free grids:** strong oscillations can appear with the Lagrange method. A smoothing of the type 'LAGRANGE_NO_NO' limit these oscillations, but can be insufficient. In this case, it is recommended to use the operand NB_POINT_FOND of CALC_G. The choice of a report of about 5 between the total number of points in bottom of crack and the number of points of calculation seems suitable to limit the oscillations with the method Theta: Lagrange. A choice of 20 points in bottom of crack is often judicious.
- **Performance** : the operand NB_POINT_FOND can also be used in order to reduce the computing time of CALC_G.

Illustration of the problems of oscillations with Legendre: Sa case hears where the solution is constant on the bottom of crack $G^{exact}(s)=\gamma_0(s)$. If the term in front of the polynomial of Legendre of degree seven is badly calculated, with a factor ε close (but all the other coefficients in front of the other polynomials are worth 0 exactly), then the digital result is: $G(s)=\gamma_0(s)+\varepsilon\cdot\gamma_7(s)$. The relative mistake made on G is thus:

$$e=\frac{G(s)-G^{exact}(s)}{G^{exact}(s)}=\varepsilon\cdot\frac{\gamma_7(s)}{\gamma_0(s)}=\varepsilon\sqrt{15}\frac{P_7\left(\frac{2s}{l}-1\right)}{P_0\left(\frac{2s}{l}-1\right)}=\varepsilon\sqrt{15}P_7\left(\frac{2s}{l}-1\right)$$

.

If the report is traced $\frac{e}{\varepsilon}$ according to the normalized curvilinear X-coordinate $\frac{s}{l}$, the following figure is obtained.

The error at the ends (in $x=0$ and in $x=\frac{s}{l}$) is approximately 2 to 3 times larger than the maximum error inside the bottom. For example, if ε is worth 10^{-2} (either 1% of error), one will make a maximum mistake of 1.5% everywhere, except at the ends where the error will reach 4%.

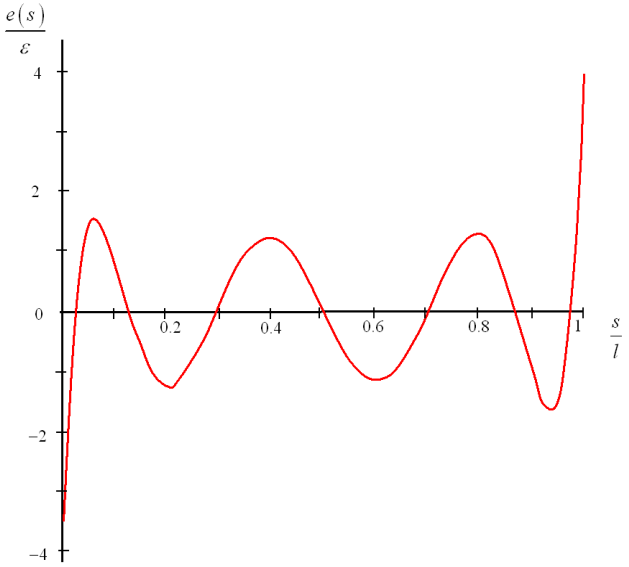


Figure 4.2.3-1 : report of the relative error on the precision according to the normalized curvilinear X-coordinate

4.3 The Councils for calculations with POST_K1_K2_K3

Choice of the distance from extrapolation: the distance from extrapolation ABSC_CURV_MAXI is the only parameter user. In general, this distance is selected equal to 3 to 5 times the size of the elements in the vicinity of the bottom of crack. In the case of a crack with a grid, the parameter ABSC_CURV_MAXI is optional. The value by default is then equal to $4h$ where h is the maximum size of the meshs connected to the nodes of the bottom of crack.
Case of the cracks with a grid: the grid must be preferably quadratic and comprise sufficient nodes perpendicular to the bottom of crack. In addition, the results are clearly improved if, if the grid is composed of quadratic elements, one moves nodes mediums (edges which touch the bottom of crack), with the quarter of these edges by bringing closer them to the bottom to crack. This is made possible by the keyword MODI_MAILLE (option 'NOEUD_QUART') order MODI_MAILLAGE [U4.23.04].
Case of the cracks nonwith a grid: Lprecision of the method has is sensitive to the choice of the zone of enrichment of method X-FEM (parameter RAYON_ENRI of DEFI_FISS_XFEM). In the ideal, the ray of enrichment and the maximum curvilinear X-coordinate ABSC_CURV_MAXI are about three times the size of stops minimal grid.
Performances: in the case X-FEM, calculations are rather consuming in time and memory if there are many points on the bottom of crack. The use of the keyword NB_POINT_FOND allows to limit postprocessing to a certain number of points équi-distribute along the bottom.

4.4 Standardisation, symmetries

4.4.1 2D plane constraints and plane deformations

In dimension 2 (plane constraints and plane deformations), the bottom of crack is tiny room to a point and the value $G(\theta)$ exit of the order CALC_G is independent of the choice of the field θ :

$$G=G(\theta) \quad \forall \theta \in \Theta$$

4.4.2 Axisymetry

Into axisymmetric it is necessary to standardize the value $G(\theta)$ obtained with *Code_Aster* for the options CALC_G, G_MAX and G_BILI :

$$G=\frac{1}{R}G(\theta)$$

where *R* is the distance from the bottom of crack to the axis of symmetry [R7.02.01 §2.4.4].

For the option CALC_K_G, values of G and of K provided in table result are directly the local values, it thus should not be standardized.

4.4.3 3D

In dimension 3, the value of $G(\theta)$ for a field θ data is such as:

$$g(\theta)=\int_{\Gamma_0} G(s)\theta(s).m(s)\,ds$$

By default, direction of the field θ in bottom of crack is the normal at the bottom of crack in the plan of the lips. By choosing a field θ unit in the vicinity of the bottom of crack, one has then:

$$\theta(s).\mathbf{m}(s)=1$$

and:

$$G(\theta)=\int_{\Gamma_0} G(s)\,ds$$

That is to say G the total rate of refund of energy, to have the value of G per unit of length, it is necessary to divide the value obtained by the length of the crack l :

$$G=\frac{1}{l}G(\theta)\,.$$

4.4.4 Symmetry of the model

It is possible to take into account a possible symmetry of with the problem dealt directly on the calculation of the rate of refund of energy G and factors of intensity of the constraints. See the keyword SYME in CALC_G [U4.82.03] and POST_K1_K2_K3 [U4.82.05].

5 Conclusions

The points to be retained are the following:

- The study of a crack in elasticity (linear or not) can be done on a crack with a grid as on a crack nonwith a grid. In both cases the grid must be sufficiently fine in the vicinity of the bottom of crack to collect the singularity of the constraints correctly.
- Calculations are possible on a nonplane crack, but the user must take care that it remains sufficiently regular so that the design assumptions are valid: one should not have a geometrical singularity on the bottom or the lips. Typically, calculation is licit for an axisymmetric crack, but not for a corner.
- In linear elasticity, several operators (CALC_G, POST_K1_K2_K3) and several methods (of extrapolation, smoothing,…) are available. It is **essential** to compare the results of the various methods and operators to make sure of good quality of the model. It is also recommended to evaluate the sensitivity of the result to the choice of the parameters of the operators (rays of integration) and to the refinement of the grid.

6 Documentation of *Code_Aster* relating to the breaking process

Reference documents:

[R7.02.01]	Rate of refund of energy in linear thermoelasticity
[R7.02.03]	Rate of refund of energy in nonlinear thermoelasticity
[R7.02.04]	Lagrangian representation of variation of field
[R7.02.05]	Calculation of the stress intensity factors in plane linear thermoelasticity
[R7.02.07]	Rate of refund of energy in thermo-élasto-plasticity
[R7.02.08]	Calculation of the coefficients of intensity of constraints by extrapolation of the field of displacements

Documents of Use:

[U4.82.01]	Operator DEFI_FOND_FISS
[U4.82.03]	Operator CALC_G
[U4.82.05]	Operator POST_K1_K2_K3
[U4.82.08]	Operator DEFI_FISS_XFEM

Case test of Validation:

		Fissure		Modélisation				Comport.				Option					
Numéro	Titre	Maillée	Non maillée	DP	CP	Axi	3D	Elas. Linéaire	Elas. Non linéaire	Elastoplastique	Grande Déf.	CALC_G	CALC_G_GLOB	CALC_K_G	POST_K1_K2_K	Autres	Particularités
hpla310	Fissure radiale dans un barreau soumis à un choc thermique	X				X		X				X					Thermique
hpla311	Fissure circulaire dans une sphère soumise à une température uniforme sur les lèvres	X				X		X				X		X			Thermique
hplp100	Plaque fissurée en thermoélasticite	X		X				X				X		X			Thermique ; Sensibilité
hplp101	Plaque fissurée en thermoélasticité	X			X			X			X	X		X			Thermique ; Sensibilité
hplp310	Fissure radial dans un cylindre épais sous pression et chargement thermique	X		X				X				X		X			Thermique ; contact
hplp311	Fissure au centre d'une plaque mince soumise à un flux thermique	X			X			X				X		X			Thermique
hplv102	G en thermoélasticité pour une fissure circulaire en milieu infini	X				X		X				X					Thermique
hplv103	G en thermoélasticité 3D pour une fissure circulaire	X					X	X				X	X	X			Thermique
hsna120	Propagation d'un réseau de fissure en fatigue thermomécanique	X				X		X							X		Thermique ; propagation
sdls114	Facteurs d'intensité des contraintes modaux	X			X		X	X							X	X	K_G_MODA
ssla310	Fissure radiale à l'interface d'un bimatériau	X				X		X				X					Bimatériau
ssla311	Fissure circulaire soumise à une charge annulaire	X				X		X				X					
sslp101	Plaque fissurée en traction	X			X			X				X		X			
sslp102	Calcul de G avec déformations initiales	X		X				X				X					Etat initial
sslp103	Calcul de K1 et K2 pour une plaque circulaire fissurée	X			X			X				X		X			Mode mixte
sslp310	Fissure pressurisée dans un domaine plan illimité	X			X			X						X			MACR_ELAS_MULT
sslp311	Fissure centrale dans une plaque à deux matériaux	X			X			X				X					Bimatériau
sslp313	Fissure oblique dans une plaque infinie en traction	X			X			X						X	X		Mode mixte ; Arlequin
sslp314	Fissure déviée à l'interface de 2 plaques élastiques	X			X			X				X		X			
sslp315	Propagation d'une fissure débouchante dans une plaque 2D		X	X				X						X	X		Propagation
sslv110	Fissure semi-elliptique dans un milieu infini	X	X				X	X				X	X	X	X		Sensibilité
sslv134	Fissure circulaire en milieu infini	X	X				X	X				X	X	X	X	X	G_BILI, G_GLOB, CALC_K_MAX
sslv310	Fissure semi-elliptique débouchante en peau interne dans un cylindre sous pression	X					X	X				X	X				
sslv311	Fiss en quart d'ellipse au coin d'un disque épais	X					X	X						X			
sslv312	Fissure elliptique perpendiculaire à l'interface entre deux matériaux	X					X	X				X	X				
sslv313	Tube sous pression fissuré	X	X				X	X				X		X	X		
sslv314	Propagation de fissure dans une plaque 3D en mode I pur		X				X	X						X	X		Propagation
sslv315	Propagation de fissure inclinée dans une plaque 3D		X				X	X						X	X		Propagation
ssnp102	Plaque entaillée en plasticité	X		X			X			X		X					
ssnp110	Fissure de bord en élastoplasticité dans une plaque rectangulaire	X	X	X	X				X			X					
ssnp311	Fissuration en mode II d'une éprouvette elastoplastique	X		X					X			X					
ssnp312	Fissure parallèle à une interface dans une CT bimétallique	X		X					X			X					
ssnv108	Eprouvette CT en non linéaire	X	X		X		X		X			X					
ssnv166	Cylindre fissuré sous chargement multiple	X					X	X						X	X		Contact
ssnv192	Eprouvette avec fissure centrale		X				X	X						X			
ssnv185	Fissure débouchante dans une plaque 3D	X	X				X	X				X		X	X		

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