Calculation of reconstituted signals and the matrix transfer transfer function

Summary

This document presents the theoretical bases having made it possible to set up the macro-order CALC_TRANSFERT [U4.53.51].

The latter makes it possible to calculate the matrix transfer function transfer between two points, but also to reconstitute the signals in a given point, knowing the signals in another point.

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1 Introduction

At the time of dynamic studies linear, the user can have to reconstitute a signal of excitation knowing the signals measured in another point of the structure. The purpose of this document is to present the theoretical elements which led to the creation of the macro-order `CALC_TRANSFERT` [U4.53.51]. In particular the equations which make it possible to determine the matrix transfer function transfer and those which make it possible to determine the reconstituted signals.

2 Theory

In this chapter, one places in a seismic example louse to present the problems where one reasons in absolute reference mark.

2.1 Principle general

At the time of an earthquake, the recorded signals are generally measured on a work. Thus, when one wishes to study the dynamic behavior of this work, the entry signal is an unknown factor of the digital model which should be determined (see figure 1).

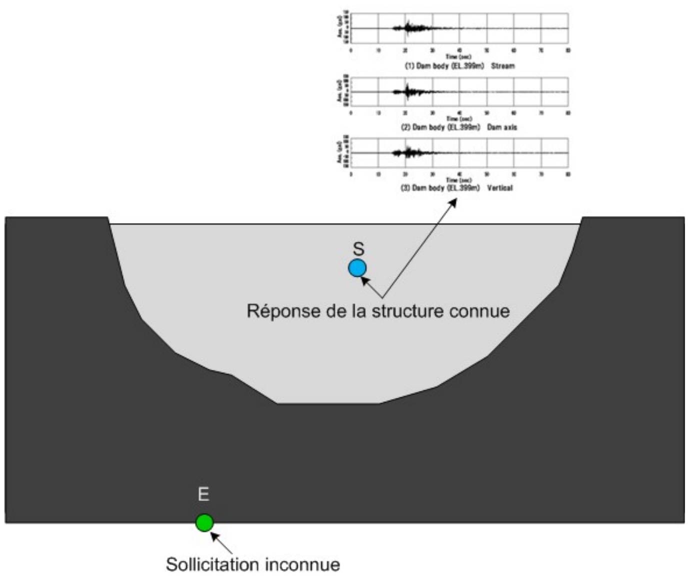


Figure 1 – Representation schematic of the problem

To reconstitute the entry signal knowing the output signal (generally measured signal), one places oneself in the frequential field and one seeks the transfer transfer function $\underline{\underline{H}}(\omega)$ between the two points of our system (see figure 2). The entrance point corresponds to the point E and the exit point at the point S on figure 1.

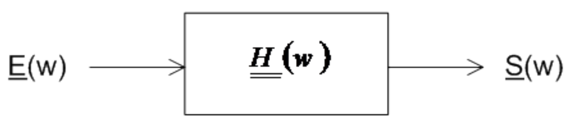


Figure 2 - Diagram of the transfer transfer function

From several dynamic calculations one-way $\gamma_i, avec\, i=x, y, z$, it is possible to reach the transfer transfer function by looking at the exit $\underline{S}(\omega)$ and the entry $\underline{E}(\omega)$.

Note:

> *DéterminationLes entry signals and of exit must be of comparable nature (ACCE/ ACCE or QUICKLY/QUICKLY or DEPL/DEPL.*
> *It is to recommend to use signals of the type white vibration in order to uniformly excite the structure on all the beach of frequency.*

Once the matrix transfer function transfer known, and knowing the output signal wished, it is possible to reconstitute the entry signal to be injected into the dynamic model to find the output signal by solving the following matric system:

$$\underline{E^{absolu}}(\omega)=\underline{\underline{H^{-1}}}(\omega)\,\underline{S^{absolu}}(\omega)$$

Where:

$\underline{S^{absolu}}(\omega)$	Vector containing the absolute accelerations measured at the exit point S of the structure at the time of the earthquake
$\underline{E^{absolu}}(\omega)$	Vector containing absolute accelerations to apply to the entrance point E to find the absolute accelerations measured at the exit point S
$\underline{\underline{H^{-1}}}(\omega)$	Matrix of opposite transfer transfer function

2.2 Determination of the matrix transfer function transfers $\underline{\underline{H}}(\omega)$

In this paragraph, one proposes to present the equations which make it possible to determine the matrix of transfer transfer function $\underline{\underline{H}}(\omega)$ in the three-dimensional case. To facilitate the setting in fact of the case, one deals with the problem in term of acceleration (this last can be also treated in term of displacement or speed).

That is to say E_i^{absolu} and S_i^{absolu} the absolute accelerations calculated respectively at the entrance points and of exit. One seeks to solve the sytème according to:

$$\underline{S^{absolu}}(\omega)=\underline{\underline{H}}(\omega)\,\underline{E^{absolu}}(\omega)$$

Where $\begin{cases} S_i^{absolu}=S_i^{relatif}+\gamma_i,\, i=x,y,z \\ E_i^{absolu}=E_i^{relatif}+\gamma_i,\, i=x,y,z \end{cases}$ and $\underline{\underline{H}}(\omega)=\begin{pmatrix} H_{xx} & H_{xy} & H_{xz} \\ H_{yx} & H_{yy} & H_{yz} \\ H_{zx} & H_{zy} & H_{zz} \end{pmatrix}$

To determine the 9 unknown factors of the problem, a system of 9 equations to 9 unknown factors must be solved. This system is determined by carrying out three calculations with an one-way loading according to directions X, there or Z in order to not to neglect possible coupling between directions.

<u>Harmonic calculation following X</u>

That is to say $X\_sortie_i^a, i=x,y,z$ the absolute acceleration calculated at the exit point and $X\_entree_i^a, i=x,y,z$ the absolute acceleration calculated at the entrance point for an one-way excitation γ_x.

The system (E1) to solve is the following:

$$(E1)\quad \begin{cases} X\_sortie_x^a = H_{xx} X\_entree_x^a + H_{xy} X\_entree_y^a + H_{xz} X\_entree_z^a \\ X\_sortie_y^a = H_{yx} X\_entree_x^a + H_{yy} X\_entree_y^a + H_{yz} X\_entree_z^a \\ X\_sortie_z^a = H_{zx} X\_entree_x^a + H_{zy} X\_entree_y^a + H_{zxz} X\_entree_z^a \end{cases}$$

<u>Harmonic calculation following Y</u>

That is to say $Y\_sortie_i^a, i=x,y,z$ the absolute acceleration calculated at the exit point and $Y\_entree_i^a, i=x,y,z$ the absolute acceleration calculated at the entrance point for an one-way excitation γ_y.

The system (E2) to solve is the following:

$$(E2)\quad \begin{cases} Y\_sortie_x^a = H_{xx} Y\_entree_x^a + H_{xy} Y\_entree_y^a + H_{xz} Y\_entree_z^a \\ Y\_sortie_y^a = H_{yx} Y\_entree_x^a + H_{yy} Y\_entree_y^a + H_{yz} Y\_entree_z^a \\ Y\_sortie_z^a = H_{zx} Y\_entree_x^a + H_{zy} Y\_entree_y^a + H_{zxz} Y\_entree_z^a \end{cases}$$

<u>Harmonic calculation following Z</u>

That is to say $Z\_sortie_i^a, i=x,y,z$ the absolute acceleration calculated at the exit point and $Z\_entree_i^a, i=x,y,z$ the absolute acceleration calculated at the entrance point for an one-way excitation γ_z.

The system (E3) to solve is the following:

$$(E3)\quad \begin{cases} Z\_sortie_x^a = H_{xx} Z\_entree_x^a + H_{xy} Z\_entree_y^a + H_{xz} Z\_entree_z^a \\ Z\_sortie_y^a = H_{yx} Z\_entree_x^a + H_{yy} Z\_entree_y^a + H_{yz} Z\_entree_z^a \\ Z\_sortie_z^a = H_{zx} Z\_entree_x^a + H_{zy} Z\_entree_y^a + H_{zxz} Z\_entree_z^a \end{cases}$$

By gathering the systems (E1), (E2) and (E3) in the form of a total system (E), one obtains a system of 9 equations to 9 unknown factors.

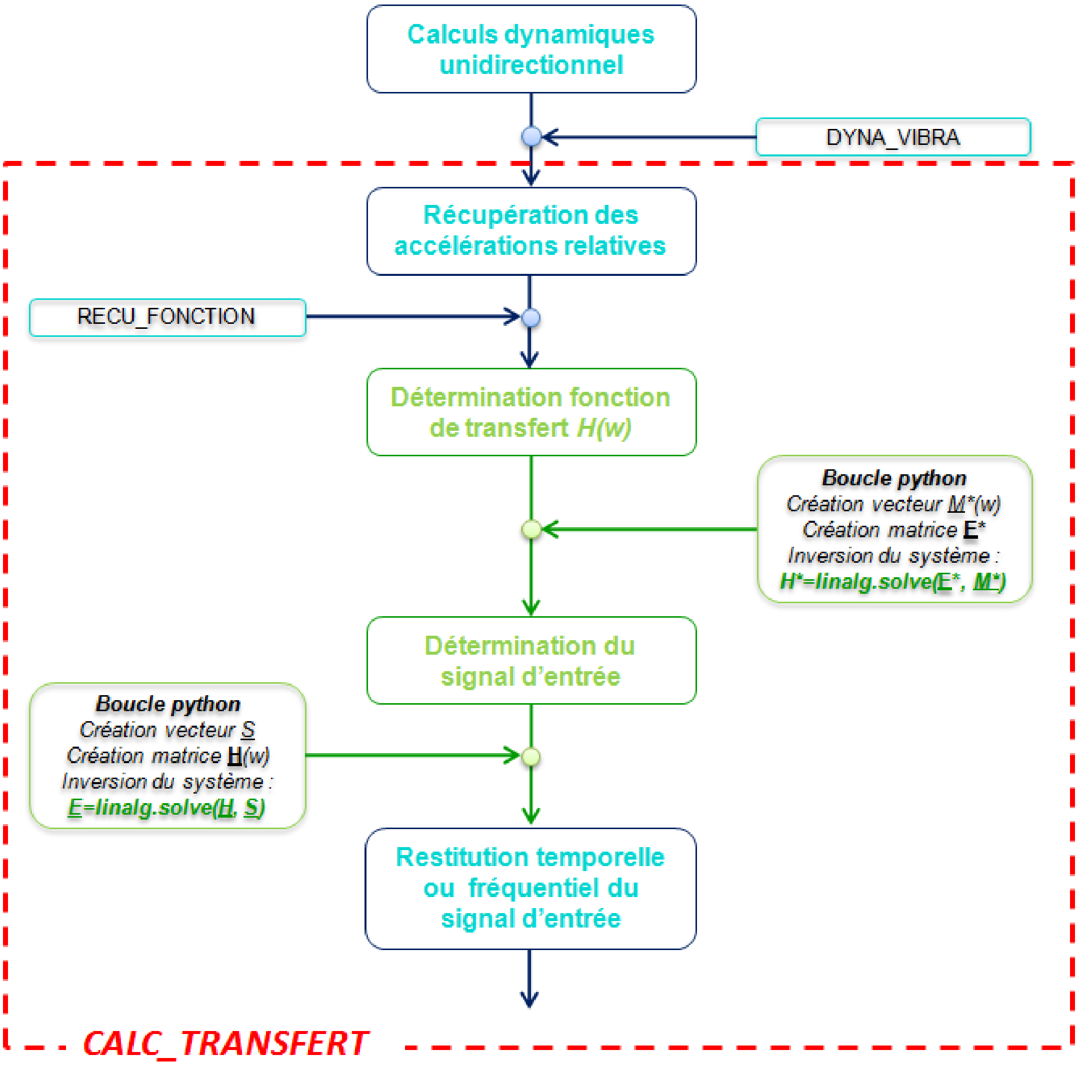
Terms H_{ij} are then determined by inversion of the matric system (E) via the operators of linear algebra of resolutions available in the library numpy of python,

2.3 Determination of the signal to be reconstituted

When terms $H_{ij}(\omega)$ matrix transfer function transfers are known, it is possible to reconstitute the entry signal knowing the output signal. With this intention it is enough to solve the following matric system:

$$\underline{E}_{reconstitué}^{absolu}(\omega)=\underline{\underline{H}}^{-1}(\omega)\underline{S}_{mesuré}^{absolu}(\omega)$$

3 Synoptic of resolution of the macro order



4 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
12.3	J, FOUQUÉ EDF-R&D/AMA	Initial text