Modeling of the rotors fissured by equivalent stiffness function of the swing angle

Summary:

This document describes a method of taking into account of the cracks in the rotors modelled in 1D.

It is based on theoretical developments described in the notes [1] and [2] and published in [3], establishing, under certain assumptions, equivalence between a nonlinear model 3D with calculation of the contact and a model of beam where nonthe linearity due to the crack is simulated by an equivalent stiffness concentrated between two elements of beam and nonlinear tabulée function of the angle of the bending moment compared to the crack. This method used, in the computer code of lines of trees CADYRO, was extended and validated compared to the experiment on the test bench EUROPE [4]. It is taken again in the operator `DYNA_VIBRA` of *Code_Aster*.

1 Introduction and objective

For reasons of performance and implementation practises the lines of trees in dynamics are not modelled by finite elements 3D but by elements of beam, which represent in a natural way the twinge of the rotors and which, in linear elasticity, approaches the real behavior of the system at a weak cost.

However one wishes to take into account in the calculation of the dynamics of the lines of trees the impact of a crack on the rotor. If calculation 3D, with fine modeling of the behavior of the crack by an algorithm of contact is conceptually possible, it is extremely demanding in computer's resources. This is why one models the behavior of the crack on the model beam usual of the line of trees by creating two nodes on the axis of the rotor fissured with the right of the crack and by introducing a “law of nonlinear behavior of crack” binding average rotations of the two nodes representing the lips of the crack to the bending moments applied to the beam.

The law of behavior of crack is tabulée starting from calculations 3D. It is shown that it is not necessary to traverse all the field of the couples (My, Mz) but that it is enough to a calculation for each angle of the bending moment compared to the crack.

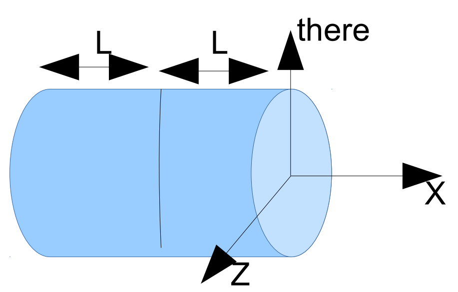
2 Assumptions

The assumptions used to build the model 1D of crack are:

- the rotor has an isotropic elastic behavior;
- the crack can be of an unspecified form but it contained in a section of beam;
- the lips of the crack are connected by a condition of contact without friction;
- the normal efforts and cutting-edges are neglected (under certain assumptions one can take into account the torsional stresses but within the framework of this element they are forgotten);
- the initial game of the crack is null;
- the behavior of the crack is quasi-static and the effects of inertia are negligible.

3 Reference mark and notations

By default the local reference mark of the beams in Code_Aster is the axis X. This convention is preserved.



The element of fissured rotor is characterized by:

- its length $2L$;
- its ray R ;
- the position of the fissured section (in the middle of the rotor)

The following mechanical characteristics are considered:

- Young modulus: E
- Inertia not fissured beam: I

One follows the classical description of the kinematics of the beams. The perpendicular sections of the beam remain right and their rotation is described by:

$$\theta = \begin{pmatrix} \theta_y(x) \\ \theta_z(x) \end{pmatrix}$$

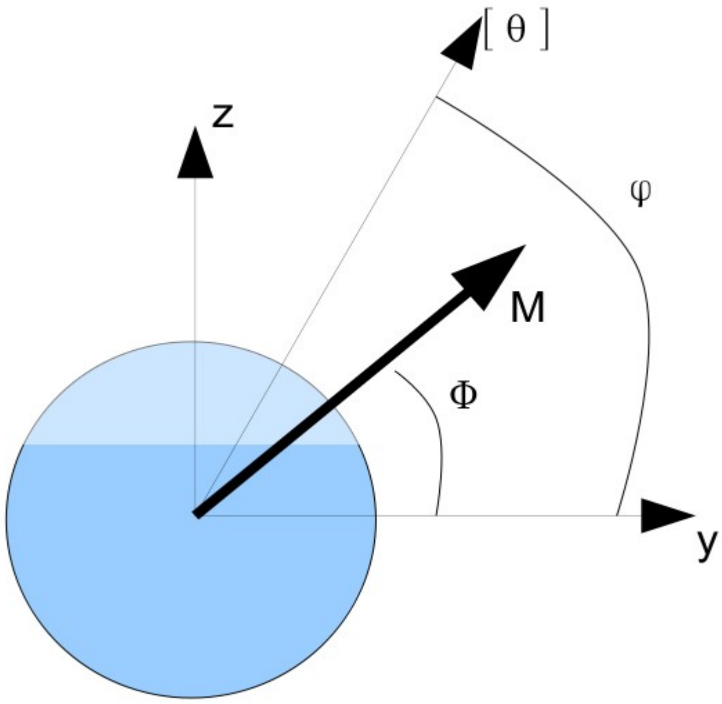
The beam is subjected to one bending moment:

$M = \begin{pmatrix} M_y \\ M_z \end{pmatrix}$, which can open or close the crack.

The deformation energy of the part of rotor fissured under the loading of inflection imposed is noted $W^f(M)$.
The deformation energy brought by the discrete element modelling the crack for a discontinuity of rotation $[\theta]$ is noted $W^d([\theta])$.

The description of the fissured section, though realised in the modeling of beam, requires however a certain attention, because the orientation of the answer to the level of the crack is not systematically the same one as that of the effort. It depends on the fissure shape.
Two angles are thus defined in the reference mark of the fissured section:

- Φ , orientation of the effort imposed in the revolving reference mark;
- φ , orientation of the answer in the revolving reference mark.

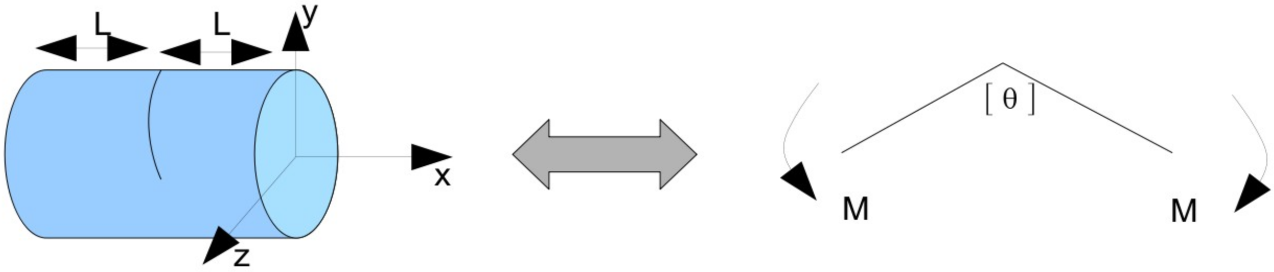


Lastly, one notes $s(M)$ the local flexibility of the element are equivalent 1D representing the crack and $k([\theta])$ its local stiffness.

4 Determination of the equivalent stiffness of the crack

4.1 Model are equivalent of crack

One seeks in this chapter to determine the law of behavior $M=f([\theta])$ connecting for the element of crack 1D the bending stresses M with the deformations expressed by the jump of rotation $[\theta]=\begin{pmatrix}[\theta_y(x)]\\ [\theta_z(x)]\end{pmatrix}$ lips of the crack represented by the two nodes borders of the crack.



4.2 Deformation energy

4.2.1 Formula of the deformation energy to imposed effort

In [1] one establishes that the energy of the model 3D into quasi-static is given by a formula of “Clapeyron”:

$$W^f(M_y, M_z)=\frac{1}{2}\left(M_y\theta_y(2\mathrm{L})+M_z\theta_z(2\mathrm{L})\right)$$

On the model finite element 3D, one can thanks to this formula calculate the energy of the system for all the possible couples (M_y, M_z) and to determine the equivalent stiffness $k(M_y, M_z)$ for the element 1D of beam fissured.

However to explore a space are equivalent to $\mathbb{R}^2$ would be too expensive for a practical application. The properties of the functional calculus of energy of the elastic problem with contact make it possible to reduce the problem to only one variable.

It is shown that W^f is convex and positively homogeneous of degree 2 compared to the moments applied:

$$\forall\lambda\geq 0,\quad W^f(\lambda M_y, \lambda M_z)=\lambda^2 W^f(M_y, M_z)$$

This property is interesting because she wants to say, in practice, that the zone of contact on the lips of the crack is independent of the amplitude M inflection. Energy is quadratic in M, as in the linear case. Its form depends only on the direction on (M_y, M_z). One can thus rewrite the function of energy with, for variables, the amplitude of the bending stress $\|M\|$ and its angle ϕ.

In the cylindrical reference mark the moment is written:

$M_y=\|M\|\cos\phi$
$M_z=\|M\|\sin\phi$
and the deformation energy: $W_f(M)=\frac{1}{2}\|M\|^2 S(\phi)$

The problem is reduced to the identification of $S(\phi)$ on the interval $[0,2\pi]$.

Energy is rewritten in a form which distinguishes the flexibility $S(\phi)$, brought by the beams (left healthy and fissured) and the flexibility $s(\phi)$, brought by the crack:

$$W^f(M)=\frac{L}{EI}\|M\|^2(1+s(\phi))$$

By calculating the deformation energy on a model finite elements 3D of the bar fissured (figure 1), one can determine $s(\phi)$.

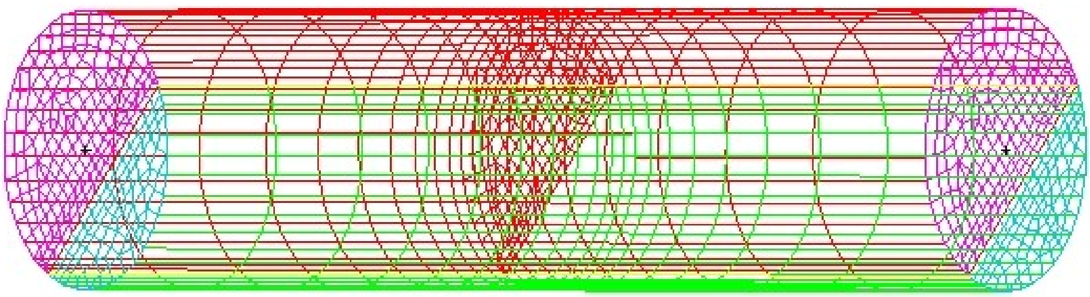


Illustration 1: Grid 3D of the fissured bar

4.2.2 Energy with imposed displacement

It now remains to establish the relation between energy and $[\theta]$. One uses for that energy with imposed displacement w^d. It has the same properties of convexity and homogeneity of degree 2 that energy with imposed effort.

While posing:

$$[\theta_x]=\|[\theta]\|\cos(\varphi)$$
$$[\theta_y]=\|[\theta]\|\sin(\varphi)$$

one obtains the following expression for the deformation energy:

$$w^d([\theta])=\frac{EI}{4\mathrm{L}}\|[\theta]\|^2 k(\varphi)$$

4.2.3 law of behavior of the fissured element

One obtains M according to $[\theta]$ by derivation of energy:

$$M_y=\frac{EI}{4\mathrm{L}}\frac{\partial(k(\varphi)([\theta_y]^2+[\theta_z]^2))}{\partial[\theta]_y}$$
$$M_z=\frac{EI}{4\mathrm{L}}\frac{\partial(k(\varphi)([\theta_y]^2+[\theta_z]^2))}{\partial[\theta]_z}$$
with $[\theta]=\sqrt{[\theta_y]^2+[\theta_z]^2}\begin{pmatrix}\cos(\varphi)\\ \sin(\varphi)\end{pmatrix}$

Some intermediate calculations lead to the following relations:

$$\begin{pmatrix} M_y \\ M_z \end{pmatrix} = \frac{EI}{2L} \begin{pmatrix} k(\varphi) & -\frac{1}{2}k'(\varphi) \\ \frac{1}{2}k'(\varphi) & k(\varphi) \end{pmatrix} \begin{pmatrix} [\theta_y(x)] \\ [\theta_z(x)] \end{pmatrix} \text{ where } \varphi = \arctan\left(\frac{[\theta_z(x)]}{[\theta_y(x)]}\right)$$

4.2.4 Relation between the rigidity of the crack and its aperture

Direction ϕ average angle of opening of the crack $[\theta]$ is not *a priori* pas la even as ϕ, that of the efforts of the moments M.

It is thus a question now of establishing a direct relationship between rigidity (or its reverse, flexibility) and the opening of the crack. It is established in [1] and [2].

By exploitation of the principle of complementary energy and thanks to the convexity of the deformation energy, one shows that:

$$w^d([\theta]) = \underset{M}{Sup}\left(M.[\theta] - \frac{L}{EI}\|M\|^2 s(\phi)\right)$$

The function $s(\phi)$ was established in the preceding paragraphs. A complementary mathematical treatment consists in applying to $s(\phi)$ a quadratic interpolation apart from the interval of nullity of $s(\phi)$ and an interpolation power at the boundaries of nullity of $s(\phi)$.

In addition, in the same way that energy at imposed time, energy with imposed deformation is convex and homogeneous of order 2. That implies that it does not depend on the standard on $[\theta]$, but only of its direction φ. One can thus limit oneself to identify $w^d([\theta])$ for $\|[\theta]\| = 1$.
Energy is written then:

$$w^d([\theta]) = \frac{EI}{4L} k(\varphi).$$

If λ is the amplitude of the moment applied and ϕ its direction, by identification one finds the formula giving the equivalent stiffness of crack:

$$k(\varphi) = \frac{4L}{EI} \underset{M=1}{Sup}\, \underset{\lambda \geq 0}{Sup}\left(\lambda M.[\theta] - \frac{L}{EI}\lambda^2 s(\phi)\right)$$

In practice that amounts finding the field $[\varphi_{1,} \varphi_2]$ what can have the direction of the crack then to maximize for all the angles of crack φ possible the following expression:

$$k(\varphi) = \underset{\phi \in [\varphi - \frac{\pi}{2,} \varphi + \frac{\pi}{2}]}{Sup} \frac{\cos^2(\phi - \varphi)}{s(\phi)}$$

The derivative $k'(\varphi) = \frac{d\,k(\varphi)}{d\,\varphi}$ is then obtained using a derivation by Finished Differences of $k(\varphi)$.

5 Bibliography

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3. Stéphane Andrieux, Christophe Varé “with 3D cracked beam model with unilateral contact. Application to rotors”; European Newspaper of Mechanics A/Solids 21 (2002) 793-810
4. Carlo Maria Stoisser, Sylvie Audebert “ With understanding theoretical, numerical and experimental approach for ace detection in power seedling rotating machinery “; Mechanical Systems and Signal Processing 22 (2008) 818-844

6 History of the versions of the document

Version Aster	Author (S) or contributor (S), organization	Description of the modifications
11.1	MR. TORKHANI	Initial version.