# Nonlocal modeling with gradients of nodal damage GVNO

**Summary**

One presents here nonlocal modeling to gradient of nodal damage entitled GVNO in Code_Aster, resulting from work of thesis of J. Beaurain. This modeling can be seen like a simplification of nonlocal modeling with gradient of internal variable GRAD_VARI.

Nonlocal modelings of type GVNO are available in 3D (3D_GVNO), axisymmetric (AXIS_GVNO) and plane deformations (D_PLAN_GVNO).

The use of GVNO is very simple, since it is enough to specify modeling X_GVNO in AFFE_MODELE, to specify a characteristic length under the keyword NON_LOCAL in DEFI_MATERIAU, and to check that the law of behavior which one wishes to use is quite available in nonlocal version.

One presents the writing and the digital processing of this model.

# Contents

# 1 Recall on the theory of the models with gradient

The models with gradient presented here were developed by E. Lorentz [bib1] in order to be able to describe the material behavior requested by strong gradients of the mechanical fields which appear in the damaged zones or in the vicinity of geometrical singularities. Indeed, in the case of strong gradients, the behavior of a material point is not independent any more of its entourage but depends on the behavior on its vicinity, from where the introduction of gradients into the models.
From a digital point of view, the calculation of a structure with a classical law of damage local shows that the damaged zone is always located on only one finite elements sleep and thus that the answer of the structure depends on the adopted grid: the models with gradient mitigate this problem.
In what follows, we make a short recall of this theory.

## 1.1 Construction of the models with gradient

This formulation is restricted with generalized standard materials (cf [Feeding-bottle 2]).

The models with gradient of internal variables consist in introducing the gradient of internal variables into a generalized standard formulation (cf [Feeding-bottle 2]).
That is to say $a$ an internal variable and $A$ its associated thermodynamic force, and is $\Delta(\dot{a})$ potential of dissipation. If it is considered that $\Delta$ also depends on the gradient on $\dot{a}$, $\Delta=\Delta(\dot{a},\nabla\dot{a})$, one cannot then write the principle of normality locally:
$A\in\partial\Delta(\dot{a},\nabla\dot{a})$

Indeed, such a writing would require the introduction of 2 variables locally independent $a$ and $a_\nabla$, to which 2 thermodynamic forces would be associated $A=-\frac{\partial\Phi}{\partial a}$, $A_\nabla=-\frac{\partial\Phi}{\partial a_\nabla}$ such as:
$(A,A_\nabla)\in\partial\Delta(\dot{a},\dot{a}_\nabla)$

If one calls F the threshold of elasticity associated with the potential $\Delta(\dot{a},\dot{a}_\nabla)$, the preceding equation is equivalent to:

$$\Delta(\dot{a},\dot{a}_\nabla)=\underset{(A,A_\nabla)\ /\ F(A,A_\nabla)\leq 0}{Sup}\left[\dot{a}\ A+\dot{a}_\nabla A_\nabla\right]$$

And one a:
$\dot{a}=\dot{\lambda}\frac{\partial F}{\partial A}$, $\dot{a}_\nabla=\dot{\lambda}\frac{\partial F}{\partial A_\nabla}$

The problem here is that the variables are not independent and are bound by the nonlocal constraint $a_\nabla=\nabla a$ so that one is not sure to check:
$\dot{a}_\nabla=\dot{\lambda}\frac{\partial F}{\partial A_\nabla}=\nabla\dot{a}$

One then proposes to forget the assumption of normal flow in each point of the structure while preserving the formalism of generalized standard materials but at the level of the structure, where the variables of state are now it field of deformation $\boldsymbol{\varepsilon}$ and it field internal variables $a$. The total free energy and the total potential of dissipation are thus defined:

$$F_\Phi(\boldsymbol{\varepsilon},a)=\int_\Omega\Phi(\boldsymbol{\varepsilon}(x),a(x),\nabla a(x))dx$$

$$D(\dot{a})=\int_{\Omega}\Delta(\dot{a}(x),\nabla\dot{a}(x))dx$$

Total potential of dissipation $D$ is now a function of field $\dot{a}$, and the writing $A\in\partial D(\dot{a})$ a direction begins again.

The relation of behavior generalized is written now:

$$\sigma=\frac{\partial F_{\Phi}}{\partial\varepsilon},\ A=-\frac{\partial F_{\Phi}}{\partial a},\ A\in\partial D(\dot{a})$$

## 1.2 Discretization in time

It will be supposed subsequently that the energy of the model regularized is the sum of the energy of the local model and an additional term depending only on the gradient of the internal variable which one regularizes:

$$F_{\Phi}(\boldsymbol{\varepsilon},a)=\int_{\Omega}\Phi^{loc}(\boldsymbol{\varepsilon}(x),a(x))+\Phi^{grad}(\nabla a(x))dx$$

This separation corresponds to the cases which we treat in Code_Aster, but it is possible to build models with gradient which do not correspond to this case. Contrary to what is made for modeling GRAD_VARI (nonlocal modeling with gradient of internal variables), also available in the code, one considers that the damage at the points of Gauss is the interpolation of the damage to the nodes. One thus avoids bringing back oneself to a mixed formulation.

While being based on the assumption of convexity compared to $(u,\alpha)$ potential $F_{\Phi}$ and while adopting a diagram of implicit Euler, the resolution of the equilibrium equations is brought back to a problem of minimization relating to the increments $(\Delta u,\Delta\alpha)$. The discretized problem is written then:

$$\min_{\Delta u}\min_{\Delta\alpha}\left(F_{\Phi}(\boldsymbol{\varepsilon}^{-}+\Delta\boldsymbol{\varepsilon},\alpha^{-}+\Delta\alpha)-W^{ext}\right)$$

where $W^{ext}$ represent the work of the external mechanical forces.

## 1.3 Space discretization by finite elements

To solve the problem of search for extrema, one carries out a space discretization by finite elements of the following sizes:

Displacement: $\boldsymbol{\varepsilon}(x)=\sum_{k\ noeuds}B_{k^u}(x)\,u_k$
Regularized damage: $\alpha(x)=\sum_{k\ noeuds}N_{k^{\alpha}}(x)\,\alpha_k$.
The gradient is expressed then thanks to the gradient of the functions of form: $\nabla\alpha(x)=\sum_{k\ noeuds}\nabla N_{k^{\alpha}}(x)\alpha_k$

where $N_{k^{\alpha}}$ are the functions of forms associated with the node $k$ for the field $\alpha$, $B_{k^u}$ are the functions of form of the deformations calculated starting from the derivative of the functions of form associated with displacements with the node $k$. One differentiates the functions from form according

to the field insofar as the degree of interpolation used can be different according to the size considered.

In the model finite elements with integration at the points of Gauss, the integral on the volume of the structure is evaluated by summation on the points of Gauss. Energy $F_\Phi$ is thus written for the problem spatially discretized:

$$F_\Phi(\boldsymbol{\varepsilon}, \alpha) = \sum_g \omega_g \underbrace{\left[\Phi^{loc}(\boldsymbol{\varepsilon}_g, \alpha_g) + \Phi^{grad}((\nabla\alpha)_g)\right]}_{\Phi_g^{nonlocal}(\boldsymbol{\varepsilon}_g, \alpha_g)}$$

where $\omega_g$ corresponds to the weight of the points of Gauss, and the index $g$ indicate that the field is evaluated at the points of Gauss starting from the values with the nodes:

$$\varepsilon_g = \sum_{k \text{ noeuds}} B_k^u(x_g) u_k$$

$$\alpha_g = \sum_{k \text{ noeuds}} N_{k^\alpha}(x_g)\, \alpha_k$$

$$(\nabla\alpha)_g = \sum_{k \text{ noeuds}} \nabla N_{k^\alpha}(x_g)\, \alpha_k$$

## 1.4 Calculation of the internal forces

The mechanical equilibrium equation as well as the equation of regularization are solved with the local nodes by search for extremum:

$$\min_{\Delta u} \min_{\Delta\alpha} \left( F_\Phi(\varepsilon^- + \Delta\varepsilon, \alpha^- + \Delta\alpha) - W^{ext} \right)$$

One locally does not solve the law of behavior, like that is done with modeling in gradient of internal variables. The criterion admits from now on a nodal resolution. The objects with which one works are the state of balance and the criterion of the selected formulation defined by the derivative first of energy compared to displacements and the damage.

For more simplicity, we leave side the external mechanical forces which are treated except for in Code_Aster. Internal forces associated with the nodal variables of the node $n$ $(u_n, \alpha_n)$ have as expressions:

$$F^u|_n = \frac{\partial F_\Phi}{\partial u_n} = \sum_g \omega_g \frac{\partial \Phi^{loc}}{\partial u_n} = \sum_g \omega_g \frac{\partial \Phi^{loc}}{\partial \varepsilon_g} : \frac{\partial \varepsilon_g}{\partial u_n} = \underbrace{\sum_g \omega_g \sigma_g : B_n^u}_{\int_\Omega B^T \sigma d\Omega}$$

$$F^\alpha|_n = \frac{\partial F_\Phi}{\partial \alpha_n} = \sum_g \omega_g \left[ \frac{\partial \Phi^{grad}}{\partial (\nabla\alpha)_g} (\nabla N)_n^\alpha + \frac{\partial \Phi^{loc}}{\partial \alpha_g} N_n^\alpha \right]$$

The tangent matrix is written in the following way:

$$K=\begin{pmatrix} \frac{\partial F^u}{\partial u} & \frac{\partial F^u}{\partial \alpha} \\ \frac{\partial F^\alpha}{\partial u} & \frac{\partial F^\alpha}{\partial \alpha} \end{pmatrix}$$

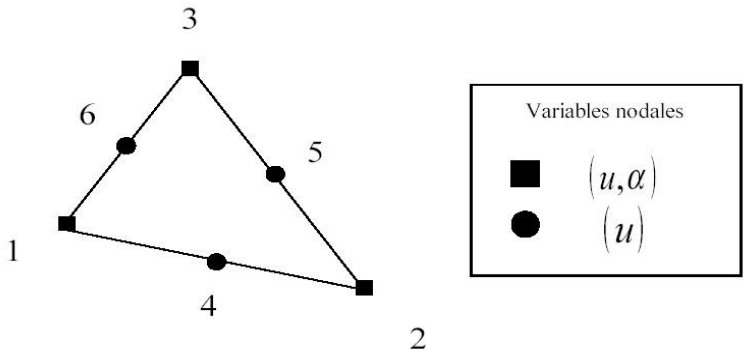
The framework of generalized standard materials in which our modeling fits ensures the symmetry of the tangent matrix. It is thus enough to calculate the lower triangular matrix:

$$\frac{\partial F^u|_n}{\partial u_m}=\sum_g \omega_g B^u_m : \frac{\partial \sigma_g}{\partial \varepsilon_g} : B^u_n$$
$$\frac{\partial F^u|_n}{\partial \alpha_m}=\sum_g \omega_g N^\alpha_m \frac{\partial \sigma_g}{\partial \alpha_g} B^u_n$$
$$\frac{\partial F^\alpha|_n}{\partial \alpha_m}=\sum_g \omega_g \left[ (\nabla N)^\alpha_m \frac{\partial^2 \Phi^{grad}}{\partial^2 (\nabla \alpha)_g} (\nabla N)^\alpha_n + N^\alpha_m \frac{\partial^2 \Phi^{loc}}{\partial^2 \alpha_g} N^\alpha_n \right]$$

# 2 Choice of the finite elements

One is in the presence of two nodal unknown factors: displacements $u$ and the damage $\alpha$.

Functions of forms are considered $P^2$ for $u$ and $P^1$ for $\alpha$.
The quadratic elements, TRIA6 and QUAD8 for the 2D, TETRA10, PENTA15 and HEXA20 for the 3D, were developed. The components of displacement are assigned to all the nodes of the element whereas the components of damage are affected only with the nodes tops. For more clearness, element TRIA6 is represented below:




One uses the families of points of Gauss of the corresponding elements linear, which results in an under-integration on displacements. The use of the families of points of Gauss of the quadratic elements would imply an on-integration for $\alpha$ causing inopportune oscillations.

# 3 Methods of resolution

If one excludes the unilateral condition from irreversibility of the damage, the algorithm of Newton such as it is currently programmed in the code is enough with the resolution. In our case, one must take account of the degradation of material. Concerning modeling GRAD_VARI, this constraint is managed locally, in each point of Gauss, by projection on the old value if it is about a behaviour of discharge. The resolution of the criterion not being more local with GVNO, it is not possible any more to carry out this kind of projection and one must impose the irreversibility at the total level.

Two methods of resolution can then be under consideration with this modeling. It is possible to work with a classical Newton, by ensuring the projection of the negative increments of damage. Maybe with an algorithm "secant" which consists in putting at 0 the extra-diagonal blocks and thus to consider the variables of independent displacement and damage. What is equivalent to the algorithm of minimization alternated in the case of quadratic laws of behavior in displacement and damage. One ensures this time the irreversibility by projection of the negative increments of damage.

In both cases, the resolution is carried out in three stages:

1) One is interested initially in the increments of damage to each iteration. When the increment is negative, one projects it to 0, of kind to respect the irreversibility.
2) One builds the matrix of rigidity (only diagonal blocks in the case of the secant) and the forces internal.
3) One definite residues of damage by the relation of complementarity:

$$R_n^{\alpha} = F^{\alpha}|_n \Delta \alpha_n$$

With respectively: $F^{\alpha}|_n \geq 0$ and $\Delta \alpha_n \geq 0$

That implies to look at the sign of the internal forces associated with the degrees of freedom of damage projected at stage 1. Indeed, if that Ci is negative one does not respect the whole of the imposed conditions and one must keep the internal force like residue. If that Ci is positive the residue is then null since the increment is null.

More precisely, the method of resolution, in the case of the secant (diagonal matrix per blocks), can arise in the following way:

Itération n :

$\mathbf{u}^-, \boldsymbol{\alpha}^-, \Delta\mathbf{u}^{\mathbf{n}}, \Delta\boldsymbol{\alpha}^{\mathbf{n}}, \mathbf{Vect} = \mathbf{0}$ (vecteur indicateur)

CALCUL DE LA NOUVELLE DIRECTION DE DESCENTE

IF ( $\Delta\alpha_i^n \leq 0$ ) THEN (i-ème terme)

$\mathbf{Vect}(\alpha_i) = 1$

$\Delta\alpha_i^n = 0$ (projection de l'incrément d'endommagement)

ENDIF

DEBUT BOUCLE SUR LES ELEMENTS

$$\mathbf{K} = \begin{bmatrix} \mathbf{K_{uu}}(\boldsymbol{\alpha}^- + \Delta\boldsymbol{\alpha}^{\mathbf{n}}) & 0 \\ 0 & \mathbf{K_{\alpha\alpha}}(\mathbf{u}^- + \Delta\mathbf{u}^{\mathbf{n}}) \end{bmatrix}, \mathbf{F} = \begin{pmatrix} \mathbf{F_u} \\ \mathbf{F_\alpha} \end{pmatrix}$$

IF ( $\Delta\alpha_i^n = 0$ ) THEN

$\mathbf{K}^{\mathbf{n}}_{(\alpha_i,\alpha_k)} = \delta_{i,k}$ (nécessaire pour l'inversion dans la résolution)

ENDIF

FIN BOUCLE SUR LES ELEMENTS

ASSEMBLAGE DE LA MATRICE GLOBALE ET DU VECTEUR RESIDU

IF ($\mathbf{Vect}(\alpha_i) \# 0$) THEN

IF ( $\mathbf{F}^{\mathbf{n}}_{\alpha_i} \geq 0$ ) THEN (l'endommagement cherche encore à diminuer)

$\mathbf{F}^{\mathbf{n}}_{\alpha_i} = 0$ (Afin de converger, comme $\dot{\alpha}_i = 0$)

ELSE (il faut laisser l'endommagement évoluer)

ENDIF

ENDIF

RESOLUTION

IF (RESIDU < CRITERE) THEN (on a convergé, fin du pas de temps)

$\mathbf{u}^- = \mathbf{u}^- + \Delta\mathbf{u}^{\mathbf{n}}$

$\boldsymbol{\alpha}^- = \boldsymbol{\alpha}^- + \Delta\boldsymbol{\alpha}^{\mathbf{n}}$

ELSE (on calcule les incréments pour l'itération suivante)

n = n +1, $\Delta\mathbf{u}^{\mathbf{n}} = \Delta\mathbf{u}^{\mathbf{n}-1} + \delta\mathbf{u}^{\mathbf{n}-1}$

$\Delta\boldsymbol{\alpha}^{\mathbf{n}} = \Delta\boldsymbol{\alpha}^{\mathbf{n}-1} + \delta\boldsymbol{\alpha}^{\mathbf{n}-1}$

ENDIF

# 4 Modelings available

These various elements are used in three types of modelings:

Calculation 2D in plane deformations: D_PLAN_GVNO
Calculation 2D into axisymmetric: AXIS_GVNO
Calculation 3D: 3D_GVNO

The mode forced plane is not yet available.

# 5 Laws of behavior available with modelings GVNO

The law of behavior currently available in the nonlocal version to gradient of nodal damage is the following one:

ENDO_CARRE Law of quadratic behavior of damage regularized

ENDO_CARRE (cf [R5.03.26])

# 6 The Councils/Procedure for the implementation of a new law of behavior to gradients of damage

The addition of a new law of behavior is very simple. It is just enough to inform the derivative first and calculated seconds locally and independently of the part in gradient which is managed in a generic way.

# 7 The Councils on the use of GVNO

It is preferable to use modeling GVNO with the traditional method of Newton in the case of calculations of damage which do not have strong instabilities (snap-back). Indeed, the method of Newton is faster but it does not make it possible to cross the snap-backs without the use of an additional algorithm, standard piloting, which was not developed yet here. In the contrary case, it is preferable to use the "secant" method, by starting one PAS_MINI_ELAS sufficient large to make sure that one checks the condition with each step of time. This method allowing to cross instabilities and thus always to converge towards a solution. However, the iteration count necessary to convergence can be rather important (ex: 800 iterations). It is thus necessary to authorize a maximum number of important iterations in the command file (approximately 1200). The speed of convergence depends on the law used and also on the temporal discretization. Steps of too large times can lead to a well too difficult convergence. This method is very effective for the quadratic laws of behavior in damage and displacements since one works then with an algorithm of alternate minimization robust.

# 8 Features and checks

Inventory of the tests in support to the development:

| Case test | Description |
|---|---|
| SSNP307a | test of D_PLAN_GVNO (cf [V6.03.307]) |
| SSNA119a | test of AXIS_GVNO (cf [V6.01.119]) |
| SSNV220a | test of 3D_GVNO (cf [V6.04.220]) |

# 9 Bibliography

[1] LORENTZ E.: "Laws of behavior to gradients of internal variables: construction, variational formulation and numeric work", Doctorate implementation of the university Paris 6, April 27th, 1999.

[2] HALPHEN B., NGUYEN Q.S.: "On generalized standard materials", Newspaper of Mechanics, vol. 14, NR$^{O}$ 1.1975.

# 10 Description of the versions of the document

| Version Aster | Author (S) Organization (S) | Description of the modifications |
|---|---|---|
| 10. | J.BEAURAIN, K.KAZYMYRENKO EDF-R&D/AMA | Initial text |