Law of damage regularized `ENDO_SCALAIRE`

Summary:

This document describes the model of elastic behavior fragile `ENDO_SCALAIRE` available only for nonlocal modeling with gradient of damage `GRAD_VARI`. The damage is modelled in a scalar way; the loadings in compression and traction are not distinguished. Unlike the other laws of damage introduced previously, the latter behaves in a regular way (not snap-back, lengthening finished with the rupture) at least in the unidimensional cases.

1 Scope of application

The law ENDO_SCALAIRE return in a broad family of the laws of damage (see for example [R5.03.18]). It aims in particular at modelling a fragile elastic behavior in nonlocal version (GRAD_VARI [R5.04.01]) so that its behavior at least in the unidimensional cases is regular. The parameters of the law were selected to ensure at the same time the absence of snap-back in the answer force-displacement, as well as the finished lengthening of the bar 1D to the rupture. This last property distinguishes it from the law ENDO_FRAGILE [R5.03.18] in nonlocal version, ENDO_SCALAIRE is more regular. The local version of the law is not implemented, because it is equivalent to that of the law ENDO_FRAGILE with a change close to parameters. The modelled material is elastic isotropic. Its rigidity can decrease in an irreversible way when the deformation energy becomes important, without distinguishing traction from compression. The width of the bands of localization is controlled by a parameter material, indicated in the operator DEFI_MATERIAU under the keyword C_GRAD_VARI keyword factor NON_LOCAL [U4.43.01].
The piloting of the type PRED_ELAS [R5.03.80] seems the mode of control of the level of the most suitable loading.

2 Variational formulation of the problem of damage

2.1 Case of a generic law

Two equivalent approaches can be used to describe the process of damage of a fragile isotropic material. On a side it is possible to derive the law from damage within the framework of generalized standard description. In this case it is necessary to define a free energy of the system, like potential of dissipation. The rule of flow then established the evolution of the internal variables.
As for the description of damage one needs only a scalar variable, preceding description is simplified and been able to be brought back towards a variational problem under constraint of increase in damage [bib2].
To define a law of behavior in gradient of damage [R5.04.01] it is thus enough to express the density of total free energy (elastique+dissipation) according to tensor to deformation $\boldsymbol{\varepsilon}$ and of variable of damage $0 \leqslant a \leqslant 1$. The space distribution of the damage is given then by a field $a(x)$. Density of free energy presents itself in general in the following form:

$$\Phi(\boldsymbol{\varepsilon}, a) = A(a) w(\boldsymbol{\varepsilon}) + \omega(a) + c/2 (\nabla a)^2 \quad \textbf{éq 2.1-1}$$

Here c is the parameter of nonlocality (C_GRAD_VARI) $w(\boldsymbol{\varepsilon})$ the elastic deformation energy, $\omega(a)$ the energy of dissipation and $A(a)$ the function of rigidity. $a=0$ corresponds to healthy material and $a=1$ corresponds to material completely damaged: $A(1)=0, A(0)=1$. The problem of evolution is from now on a simple problem of minimization of free energy of Helmholtz $F \equiv \int \Phi(\boldsymbol{\varepsilon}, a) d\Omega$ under constraint $\dot{a} \geq 0$ [1].

$$min_{(\boldsymbol{\varepsilon}, a)} F(\boldsymbol{\varepsilon}, a), \text{ où } \qquad F(\boldsymbol{\varepsilon}, a) = \int \left[A(a) \boldsymbol{\varepsilon} : \boldsymbol{E} : \boldsymbol{\varepsilon} + \omega(a) + c/2 (\nabla a)^2 \right] d\Omega$$

where one replaced $w(\boldsymbol{\varepsilon}) = \boldsymbol{\varepsilon} : \boldsymbol{E} : \boldsymbol{\varepsilon}/2$ by using the definition of the tensor of Hooke. Two equations derive from the variational problem of minimization: $\delta F(\boldsymbol{\varepsilon}, a)/\delta \boldsymbol{\varepsilon} = 0$ [2] and $\delta F(\boldsymbol{\varepsilon}, a)/\delta a \geq 0$. The inequality in the second equation is related to the presence of imposed constraint. These two equations

1 One notes by ∇a the space derivative of the field of damage and by $\dot{a}$ that related to the temporal evolution

2 $\delta F(\boldsymbol{\varepsilon}, a)/\delta \boldsymbol{\varepsilon}$ is the variational derivative partial according to the direction of the space field $\boldsymbol{\varepsilon}(x)$, field $a(x)$ remaining fixed.

must be satisfied everywhere in the field with integration Ω. They are supplemented by an equation of coherence of Kuhn-Tucker $\dot{a}\cdot\delta F(\boldsymbol{\varepsilon},a)/\delta a=0$. On the edges $\partial\Omega$ we obtain an additional condition of normality $\nabla a\cdot\boldsymbol{n}=0$, where $\boldsymbol{n}$ is vector-normal. Finally the variable of damage and its gradient must be continuous at interior of the field of integration to carry out the minimum of functional calculus in question (see [bib2,4] for more details).

2.2 Relations of behavior

The link between the variational formulation and the usual laws of evolution is direct. The state of material is characterized by the deformation $\boldsymbol{\varepsilon}$ and the damage a, ranging between 0 and 1. The relation stress-strain is defined, which remains elastic, and rigidity is affected by the damage:

$$\boldsymbol{\sigma}=\delta F(\boldsymbol{\varepsilon},a)/\delta\boldsymbol{\varepsilon}=A(a)\boldsymbol{E}:\boldsymbol{\varepsilon}$$ **éq 2.2-1**

with $\boldsymbol{E}$ the tensor of Hooke. The evolution of the damage, always increasing, is controlled by the following function threshold:

$$f(\boldsymbol{\varepsilon},a)=-\delta\Phi(\varepsilon,a)/\delta a=-\frac{1}{2}A'(a)\boldsymbol{\varepsilon}:\boldsymbol{E}:\boldsymbol{\varepsilon}-\omega'(a)+c\,\Delta a$$ **éq 2.2-2**

The condition of coherence takes its usual form then:

$$f(\boldsymbol{\varepsilon},a)\le 0 \quad \dot{a}\ge 0 \quad \dot{a}\,f(\boldsymbol{\varepsilon},a)=0$$ **éq 2.2-3**

Two characteristics of this formulation are noted. Firstly, the function threshold is not-local because of the presence of the Laplacian of damage. Then, the absence of condition of flow is justified by the double role of the damage a, on a side it is presented in the form of an internal variable of evolution, other side it fulfills the mission of the parameter of Lagrange $\lambda\equiv a$.
One sees also the advantage of presentation of the laws of damage in their variational form. It is enough to describe the density of total free energy (éq.2.1-1), which includes dissipation, to define the law of evolution completely.

2.3 Identification of the parameters pour the law ENDO_SCALAIRE

In the law ENDO_SCALAIRE the functions of rigidity and dissipation are selected as follows:

$$\omega(a)=ka,\quad A(a)=\left(\frac{1-a}{1+\gamma a}\right)^2$$

The parameters of this law of behavior are then five. On the one hand, the Young modulus E and the Poisson's ratio ν who determine the tensor of Hooke by:

$$\mathbf{E}^{-1}\cdot\boldsymbol{\sigma}=\frac{1+\nu}{E}\boldsymbol{\sigma}-\frac{\nu}{E}(\mathrm{tr}\,\boldsymbol{\sigma})\,\mathbf{Id}$$ **éq 2.2-1**

In addition, k,γ,c who define the lenitive behavior, as well as the width characteristic of the band of damage. The latter can be readjusted with the macroscopic parameters starting from the unidimensional model, which admits an semi-analytical solution (feeding-bottle 6.7). By noting the constraint with the peak by σ_y, the energy of the rupture of Griffith by G_f and zone damaged with the rupture cuts it by D one obtains:

$$k=\frac{3G_f}{4D},\quad c=\frac{3}{8}DG_f,\quad \gamma=\frac{3EG_f}{4\sigma_y^2 D}-1$$

The digital tests showed that to avoid the presence of snap-back in the answer force-displacement in 1D, it would be necessary to have $\gamma\ge 2.8$. For this reason the choice was made to simplify the entry of the data of the model, one not informs the complete game of macroscopic parameters σ_y, G_f, D, but directly parameters of the model γ, c and the constraint with the peak σ_y, given under the keywords factors ENDO_SCALAIRE (GAMMA, SY) and NON_LOCAL (C_GRAD_VARI) of the operator DEFI_MATERIAU. As for E and ν, they are given classically under the keyword factor ELAS or ELAS_FO. The reasoning which follows, is valid in a strict sense only for modeling 1D, but can be useful for the not informed users. If parameters E, ν, G_f, σ_y are a priori defined, the user can vary the parameter D in order to satisfy the condition with absence of snap-backs local $\gamma \geq 2.8$. It must make sure thereafter that the size of the system considered is higher than the bandwidth of damage D.

Example of concrete in traction	$E=30\text{GPa},\ \nu=0.2$ $G_f=100\text{N/m}$ $\sigma_y=3\text{MPa}$	ELAS(E=3e10,NU=0.2) ENDO_SCALAIRE(GAMMA=1/(4D)−1,SY=3e6) NON_LOCAL(C_GRAD_VARI=37.5D)

The bandwidth of damage is to be chosen while respecting $\gamma \geq 2.8$ <=> $D \leq 66\text{mm}$

2.4 Integration of the law of behavior locally

We present here the method of integration of the law ENDO_SCALAIRE in its local version ($c=0$), so that the user can make a generalization for the case not-room, which it is generic and rests entirely on the algorithm presented in Doc. [R5.04.01].

Temporal discretization of the equations [éq 2.2-1] with [éq 2.1-3] on a step of time $[t^- \ t]$ is realized by a diagram of implicit Euler. To integrate in time the law of behavior consists in determining the state of constraint and damage of the solution of the following nonlinear system:

$$\boldsymbol{\sigma}=A(a)\boldsymbol{E}:\boldsymbol{\varepsilon} \qquad \text{éq 2.4-1}$$

$$f_{\text{loc}}(\varepsilon, a)\leq 0 \quad a-a^-\geq 0 \quad (a-a^-)\cdot f_{loc}(\varepsilon, a)=0 \qquad \text{éq 2.4-2}$$

where the variables without indices correspond to the step of final time t, such as for example the deformation $\boldsymbol{\varepsilon}$; the state of material at the beginning of the step of time $(\boldsymbol{\varepsilon}^-, a^-)$ is indicated by the index "-". The local function threshold is given by (éq. 2.2-2):
$f_{\text{loc}}(\boldsymbol{\varepsilon}, a)=(1+\gamma)\dfrac{(1-a)}{(1+\gamma a)^3}\boldsymbol{\varepsilon}:\boldsymbol{E}:\boldsymbol{\varepsilon}-k$

A method of resolution was proposed by [bib3]. It starts by examining the solution without evolution of the damage (also called elastic test) then, if necessary, carries out a correction to check the condition of coherence. In this case, the existence and the unicity of the solution guarantee the good performance of the method. Let us consider the elastic test:

$$a=a^- \text{ solution if } f^{\text{el}}(\boldsymbol{\varepsilon})\equiv f_{\text{loc}}(\boldsymbol{\varepsilon},\ a^-)\leq 0 \qquad \text{éq 2.4-3}$$

In the contrary case, the damage is obtained while solving $f_{\text{loc}}(\boldsymbol{\varepsilon},\ a)=0$ (polynomial of order 3).

$$(1-a)(1+\gamma)\boldsymbol{\varepsilon}:\boldsymbol{E}:\boldsymbol{\varepsilon}/k=(1+\gamma a)^3 \qquad \text{éq 2.4-4}$$

It is the largest root which is selected among the three existing.

It still remains to be made sure that the damage does not exceed value 1. In fact, when $a=1$, the rigidity of the material point considered is cancelled $A(1)=0$. Insofar as no technique of suppression of finite elements "broken" is put in work (technical possibly delicate when the finite elements have several points of Gauss), of the worthless pivots can appear in the matrix of rigidity. This is why one introduces a digital threshold of elastic residual rigidity for the tangent matrix, which can be indicated under the keyword factor COEF_RIGI_MINI of the operator DEFI_MATERIAU. This value without dimension is a multiplying coefficient of elastic module of an isotropic linear model. To preserve a reasonable conditioning of the matrix of rigidity, the value by default is chosen $min\, A(a)=10^{-5}$.

An indicator χ , arranged in the second internal variable, the behavior specifies then during the step of current time:

- $\chi=0$ elastic behavior (deformation energy lower than the threshold)
- $\chi=1$ evolution of the damage
- $\chi=2$ saturated damage ($a=1$).
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2.5 Integration of the law of behavior in nonlocal

We present here only the method of integration of the law ENDO_SCALAIRE in its local version ($c=0$), because generalization for the case not-room is generic and rests entirely on the algorithm presented in Doc. [R5.04.01]. It is noted that for the nonlocal version the function threshold is shifted, we thus obtain a polynomial of order 4 to solve. As for the constraint, it is given by [éq 2.4-1] in all the cases.

2.6 Description of the internal variables

The internal variables are three:

- $VI(1)$ damage a
- $VI(2)$ indicator χ
- $VI(3)$ residual rigidity $1-A(a)$

3 Piloting by elastic prediction

The piloting of the type PRED_ELAS standard controls the intensity of the loading to satisfy a certain equation related to the value with the function threshold f^{el} during the elastic test [bib5]. Consequently, only the points where the damage is not saturated are taken into account. The algorithm which deals with this mode of piloting, cf [R5.03.80], requires the resolution of each one of these points of Gauss of the following scalar equation in which $\Delta\tau$ is a data and η the unknown factor:

$$f^{el}(\boldsymbol{\varepsilon}_{\text{impo}}+\eta\,\boldsymbol{\varepsilon}_{\text{pilo}},a^{-})=\Delta\tau \quad \textbf{éq 3-1}$$

Let us note that this equation is modified for piloting PRED_ELAS in ENDO_SCALAIRE in order to have the parameter $\Delta\tau$ who corresponds to the increment of damage which one seeks to obtain for at least a point of the structure. One then does not seek any more one parameter of piloting η who makes leave the criterion a value $\Delta\tau$ with the damage resulting from the step of previous time (cf Eq 3-1), but a parameter η who brings back for us on the criterion with a damage increased by $\Delta\tau$:

$$f^{el}(\boldsymbol{\varepsilon}_{\text{impo}}+\eta\,\boldsymbol{\varepsilon}_{\text{pilo}},a^{-})=\Delta\tau \quad \Rightarrow \quad f^{el}(\boldsymbol{\varepsilon}_{\text{impo}}+\eta\,\boldsymbol{\varepsilon}_{\text{pilo}},a^{-}+\Delta\tau)=0 \quad \textbf{éq 3-2}$$

4 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
10.0	K.KAZYMYRENKO, E.LORENTZ, S.CUVILLIEZ EDF-R&D/AMA	Initial text

10.2	K.KAZYMYRENKO, EDF-R&D/AMA	Minor corrections of the notations

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