Model of great deformations GDEF_LOG

Summary:

This document presents a hypoelastic formulation of great deformations for the laws of behavior of the type von Mises called GDEF_LOG in Code_Aster.
This model, due to C.Miehe, N.Appel and M.Lambrecht [13] is a model great deformations based to a logarithmic curve measure, with a tensor of constraints in duality individual. It is valid whatever the behavior in small deformations and has the advantage of providing a symmetrical tangent matrix. No modification of the internal variables kinematic is necessary. It pemet an integration incrémentalement objectifies laws of behavior (like the model SIMO_MIEHE). However, like all the hypoelastic laws, the laws of behavior in any rigour are limited to the weak elastic strain .

One illustrates in this document the capacities of this model and the advantages compared to the approximation PETIT_REAC.

Contents

1 Introduction

Most laws of behavior of Code_Aster are usable under the assumption of the small disturbances (HP), which makes it possible to confuse the geometrical configurations initial and current. However, when the deformations become important (one in general fixes the limit at 5%), this assumption is not checked any more. The concepts of particulate and partial derivative are then different, and of this fact the laws of behavior formulated incrémentalement lose their objective character (independence of the mechanical state according to the observer); a tedious consequence is the possible evolution of the constraints for movement of a rigid, contrary body with physics.

In order to find objectivity, essential thus to guarantee a good reliability of the result, a strategy great deformations is possible. The object of this document is to present the formalism set up in Code_Aster to treat the laws of behavior with work hardenings isotropic and kinematic and criterion of Von Mises.

The formalism is presented GDEF_LOG, due to C.Miehe, N.Appel and M.Lambrecht [13] which is a model great deformations based to a logarithmic curve measure, with a tensor of constraints in duality individual. He is valid some is the behavior in small deformations and present the advantage of providing a symmetrical tangent matrix. No modification of the internal variables kinematic is necessary.

2 Writing and assumptions

This algorithm, due to C.Miehe; N.Appel and M.Lambrecht [13] are based on an energy formulation and the matrix of rigidity is provided in [13].

2.1 Elements of kinematics

The elements kinematics in the continuous case can be found for example in [3]. One will be interested here in the case directly discretized in time allowing to define the sizes used in the formalism presented in this document. A closed initial continuous field is considered $\Omega_0 \subset \mathbb{R}^3$, of which each point is located by its coordinates $X \in \Omega_0$, undergoing a field of deformation φ making pass in the configuration Ω :

$$\varphi : \Omega_0 \rightarrow \Omega \subset \mathbb{R}^3 \quad (1)$$

One will note $\boldsymbol{x} \in \Omega$ coordinates of this point in the current configuration.
The deformation evolving in the course of time, one actually defines, by the means of the temporal discretization, a family of field φ_n corresponding each one to one moment t_n history of evolution of the field.
In the case of the formalism great deformations treated here, it is necessary to introduce four configurations for the field and its evolution (cf Figure 1): configuration Ω_0 initial of reference (i.e. for which the deformations are worthless), configuration Ω_n at the beginning of the step of current time $t_{n+1} = t_n + \Delta t$, configuration Ω_{n+1} at the end of this step of time, and a configuration medium of the step of time, $\Omega_{n+\frac{1}{2}}$, formalism being integrated with a rule of point medium.

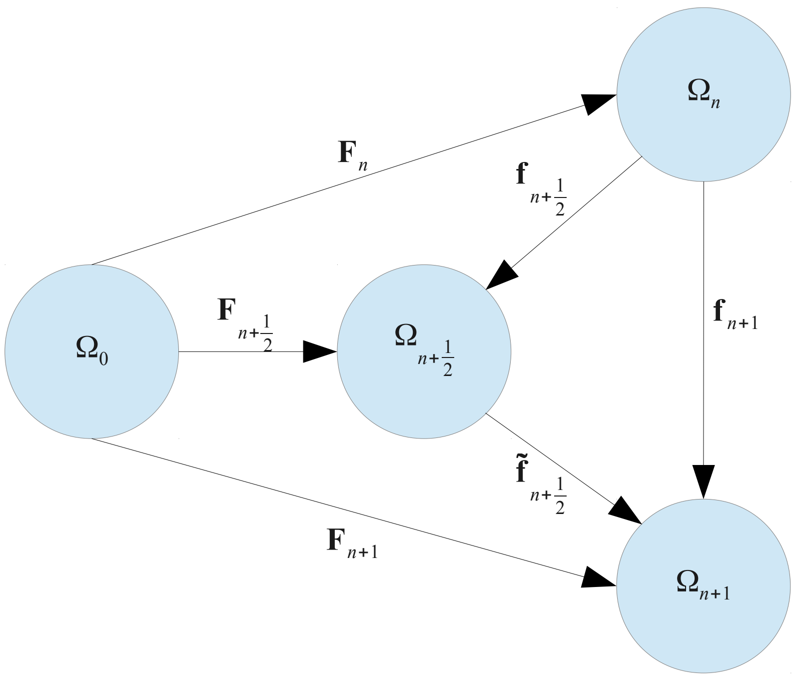


Figure 1: Configurations necessary and gradients of transformation

From these configurations, one defines the fields of displacements and the gradients of transformation to pass from the one to the other. Sizes making pass from the initial configuration to a given configuration are noted in capital letter $(\boldsymbol{U},\mathbf{F})$ and the sizes connecting two configurations deformed between them are noted into tiny $(\boldsymbol{u},\mathbf{f})$. Table 1 recapitulates the various sizes and their expressions.

Starting configuration	Configuration of arrival	Displacement	Gradient of transformation
Ω_0	Ω_n	$\boldsymbol{U}_n$	$\mathbf{F}_n=\mathbf{I}_d+\mathbf{grad}_0\boldsymbol{U}_n$
Ω_0	Ω_{n+1}	$\boldsymbol{U}_{n+1}$	$\mathbf{F}_{n+1}=\mathbf{I}_d+\mathbf{grad}_0\boldsymbol{U}_{n+1}$
Ω_0	$\Omega_{n+\frac{1}{2}}$		$\mathbf{F}_{n+\frac{1}{2}}=\frac{1}{2}(\mathbf{F}_n+\mathbf{F}_{n+1})$
Ω_n	$\Omega_{n+\frac{1}{2}}$		$\mathbf{f}_{n+\frac{1}{2}}=\boldsymbol{I}_d+\frac{1}{2}\mathbf{grad}_n\boldsymbol{u}$ $\mathbf{f}_{n+\frac{1}{2}}=\mathbf{F}_{n+\frac{1}{2}}\cdot\mathbf{F}_n^{-1}$ $\mathbf{f}_{n+\frac{1}{2}}=\frac{1}{2}(\mathbf{f}_n+\mathbf{I}_d)$
Ω_n	Ω_{n+1}	$\boldsymbol{u}$	$\mathbf{f}_{n+1}=\boldsymbol{I}_d+\mathbf{grad}_n\boldsymbol{u}$ $\mathbf{f}_{n+1}=\mathbf{F}_{n+1}\cdot\mathbf{F}_n^{-1}$ $\mathbf{f}_{n+1}=\Delta\mathbf{F}$
$\Omega_{n+\frac{1}{2}}$	Ω_{n+1}		$\tilde{\mathbf{f}}_{n+\frac{1}{2}}=\mathbf{f}_{n+1}\cdot\mathbf{f}_{n+\frac{1}{2}}^{-1}$ $\tilde{\mathbf{f}}_{n+\frac{1}{2}}=\mathbf{F}_{n+1}\cdot\mathbf{F}_{n+\frac{1}{2}}^{-1}$

Table 1: summary of displacements and gradients of transformation

From these gradients of transformation, it is possible to define the rates of deformation $\mathbf{L}$:

$$\mathbf{L}_\circ=\dot{\mathbf{F}}_\circ\dot{\mathbf{F}}_\circ^{-1} \tag{2}$$

Tensor rates of rotation $\boldsymbol{\omega}$:

$$\mathbf{d}_\circ=\frac{1}{2}(\mathbf{L}_\circ+\mathbf{L}_\circ^T) \tag{3}$$

and tensor rates of deformation $\mathbf{d}$:

$$\boldsymbol{\omega}_\circ=\frac{1}{2}(\mathbf{L}_\circ-\mathbf{L}_\circ^T) \tag{4}$$

with $\circ$ indicating the configuration n, $n+1$ or $n+\frac{1}{2}$. The tensor of Eulérien deformation enters the configurations Ω_n and Ω_{n+1} results from these definitions:

$$\mathbf{e}_{n+1}=\frac{1}{2}\left[\mathbf{I}_d-\left(\mathbf{f}_{n+1}\cdot\mathbf{f}_{n+1}^T\right)^{-1}\right] \tag{5}$$

The rate of deformation is then quite related to the deformation eulérienne:

$$\mathbf{d}_{n+\frac{1}{2}}=\frac{1}{\Delta t}\tilde{\mathbf{f}}_{n+\frac{1}{2}}^{-1}\cdot\mathbf{e}_{n+1}\cdot\tilde{\mathbf{f}}_{n+\frac{1}{2}} \tag{6}$$

The last size to be introduced for our algorithm is the gradient of incremental displacement, relating to the configuration $\Omega_{n+\frac{1}{2}}$ and defined by:

$$\mathbf{h}_{n+\frac{1}{2}} = \mathbf{grad}_n \boldsymbol{u} \cdot \mathbf{f}^{-1}_{n+\frac{1}{2}} \quad (7)$$

This last makes it possible to determine the rate of rotation in the same configuration by the relation:

$$\boldsymbol{\omega}_{n+\frac{1}{2}} = \frac{1}{2\Delta t}\left[\mathbf{h}_{n+\frac{1}{2}} - \mathbf{h}^T_{n+\frac{1}{2}}\right] \quad (8)$$

From these elements of kinematics, it is possible to define hypoelastic laws of behavior whose integration is objective in great deformations. The following paragraph presents this kind of formulation of the laws of behavior.

2.2 Hypo-elastoplastic laws of behavior

In this section, the phenomenologic class of model of plasticity (here independent of time) with hypoelasticity is considered. It constitutes an extension *ad hoc* writing of the laws in small deformations, which allows certain generics and represents an advantage in the context of a computer code: one will see in the chapter according to whether it is possible to carry out its digital integration in a way equivalent to that of the small deformations.

This class of models is to be opposed to the hyperelastic class, based on the thermodynamic approach of the continuous mediums. In this context, a free energy, being able for example to be regarded as a function of the temperature and the deformation of Green-Lagrange, is defined; the evolutions of the constraints and possibly of the internal variables result from this. One can quote for example the case of the hyperelastic law of Signorini (cf [4]) in elasticity and the Simo-Miehe formalism in hyperelastoplasticity (cf [3]).

An hypo-elastoplastic law of behavior is generally built in five stages.

1. Following the example of the additive decomposition of the small deformations, the rate of deformation $\mathbf{d}$ is first of all broken up into an elastic part and a plastic part:

$$\mathbf{d} = \mathbf{d}^e + \mathbf{d}^p \quad (9)$$

2. A derivative of the constraint of Kirchhoff $\boldsymbol{\tau} = det(\mathbf{F})\boldsymbol{\sigma}$ by an incremental relation function of the elastic rate of deformation is then determined, with $\mathring{\boldsymbol{x}}$ a derivative objectifies to define and $\mathbf{C}$ the tensor of elasticity.:

$$\mathring{\boldsymbol{\tau}} = \mathbf{C} : \left[\mathbf{d} - \mathbf{d}^p\right] \quad (10)$$

3. One builds a field of reversibility convex defining acceptable space of the constraints starting from a function f, with $\boldsymbol{S}$ the space of the constraints and $\mathbf{q}$ the whole of m internal variables representing the kinematic work hardening of material and α scalar variables (including isotropic work hardening):

$$\mathrm{E}_{\boldsymbol{\tau}} = \left\{(\boldsymbol{\tau}, \mathbf{q}, \alpha) \in \boldsymbol{S} \times \mathbb{R}^{m+1} \,|\, f(\boldsymbol{\tau}, \mathbf{q}, \alpha) \leqslant 0\right\} \quad (11)$$

4. The laws of evolution of these internal variables follow a principle of normality (here only the associated laws of behavior are considered), with $\gamma \geqslant 0$ the plastic multiplier, $\frac{\partial f}{\partial \boldsymbol{\tau}}(\boldsymbol{\tau}, \mathbf{q}, \alpha)$ defining the plastic direction of flow and $g(\boldsymbol{\tau}, \mathbf{q}, \alpha)$ evolution of the other internal variables:

$$\begin{aligned} \mathbf{d}^p &= \gamma \frac{\partial f(\boldsymbol{\tau}, \mathbf{q}, \alpha)}{\partial \boldsymbol{\tau}} \\ \mathring{\boldsymbol{q}} &= -\gamma\, g(\boldsymbol{\tau}, \mathbf{q}, \alpha) \end{aligned} \quad (12)$$

5. The writing of the conditions of load/discharge, classically represented by Kuhn-Tucker and the condition of coherence:

$$\begin{cases} \gamma \geqslant 0 \\ f(\boldsymbol{\tau}, \boldsymbol{q}, \alpha) \leq 0 \\ \gamma f(\boldsymbol{\tau}, \boldsymbol{q}, \alpha) = 0 \end{cases} \quad (13)$$

This class of model is thus characterized by a strong analogy with the formalisms small deformations, with an incremental writing of the constraints which is not without raising some difficulties of digital integration: indeed, in order to prevent the evolutions of constraints by a rigid movement of body, it is advisable to have an objective integration of the equation (10).

2.3 Tensors of strain and stress

The model is based on the deformation logarithmic curve defined by:

$$\mathbf{E}=\frac{1}{2}\log\left[\mathbf{F}^{\mathbf{T}}.\mathbf{F}\right] \tag{14}$$

The definition of this expression is provided in appendix 2. The constraint $\mathbf{T}$ is defined in space logarithmic curve like dual of $\mathbf{E}$, so that density of power mechanical p_m express yourself by their product $\mathbf{T}:\dot{\mathbf{E}}$. It is not a classical tensor of constraints, but one can connect it to the usual tensors. Indeed mechanical power being written:

$$p_m=\mathbf{T}:\dot{\mathbf{E}}=\mathbf{\Pi}:\dot{\mathbf{F}} \tag{15}$$

One obtains:

$$p_m=\mathbf{T}:\dot{\mathbf{E}}=\mathbf{T}:\frac{\partial\mathbf{E}}{\partial\mathbf{F}}:\dot{\mathbf{F}}=\mathbf{\Pi}:\dot{\mathbf{F}}=\mathbf{T}:\mathbf{P}_{\Pi}:\mathbf{F} \tag{16}$$

With $\mathbf{\Pi}$ the tensor of the constraints of Piola-Kirchhoff of first species. What defines the tensor (order four in 3D) of projection $\mathbf{P}_{\Pi}$:

$$\mathbf{P}_{\Pi}=\frac{\partial\mathbf{E}}{\partial\mathbf{F}} \tag{17}$$

One thus has:

$$\mathbf{\Pi}=\mathbf{T}:\mathbf{P}_{\Pi} \tag{18}$$

Tensors of Cauchy $\boldsymbol{\sigma}$ and of Kirchhoff $\boldsymbol{\tau}$ will be written in a usual way:

$$J\boldsymbol{\sigma}=\boldsymbol{\tau}=\mathbf{\Pi}\,\mathbf{F}^{T} \tag{19}$$

One can also calculate the second tensor of Piola-Kirchhoff $\mathbf{S}$ according to $\mathbf{T}$:

$$p_m=\mathbf{T}:\dot{\mathbf{E}}=\mathbf{S}:\dot{\mathbf{\Delta}}=\mathbf{S}:\frac{1}{2}\dot{\mathbf{C}} \tag{20}$$

With $\mathbf{\Delta}$ the tensor of the deformations of Green-Lagrange such as:

$$\mathbf{\Delta}=\frac{1}{2}(\mathbf{C}-\mathbf{I})=\frac{1}{2}(\mathbf{F}^{T}\mathbf{F}-\mathbf{I}) \tag{21}$$

There is a new form of the mechanical power:

$$p_m=\mathbf{T}:\dot{\mathbf{E}}=\mathbf{T}:\frac{\partial\mathbf{E}}{\partial\mathbf{C}}:\dot{\mathbf{C}}=\mathbf{S}:\frac{1}{2}\dot{\mathbf{C}} \tag{22}$$

One obtains for the second tensor of Piola-Kirchhoff $\mathbf{S}$:

$$\mathbf{S}=\mathbf{T}:\mathbf{P}\ \text{ with }\ \mathbf{P}=2\frac{\partial\mathbf{E}}{\partial\mathbf{C}} \tag{23}$$

If selected physics is particular, the model makes it possible however to keep the additive decomposition of the elastic strain and plastics classic in HP with:

$$\mathbf{E}^{p}=\frac{1}{2}\log\left(\mathbf{F}^{p,T}.\mathbf{F}^{p}\right) \tag{24}$$

Such a choice is always licit. That simply amounts adopting a definition for the elastic strain. However, this one proves to be coherent with a multiplicative decomposition, in the absence of rotation (coaxial situation). Moreover, the plastic incompressibility is assured because:

$$tr\,\mathbf{E}^{p}=\log J^{p} \tag{25}$$

Elastic energy ψ^e model also takes the same shape as that of the small deformations, but by adopting the concepts of constraint and deformation specific to this formalism, one a:

$$\psi^e=\frac{1}{2}\|\mathbf{E}-\mathbf{E}^p\|_E^2=\frac{1}{2}\mathbf{T}:\mathbf{C}^{-1}:\mathbf{T} \quad (26)$$

This formulation has certain advantages:

- Dkinematic imension of the model is confined upstream and downstream from the integration of the behavior; this was one of the principal elements for the choice of the formalism; all models of behavior available in small deformations are *a priori* available, in condition of course that has a physical direction (the great hypoelastic deformations are well adapted to metal behaviors, and not with the behaviors concrete);
- if the model HP admits an energy expression, it will be the same for the model great deformations: the tangent matrix is thus symmetrical;
- the only difficulty seems *a priori* concentrated in the definition of the deformation logarithmic curve, but the article [13] provides a calculation algorithm distinguishing the difficult cases (multiple clean entities);
- the model, according to the examples presented by the authors, give results very close to those obtained by a classical formalism to multiplicative decomposition;
- the model can be wide with the cases of anisotropy (initial or induced).

Moreover, the article [13] provides a form of the tangent matrix in the configuration using $\mathbf{\Pi}$ (called nominal); however, as it is based on a writing starting from the constraints of Piola-Kirchhoff of first species, not-symmetrical, which classical and is never carried out in *Code_Aster*, one prefers here to use the second tensor of Piola-Kirchhoff to calculate the internal forces and the matrix tangent on the initial configuration, while referring to [15] for example.

3 Algorithm

3.1 Preprocessing

The tensor of the deformations logarithmic curves calculated by spectral decomposition:

$$\mathbf{E}_{n+1}=\frac{1}{2}\log\left[\mathbf{F}_{n+1}^{T}\cdot\mathbf{F}_{n+1}\right]=\frac{1}{2}\log\mathbf{C}_{n+1} \quad (27)$$

I.e., if them $\lambda^{(i)}$ are eigenvalues of $\mathbf{C}_{n+1}$ and $\boldsymbol{N}^{(i)}$ the associated clean vectors, then the measurement of selected deformation is written:

$$\mathbf{E}_{n+1}=\frac{1}{2}\sum_{i=1,3}\log\left(\lambda^{(i)}\right)\boldsymbol{N}^{(i)}\otimes\boldsymbol{N}^{(i)} \quad (28)$$

This measurement makes it possible to obtain an additive decomposition:

$$\mathbf{E}=\mathbf{E}^{e}+\mathbf{E}^{p} \quad (29)$$

With:

$$\mathbf{E}^{p}=\frac{1}{2}\log\left[\mathbf{F}^{p,T}\cdot\mathbf{F}^{p}\right] \text{ and } \mathbf{E}^{e}=\frac{1}{2}\log\left[\mathbf{F}^{e,T}\cdot\mathbf{F}^{e}\right] \quad (30)$$

Moreover one can also write, between the moment n and the moment $n+1$:

$$\mathbf{E}_{n+1}=\mathbf{E}_{n}+\Delta\mathbf{E} \quad (31)$$

3.2 Connection with the law of behavior HP

The law of behavior HP must provide the tensor of the constraints $\mathbf{T}$, defined by:

$$\mathbf{T}_{n+1}=\hat{\mathbf{T}}\left(\Delta\mathbf{E};\mathbf{E}_{n},\mathbf{T}_{n},\boldsymbol{\beta}_{n}\right) \quad (32)$$

where $\boldsymbol{\beta}_{n}$ represent the whole of the internal variables at the moment n and $\mathbf{T}_{n}$ constraints at the moment n . It is thus necessary to recompute $\mathbf{T}_{n}$ according to the constraints of Cauchy $\boldsymbol{\sigma}_{n}$ stored at the moment n . These constraints are written $\mathbf{T}_{n}=\mathbf{S}_{n}:\mathbf{P}_{n}^{-1}$. However that requires the transformation of $\boldsymbol{\sigma}_{n}$ in $\mathbf{S}_{n}$ and the calculation of $\mathbf{P}_{n}^{-1}$ who can be expensive. One thus chooses to store the tensors $\mathbf{T}$ as internal variables.

Notice

Tensors $\mathbf{T}$ being stored as internal variables, the user wishing to impose a state of initial stress will have to use the operands `VARI` and `DEPL` keyword factor `ETAT_INIT` order `STAT_NON_LINE`. Indeed, it is necessary to give as starter the tensor of constraint defined in space logarithmic curve $\mathbf{T}$ (and not that of Cauchy $\boldsymbol{\sigma}$). The user wishing to use the formalism `GDEF_LOG` with an initial stress field (`ETAT_INIT`) is advised to refer to the case test *ssnp159b (V6.03.159).*

3.3 Postprocessing

The tensor of the constraints of Piola-Kirchhoff of second species is obtained by:

$$\mathbf{S}_{n+1}=\mathbf{T}_{n+1}:\mathbf{P}_{n+1} \quad (33)$$

With $\mathbf{P}_{n+1}=\partial_{\mathbf{C}}\left(\mathbf{E}_{n+1}\right)$. This quantity is calculated *via* an algorithm presented in [13] and quoted in appendix 2. It is also pointed out that the tensor of the constraints of Cauchy is obtained by:

$$J\boldsymbol{\sigma}=\boldsymbol{\tau}=\mathbf{F}\cdot\mathbf{S}\cdot\mathbf{F}^{T}=\mathbf{F}\cdot\left(\mathbf{T}:\mathbf{P}\right)\cdot\mathbf{F}^{T} \quad (34)$$

One obtains the tangent module, in configuration known as “Lagrangian”, by derivation of $\mathbf{S}_{n+1}$:

$$\dot{\mathbf{S}}_{n+1}=\mathbf{C}_{n+1}^{ep}:\frac{1}{2}\dot{\mathbf{C}}_{n+1} \text{ with } \mathbf{C}_{n+1}^{ep}=\mathbf{P}_{n+1}^{T}:\mathbf{E}_{n+1}^{ep}:\mathbf{P}_{n+1}+\mathbf{T}_{n+1}:\mathbf{L}_{n+1} \quad (35)$$

$\mathbf{E}_{n+1}^{ep}=\frac{\partial\mathbf{T}_{n+1}}{\partial\mathbf{E}_{n+1}}$ represent the tangent operator resulting from the law of behavior and $\mathbf{L}_{n+1}$ is the tensor of a nature six (in 3D) defined by:

$$\mathbf{L}_{n+1}=4\,\partial^2_{\mathbf{C}\mathbf{C}}\left(\mathbf{E}_{n+1}\right) \quad (36)$$

One can then calculate on this configuration the internal forces and the tangent matrix, on the initial configuration, as in [14]. L'objectivity is preserved (forced invariant by rotation in work hardenings isotropic and kinematic) and the good precision (results identical to SIMO_MIEHE into isotropic).

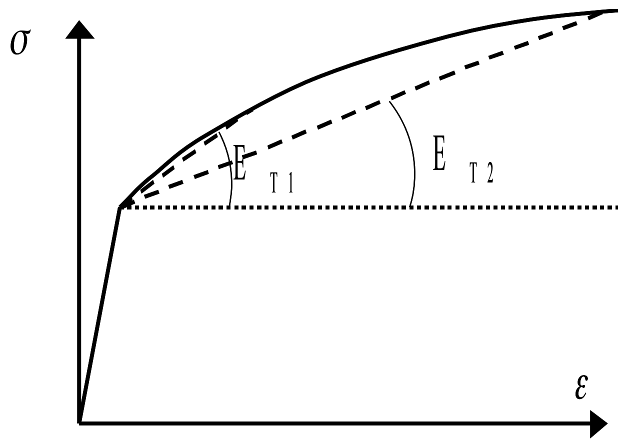
4 Validity of the models of great deformations

4.1 Identification of the parameters

It also should be specified that the formalism here presented and studied does not extend the validity of the laws of behavior in the field of the great deformations, it nothing but does propose an objective derivation of it. To clarify this matter, it is possible to consider the case of an elastoplastic law with linear isotropic work hardening. This kind of law of plasticity is valid physically in small deformations; its use is extended to the great deformations, but its physical validity can be precisely called in question.

Moreover, one identification made on tests in small deformations must be potentially reconsidered; on figure 2, one presents a traction diagram modelled by a linear isotropic work hardening: the tangent module must inevitably be defined compared to the beach of deformation considered. In small deformations, it seems more judicious to use E_{T1} ; if the deformations are more important, it seems more judicious to use E_{T2} like value of the slope of work hardening. But it is felt well that it is the physical validity of the law itself which should be reconsideration.

Figure 2: Identification in great or small deformations



5 Comparison with PETIT_REAC

5.1 Approximation of the great deformations by PETIT_REAC

The principle of the formulation `PETIT_REAC` simply consist in reactualizing the geometry of the problem during iterations of Newton (and not at the end of each step of time). This means that all the quantities intervening in the equations of the problem are evaluated on the current configuration. Anything else is not modified compared to the case small disturbances.

5.1.1 Kinematic description

Kinematic description is the same one as that of the small disturbances. This means that increment of deformation is calculated by:

$$\Delta\varepsilon=\frac{1}{2}(\nabla_{\Omega_i}(\Delta u)+\nabla_{\Omega_i}^T(\Delta u)) \quad (37)$$

Ω_i being reactualized configuration. The total deflection is then the sum of each one of these increments of linearized deformation, calculated on different configurations. It is thus delicate to give him a physical direction and better is worth to use it like an indicator of the level of deformation reached. The assumption of additive decomposition of the deformations is applied.

5.1.2 Elastoplastic relation of behavior

In the expression of the relation elastic stress-strains, one saw the need for using an objective derivative: $\overset{\circ}{\boldsymbol{\sigma}}=\mathbf{C}:[\dot{\boldsymbol{\varepsilon}}-\dot{\boldsymbol{\varepsilon}}^p]$. With `PETIT_REAC` one replaces the objective derivative by the simple derivative in time: it is thus not objective. Consequently, the employment of `PETIT_REAC` is thus not appropriate to great rotations but it is it with the great deformations, under certain conditions [10]:

- very small increments;
- very small rotations (what implies a quasi-radial loading);
- elastic strain small in front of the plastic deformations;
- isotropic behavior.

5.1.3 Balance and tangent matrix

In term of finite elements, the resolution by `PETIT_REAC` imply with each step of load the resolution of the same nonlinear system as in small deformations [11]:

$$\begin{cases} L^{\text{int}}(u_i,t_i)+B^T.\lambda_i=L^{ext}(t_i) \\ B.u_i=u^d(t_i) \end{cases} \quad (38)$$

With the difference close the internal forces are formally calculated by:

$$L^{\text{int}}(u_i,t_i)=Q^T(u_i):\sigma \quad (39)$$

where the operator Q depends on displacements. Within this framework, the calculation of the tangent matrix carries out to:

$$K_i^n=\frac{\partial L^{\text{int}}}{\partial u}|_{(u_i^n,t_i)}=Q(u):\frac{\partial\sigma}{\partial u}|_{(u_i^n,t_i)}+\frac{\partial Q(u)}{\partial u}|_{(u_i^n,t_i)}:\sigma \quad (40)$$

The first term is the contribution of the behavior, similar to what was presented in small transformations, to the difference which this contribution is evaluated here in current configuration. The second term is the contribution of the geometry which is not present in small transformations.
Within the framework of the resolution `PETIT_REAC`, this term is not present in the calculation of the tangent matrix. One thus has:

$$K_i^n=Q(u):\frac{\partial\sigma}{\partial u}|_{(u_i^n,t_i)} \quad (41)$$

The absence of the geometrical contribution in the tangent matrix can sometimes make convergence difficult.

5.2 Comparison on an example

Formalism PETIT_REAC (cf [6]) bases itself on an actualization of the geometry for the calculation of the increment of deformation before integrating the behavior of way identical to the small deformations. This allows a simple treatment of the great deformations, but in a very approximate way, not objectifies and being able to generate great errors.
To be convinced some, let us consider the alternate traction-rotation of a cube; for more details on the test, one will refer to [7] for example.

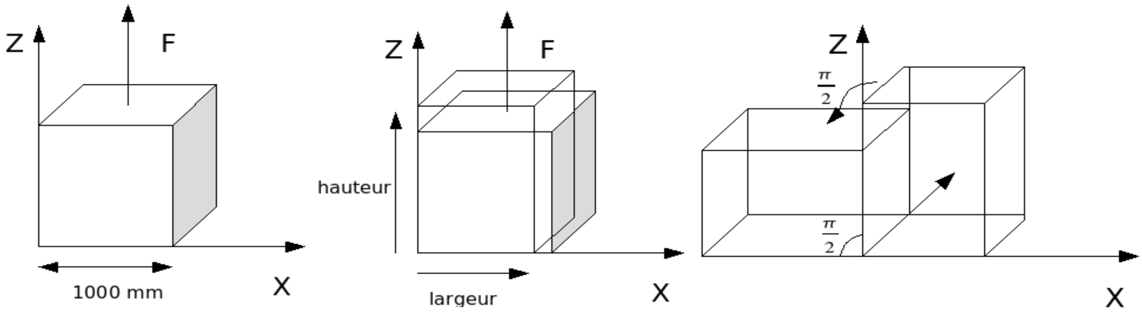


Figure 3: Example of traction rotation of a cube

During the phases of rotation, the constraint must remain constant: a rigid movement of body does not generate constraints (in statics all at least and without viscosity).

If one considers the answer obtained with a behavior VMIS_ISOT_LINE for the deformations PETIT_REAC and GDEF_LOG , the type of deformation PETIT_REAC is put at fault whereas GDEF_LOG is valid (and provides an answer identical to SIMO_MIEHE).

6 Bibliography

1 Simo J.C., Hughes T.J.R., Computational Inelasticity, Mechanics and Materials, Interdisciplinary Applied Mathematics vol. 7, Springer-Verlag Edition, New York, 1998.

2 Handbook use of U4.51.11 Aster, Behavior Non Linéaires

3 Handbook of reference R5.03.21 Aster, Modeling élasto- (visco) - plastic with isotropic work hardening in great deformations,

4 Handbook of reference R5.03.19 Aster, hyperelastic Law of behavior. Material almost incompressible.

5 Note interns EDF HI-74/98/006/0, Introduction into Code_Aster of a model of behavior in great deformations elastoplastic with isotropic work hardening, V. Cano E. Lorentz, 1998.

6 Handbook use U4.51.11 Aster, Behaviors Non Linéaires.

7 Handbook validation Aster ssna106, alternate Traction-rotation of an element. Isotropic plastic law.

8 Great transformations, plane constraints, generalized plane deformations, Leblond, 1989.

9 Handbook of references R5.03.40 Aster, static and dynamic Modeling of the beams in great rotations.

10 Report EDF/DER-MMN 1536/07: Great plastic deformations. Modeling in Aster by `PETIT_REAC`. E.Lorentz, June 24th, 1996

11 Handbook of references R5.03.01 Aster, Algorithm of `STAT_NON_LINE`

12 CR-AMA-09.201. “Model of great deformations compatible with most behaviors”. Report of internship of Julie Coloigner. R. Bargellini, J. - M. Proix.

13 Miehe C., Call NR., Lambrecht Mr.: Additive Anisotropic plasticity in the logarithm strain space: modular kinematic formulation and implementation based one incremental minimization principles for standard materials., Methods Computer in Applied Mechanics and Engineering, 191, pp.5383-5425, 2002.

14 Handbook of reference R5.03.20 Aster, relation of nonlinear elastic behavior in great displacements.

15 “Computational methods for plasticity” Wiley 2008, of EA of Souza Neto, D. Peric, DRJ. Owen

7 Description of the versions of the document

Version Aster	Author (S) Organization (S)	Description of the modifications
10.2	R.BARGELLINI, J.M.PROIX R & D /AMA J.COLOIGNER Univ. Rennes	Initial text
10.4	R.BARGELLINI, J.M.PROIX R & D /AMA	Addition of the description of the algorithm using the derivative of Dregs, of the calculation of the tangent operator, and the algorithm `GDEF_LOG`
13.1	M.Abbas	Suppression of `GDEF_HYPO_ELAS`

8 Appendix 1: Calculation of the deformations logarithmic curves

8.1 Notations:

$\mathbf{A}$ indicate a tensor of order two, and $\bar{\mathbf{A}}$ a tensor of order four. We adopt the notation of Voigt, (see for example [16]) definite for a tensor of order two:

$$\mathbf{A}=\begin{Bmatrix} A_{11} \\ A_{22} \\ A_{33} \\ \sqrt{2}A_{12} \\ \sqrt{2}A_{13} \\ \sqrt{2}A_{23} \end{Bmatrix} \quad (42)$$

And for a tensor of order four:

$$\bar{\mathbf{A}}=\begin{Bmatrix} A_{1111} & A_{1122} & A_{1133} & \sqrt{2}A_{1112} & \sqrt{2}A_{1123} & \sqrt{2}A_{1113} \\ A_{2211} & A_{2222} & A_{2233} & \sqrt{2}A_{2212} & \sqrt{2}A_{2223} & \sqrt{2}A_{2213} \\ A_{3311} & A_{3322} & A_{3333} & \sqrt{2}A_{3312} & \sqrt{2}A_{3323} & \sqrt{2}A_{3313} \\ \sqrt{2}A_{1211} & \sqrt{2}A_{1222} & \sqrt{2}A_{1233} & 2A_{1212} & 2A_{1223} & 2A_{1213} \\ \sqrt{2}A_{1311} & \sqrt{2}A_{1322} & \sqrt{2}A_{1333} & 2A_{1312} & 2A_{1323} & 2A_{1313} \\ \sqrt{2}A_{2311} & \sqrt{2}A_{2322} & \sqrt{2}A_{2333} & 2A_{2312} & 2A_{2323} & 2A_{2313} \end{Bmatrix} \quad (43)$$

Where the components relating to the notation of Voigt will be indicated by a Greek letter:

$$\|A_{ij}\|=\|A_\alpha\| \quad (44)$$

There are then the following properties:

$$\begin{aligned} \mathbf{A}:\mathbf{B}&=A_{ij}B_{ij}=A_\alpha B_\alpha \\ \bar{\mathbf{A}}:\mathbf{B}&=A_{ijkl}B_{kl}=A_{\alpha\beta}B_\beta \\ \bar{\mathbf{A}}:\bar{\mathbf{B}}&=A_{ijkl}B_{klmn}=A_{\alpha\beta}B_{\beta\gamma} \end{aligned} \quad (45)$$

The reverse of a tensor of a nature qutre comprising minor symmetries ($A_{ijrs}=A_{jirs}=A_{ijsr}$) is written:

$$\begin{aligned} \bar{\mathbf{A}}:\bar{\mathbf{A}}^{-1}&=\bar{\mathbf{I}}_d \\ A_{ijrs}A^{-1}_{rskl}=I_{ijkl}&=\frac{1}{2}\left(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}\right) \\ A_{\alpha\gamma}A^{-1}_{\gamma\beta}=I_{\alpha\beta}&=\delta_{\alpha\beta} \end{aligned} \quad (46)$$

8.2 Expression of the constraints in Lagrangian configuration

The power of the interior efforts is written:

$$p_{\text{int}}=\mathbf{T}:\dot{\mathbf{E}}=\mathbf{S}:\bar{\mathbf{P}}^{-1}\dot{\mathbf{E}} \quad (47)$$

with $\bar{\mathbf{P}}=2\frac{\partial\mathbf{E}}{\partial\mathbf{C}}$ what makes it possible to calculate $\mathbf{S}=\mathbf{T}:\mathbf{P}$ (or $S_{ij}=T_{kl}:P_{klij}$). To calculate the tensor of the constraints of Cauchy, it is enough to write:

$$\boldsymbol{\sigma}=\frac{1}{det\,\mathbf{F}}\mathbf{F}\cdot\mathbf{S}\cdot\mathbf{F}^T \quad (48)$$

Typical case of the plane constraints:

In this case, one entirely does not know $det\,\mathbf{F}$. Indeed, the component zz tensor of deformations logarithmic curves $\mathbf{E}$ is unknown, because dependent on the law of behavior. While limiting itself to the behaviors such as $det\,\mathbf{F}^p=0$ (plastic incompressibility), one has then $det\,\mathbf{F}=det\,\mathbf{F}^e$.
According to [16] one can calculate this expression:

$$det\,\mathbf{F}^e=e^{E^e_{xx}+E^e_{yy}+E^e_{zz}} \tag{49}$$

Where $\mathbf{E}^e$ represent the elastic of the deformations logarithmic curves, known part for any law of elastoplastic or élasto-viscoplastic behavior by the law of Hooke $\mathbf{E}^e=\mathbf{\Lambda}^{-1}\cdot\mathbf{T}$.
One will show this relation. One recalls the definition of the deformations logarithmic curves:

$$E_{ij}=\frac{1}{2}\sum_{k=1,3}\log\left(\lambda^{(k)}\right)\boldsymbol{N}_i^{(k)}\otimes\boldsymbol{N}_j^{(k)} \tag{50}$$

What leads to this expression of the determinant:

$$det\,\mathbf{F}^e=\sqrt{det\,\mathbf{F}^T\mathbf{F}}=\sqrt{\lambda_1\lambda_2\lambda_3} \tag{51}$$

λ_i being eigenvalues of $\mathbf{F}^T\cdot\mathbf{F}$. Thus:

$$\log\left(det\,\mathbf{F}^e\right)=\frac{1}{2}\left\{\log\left(\lambda_1\right)+\log\left(\lambda_2\right)+\log\left(\lambda_3\right)\right\} \tag{52}$$

By applying the exponential function, there is the result (49).

8.3 Expression of the tangent operator in Lagrangian configuration

By deriving the expression $\mathbf{S}=\mathbf{T}:\mathbf{P}$ compared to time:

$$\dot{\mathbf{S}}=\dot{\mathbf{T}}:\bar{\mathbf{P}}+\mathbf{T}:\dot{\bar{\mathbf{P}}}=\left(\frac{\partial\mathbf{T}}{\partial\mathbf{E}}:\dot{\mathbf{E}}\right):\bar{\mathbf{P}}+\mathbf{T}:\left(\frac{\partial\bar{\mathbf{P}}}{\partial\mathbf{C}}:\dot{\mathbf{C}}\right)=\left[\frac{\partial\mathbf{T}}{\partial\mathbf{E}}:\left(\frac{\partial\mathbf{E}}{\partial\mathbf{C}}:\dot{\mathbf{C}}\right)\right]:\bar{\mathbf{P}}+\mathbf{T}:\left(\frac{\partial\bar{\mathbf{P}}}{\partial\mathbf{C}}:\dot{\mathbf{C}}\right) \tag{53}$$

That is to say:

$$\dot{\mathbf{S}}=\left(\bar{\mathbf{P}}^T:\bar{\mathbf{E}}^p:\bar{\mathbf{P}}+\mathbf{T}:\bar{\bar{\mathbf{L}}}\right):\frac{1}{2}\dot{\mathbf{C}} \tag{54}$$

with $\bar{\bar{\mathbf{L}}}=4\frac{\partial^2\mathbf{E}}{\partial\mathbf{C}\partial\mathbf{C}}$ and $\bar{\mathbf{E}}^p=\frac{\partial\mathbf{T}}{\partial\mathbf{E}}$. What defines the tangent operator $\bar{\bar{\mathbf{C}}}^{ep}=\left(\bar{\mathbf{P}}^T\cdot\bar{\mathbf{E}}^p\cdot\bar{\mathbf{P}}+\mathbf{T}:\bar{\bar{\mathbf{L}}}\right)$ who checks $\dot{\mathbf{S}}=\bar{\bar{\mathbf{C}}}^{ep}:\frac{1}{2}\dot{\mathbf{C}}$. Or, according to the deformations of Green-Lagrange $\mathbf{\Delta}=\frac{1}{2}\left(\mathbf{C}-\mathbf{I}_d\right)$:

$$\frac{\partial\mathbf{S}}{\partial\mathbf{\Delta}}=\frac{\partial\mathbf{S}}{\partial\mathbf{C}}:\frac{\partial\mathbf{C}}{\partial\mathbf{\Delta}}=2\frac{\partial\mathbf{S}}{\partial\mathbf{C}}=\bar{\bar{\mathbf{C}}}^{ep} \tag{55}$$

The expression of this tangent operator as well as the tensor of the constraints, both in Lagrangian configuration, allow, for the calculation of the internal forces, to use a variational formulation in initial configuration, as in [R5.03.20] for example. One writes balance in variational form on the initial configuration:

$$\delta W_{\text{int}}\cdot\delta\mathbf{v}+SW_{\text{ext}}\cdot\delta\mathbf{v}=0\quad\forall\delta\mathbf{v}\ \text{kinematically acceptable} \tag{56}$$

Under the assumption that the loading does not depend on the geometrical transformation, the virtual work of the external efforts is written like a linear form:

$$\delta W_{\text{ext}}\cdot\delta\mathbf{v}=\int_{\Omega_0}\rho_0 F_i\delta v_i\,d\Omega_0+\int_{\partial_F\Omega_0}T_i^d\delta v_i\,dS_o \tag{57}$$

With $\mathbf{F}$ the voluminal loading and $\mathbf{T}^d$ a surface loading being exerted on the edge $\partial_F\Omega_0$. There still, we choose the initial configuration like configuration of reference, to express the work of the interior efforts [R5,03,20]. [R7.02.03]:

$$SW_{\text{int}}\cdot\delta\mathbf{v}=-\int_{\Omega_0}F_{ik}S_{kl}\delta v_{i,l}\,d\Omega_o\quad\text{with}\quad\delta v_{i,l}=\frac{\partial dv_i}{\partial X_l} \tag{58}$$

In the optics of a resolution by a method of Newton, it is important to also express the variation second of the virtual work of the interior efforts, namely, geometrical rigidity:

$$d^2 W_{\text{int}}\cdot\delta\mathbf{u}\cdot\delta\mathbf{v}=-\int_{\Omega_0}\delta u_{i,k}\, S r_{kl}\,\delta v r_{i,l}\, d\Omega_0 \qquad (59)$$

And elastic rigidity:

$$-\int_{\Omega_0}\delta u_{i,q}\, F_{ip}\left(\frac{\partial\mathbf{S}}{\partial\boldsymbol{\Delta}}\right)_{pqkl} F_{jk}\,\delta v_{j,l}\, d\Omega_0 \qquad (60)$$

8.4 Effective calculation of the deformations logarithmic curves

The deformations logarithmic curves are defined by:

$$\mathbf{E}=\frac{1}{2}\log(\mathbf{C})=\frac{1}{2}\log(\mathbf{F}^T\cdot\mathbf{F}) \qquad (61)$$

In any rigour, it would be necessary to add the metric tensor in the case of an initial configuration defined in a space different from Euclidean space (case of the hulls for example). To simplify the writings, we will in the case of place ourselves an Euclidean initial configuration, the components of the vectors and tensors being written in an orthonormal reference mark 3D. The restriction on the 2D is immediate. The calculation of the deformation logarithmic curve can be done only in the clean reference mark. The three eigenvalues thus should be determined $\lambda^{(i)}$ and clean vectors $\boldsymbol{N}^{(i)}$ solutions of the problem to the eigenvalues according to:

$$\mathbf{C}\boldsymbol{N}^{(i)}=\lambda^{(i)}\boldsymbol{N}^{(i)} \qquad (62)$$

One can then calculate the three values in “clean” space:

$$e^{(i)}=\frac{1}{2}\log(\lambda^{(i)}) \qquad (63)$$

The deformations logarithmic curves are then transported within the space of origin by:

$$E_{ij}=\frac{1}{2}\sum_{k=1,3}\log(\lambda^{(k)})\boldsymbol{N}_i^{(k)}\otimes\boldsymbol{N}_j^{(k)} \qquad (64)$$

Because the function logarithmic curve is an isotropic function of the tensor $\mathbf{C}$ [16]. For postprocessing (see §3.3), i.e. the calculation of the tensor of the constraints and the tangent operator, the quantities should be calculated $\bar{\mathbf{P}}=2\frac{\partial\mathbf{E}}{\partial\mathbf{C}}$:

$$\bar{\mathbf{P}}=\sum_{i=1,3}\frac{1}{2}d^{(i)}\boldsymbol{N}^{(i)}\otimes\boldsymbol{N}^{(i)}\otimes\boldsymbol{M}^{(ii)}+\sum_{i=1,3}\sum_{j\neq i}^{3}\theta_{ij}\boldsymbol{N}^{(i)}\otimes\boldsymbol{N}^{(j)}\otimes\boldsymbol{M}^{(ij)} \qquad (65)$$

And quantity $\mathbf{T}:\bar{\mathbf{L}}$:

$$\begin{aligned}\mathbf{T}:\bar{\bar{\mathbf{L}}}=&\sum_{i}^{3}\frac{1}{4}f^{(i)}\zeta^{(ii)}\mathbf{M}^{(ii)}\otimes\mathbf{M}^{(ii)}+\sum_{i}^{3}\sum_{j\neq i}^{3}\sum_{k\neq i,k\neq j}^{3}2\eta\zeta^{(ij)}\mathbf{M}^{(ik)}\otimes\mathbf{M}^{(ij)}\\&+\sum_{i}^{3}\sum_{j\neq i}2\xi^{(ij)}\left[\zeta^{(ij)}\left(\mathbf{M}^{(ij)}\otimes\mathbf{M}^{(jj)}+\mathbf{M}^{(jj)}\otimes\mathbf{M}^{(ij)}\right)+\zeta^{(jj)}\mathbf{M}^{(ij)}\otimes\mathbf{M}^{(ij)}\right]\end{aligned} \qquad (66)$$

With $d^{(i)}=\frac{1}{\lambda^{(i)}}$, $f^{(i)}=\frac{-2}{(\lambda^{(i)})^2}$, $\zeta^{(ij)}=\mathbf{T}:\boldsymbol{N}^{(i)}\otimes\boldsymbol{N}^{(j)}$, $\mathbf{M}_{ab}^{(ij)}=\boldsymbol{N}_a^{(i)}\boldsymbol{N}_b^{(j)}+\boldsymbol{N}_a^{(j)}\boldsymbol{N}_b^{(i)}$. $\theta^{(ij)}$, $\xi^{(ij)}$ and $\eta^{(ij)}$ are defined by:

- if all the eigenvalues are different:

$$\theta^{(ij)}=\frac{\left(e^{(i)}-e^{(j)}\right)}{\left(\lambda^{(i)}-\lambda^{(j)}\right)}$$
$$\xi^{(ij)}=\frac{\left(\theta^{(ij)}-\frac{1}{2}d^{(j)}\right)}{\left(\lambda^{(i)}-\lambda^{(j)}\right)} \quad (67)$$
$$\eta=\sum_{i}^{3}\sum_{j\neq i}^{3}\sum_{k\neq i,k\neq j}^{3}\frac{e^{(i)}}{2(\lambda^{(i)}-\lambda^{(j)})(\lambda^{(i)}-\lambda^{(k)})}$$

- if two eigenvalues are equal $\lambda^{(i)}=\lambda^{(j)}\neq\lambda^{(k)}$:

$$\theta^{(ij)}=\theta^{(ji)}=\frac{1}{2}d^{(j)},\ \xi^{(ij)}=\xi^{(ji)}=\frac{1}{8}f^{(j)},\ \eta=\xi^{(ki)}$$
for $n=k,\ m\in\{i,j\}$ or $m=k,\ n\in\{i,j\}$
$$\theta^{(mn)}=\frac{\left(e^{(m)}-e^{(n)}\right)}{\left(\lambda^{(m)}-\lambda^{(n)}\right)} \text{ and } \xi^{(mn)}=\frac{\left(\theta^{(mn)}-\frac{1}{2}d^{(n)}\right)}{\left(\lambda^{(m)}-\lambda^{(n)}\right)} \quad (68)$$

- if the three eigenvalues are equal $\lambda^{(i)}=\lambda^{(j)}=\lambda^{(k)}$:

$$\theta^{(ij)}=\frac{1}{2}d^{(j)},\ \xi^{(ij)}=\frac{1}{8}f^{(j)},\ \eta=\frac{1}{8}f^{(j)} \quad (69)$$