AHLV101 - Guide wave at anechoic exit

Abstract:

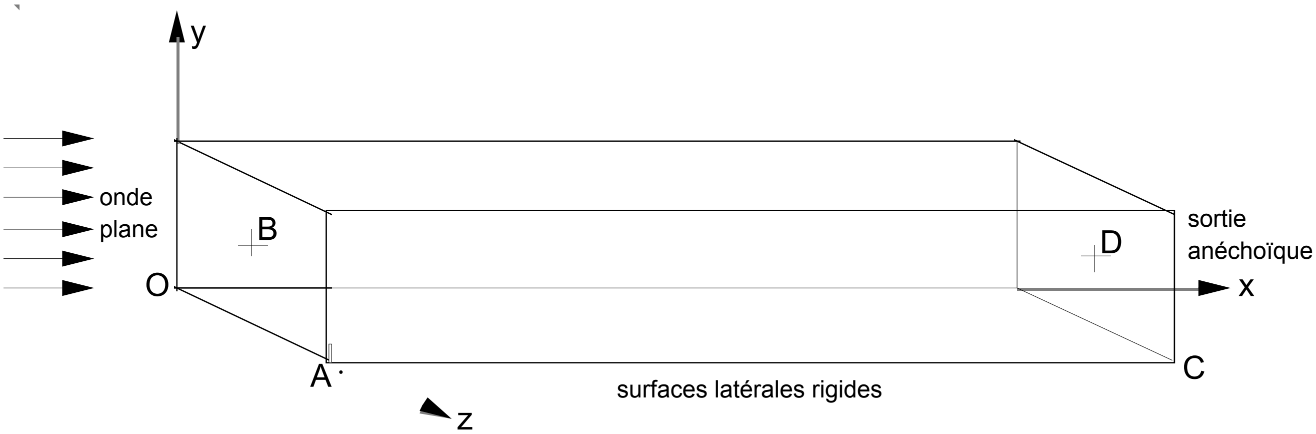
A guide of wave rectilinear at anechoic exit, with rigid walls, whose propagation medium is “normal” air, is excited by one incident wave harmonic, normal with the face of entry. One by means of calculates the acoustic field of pressure of the harmonic response the élasto-acoustics formulation in pressure - displacement-potential of displacements.

The tests relate to 3 different modelizations (finite elements élasto-acoustics three-dimensional, two-dimensional and axisymmetric), they make it possible to validate the stiffness matrixes, of mass, impedance and the vector source for the 3 modelizations.

Result from reference comes from an analytical computation.

1 Problem of reference

1.1 Geometry



Tubes with rectangular section:

length: $L=l_x=1.0\,m$
height: $h=l_y=0.1\,m$
width: $l=l_z=0.2\,m$

Coordinates of the points (in m):

	A	B	C	D
x	0.	0.	1.00	1.00
y	0.	0.05	0.	0.05
z	0.20	0.10	0.20	0.10

1.2 Properties of the materials

Air:

$\rho=1.3\,kg.m^{-3}$
$c=343.\,m.s^{-1}$

1.3 Boundary conditions and loading

Pressure of normal incident wave at the entry	$P_i=P_0 * e^{i\omega t}$ With $P_0=1.0\,Pa$
Frequency	$f=500\,Hz$
Impedance at the Reference solution CD	$Z=\rho c=445.9\,kg.m^{-2}s^{-1}$

2 end

2.1 Method of calculating used for the reference solution

the frequencies of the excitation are rather low and jointly the guide of wave is sufficiently long compared to its side dimensions so that one limits oneself to the plane waves: the phenomenon is then identical in all points of a plane of wave, i.e. does not depend on the coordinates describing the points of this plane, y and z for example.

One gives on this assumption the well-known general solution of the equations of the acoustics for the two quantities **pressure** p and **acoustic velocity** v :

$$v=f\left(t-\frac{x}{c}\right)+g\left(t+\frac{x}{c}\right) \quad \text{éq 2.1-1}$$

$$p=\rho c\left[f\left(t-\frac{x}{c}\right)-g\left(t+\frac{x}{c}\right)\right] \quad \text{éq 2.1-2}$$

the guide is supposed to be closed at the end of X-coordinate L on an impedance Z_L ; it occurs a reflection on the level of this impedance, which gives one wave of return g .

In each point of the guide, there is then superposition of the two functions f and g ; by definition even the final impedance Z_L imposes on the point of X-coordinate L , enters p and v the relation.

$$\frac{p_L}{v_L}=Z_L$$

In the harmonic case f and g are written:

$$f\left(t-\frac{x}{c}\right)=I\,e^{i\omega\left(t-\frac{x}{c}\right)}$$

$$g\left(t+\frac{x}{c}\right)=R\,e^{i\omega\left(t+\frac{x}{c}\right)}$$

where I and R are determined by the boundary conditions.

In the computation of the impedance $Z=\frac{p}{v}$ in any point x variable time this time is eliminated, in accordance with the computation even of the impedances and is written:

$$Z(x)=Z_0\frac{I\,e^{-i\omega\frac{x}{c}}-R\,e^{i\omega\frac{x}{c}}}{I\,e^{-i\omega\frac{x}{c}}+R\,e^{i\omega\frac{x}{c}}}$$

The final impedance becomes:

$$Z_L = Z_0 \frac{I\,e^{-i\omega\frac{L}{c}} - R\,e^{i\omega\frac{L}{c}}}{I\,e^{-i\omega\frac{L}{c}} + R\,e^{i\omega\frac{L}{c}}}$$

$Z_0 = \rho c$ The iterative impedance is called.

On the fluid border at the entrance of guide the limiting condition of standard incident wave imposed on $P_i = P_0 e^{i\omega t}$, is obtained by writing at the border the following linear relation:

$$p - \rho c v_n = P_i \quad \text{éq 2.1-3}$$

where $v_n = \boldsymbol{v.n}$ is the velocity according to the outgoing unit $\boldsymbol{n}$ **norm** of the fluid.

One imposes moreover on the output of the guide a value of final impedance $Z_L = Z_0$ which in fact an anechoic end.

The final impedance is equal to the iterative impedance Z_0 when $R = 0$, i.e. when there is no wave of return; one then has a pure **travelling wave** in the meaning of the incident wave, that is to say:

$$v = I\,e^{i\omega\left(t - \frac{x}{c}\right)}$$

$$p = \rho c I\,e^{i\omega\left(t - \frac{x}{c}\right)}$$

thus the relation of imposed incident wave [éq 2.1-3] is written:

$$p - \rho c v_n = p(x=0) + \rho c v(x=0) = 2\rho c I\,e^{i\omega t}$$

from where one identifies $2\rho c I\,e^{i\omega t} = P_i$; one from of deduced the statement from the travelling wave of pressure in the guide when one imposes P_i at the entrance of guide:

$$p = \frac{P_i}{2} e^{-i\omega\frac{x}{c}} = \frac{P_0}{2} e^{i\omega\left(t - \frac{x}{c}\right)}$$

2.2 Results of reference

Pressure to the points A B C , D (for the modelizations A, B, C).

2.3 Uncertainty on the analytical

solution Solution.

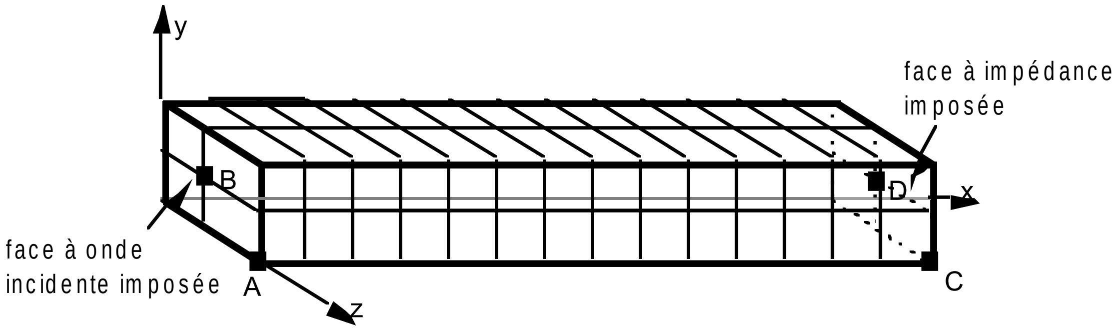
2.4 Bibliographical references

1.F. STIFKENS “Introduction into *the Code_Aster* of condition limits of standard incident wave into vibro-acoustic - Ratio HP-61/95/026/

3 Modelization A

3.1 Characteristic of the modelization

pressure-potential Formulation of displacements elements `"3D_FLUIDE"` (`MEFL_HEXA20` and `MEFL_FACE8`)



Cutting =	15	meshes HEXA20 according to the axis of x
	the 2	meshes HEXA20 according to the axis of y
	the 2	meshes HEXA20 according to the axis of z

the limiting Conditions:

```
ONDE_FLUI:   (GROUP_MA: EntréePRES     : 1.0  )
IMPE_FACE:   (GROUP_MA: SortieIMPE     : 445.9)
```

Name of the nodes $A=No1$ $B=No780$ $C=No751$ $D=No763$

3.2 Characteristic of the mesh

Many nodes:	471
Number of meshes and types:	60 HEXA20 8 QUAD8

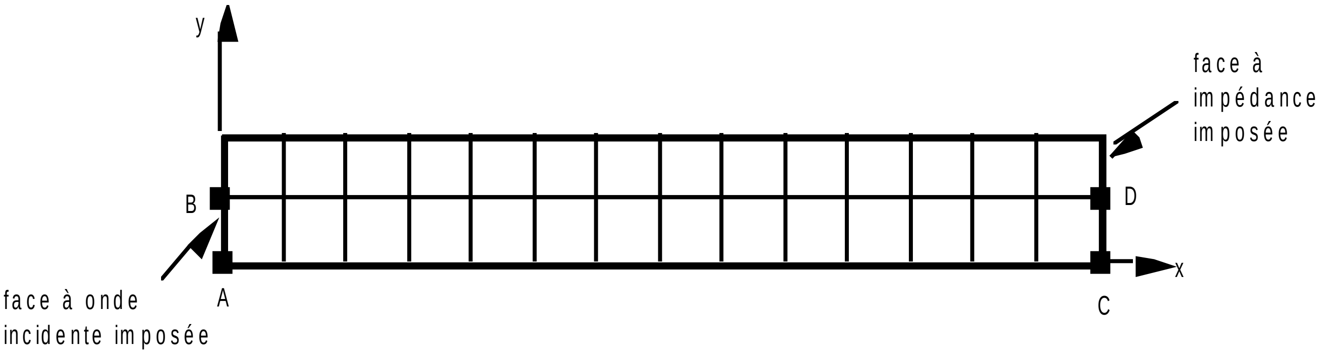
3.3 Values tested

Localization	Quantities	Reference
A	p (reality) p (imag)	0.5.0.0
B	p (reality) p (imag)	0.5.0.0
C	p (reality) p (imag)	– 0.482466 – 0.131252
D	p (reality) p (imag)	– 0.482466 – 0.131252

4 Modelization B

4.1 Characteristic of the modelization

Formulation pressure potential of displacements elements "`2D_FLUIDE`" (`MEFLSE3` and `MEFLQU8`)



Cutting = 15 meshes QUAD8 according to the axis of x
the 2 meshes QUAD8 according to the axis of y

the limiting Conditions:

```
ONDE_FLUI:   (GROUP_MA: EntréePRES     : 1.0  )
IMPE_FACE:   (GROUP_MA: SortieIMPE     : 445.9)
```

Name of the nodes $A=No1$ $B=No3$ $C=No751$ $D=No153$

4.2 Characteristic of the mesh

Many nodes: 125
Number of meshes and types: 30 QUAD8 4 SEG3

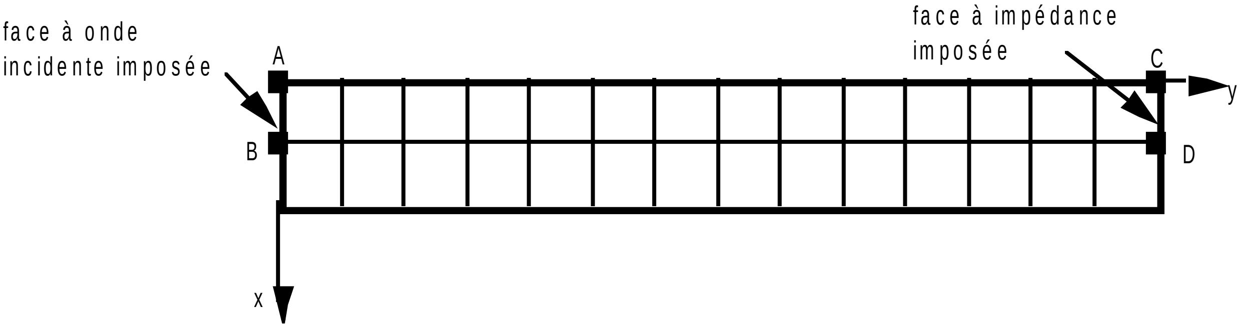
4.3 Values tested

Localization	Quantities	Reference	Aster	% difference
A	p (real)	0.5.0.0	0.499997	6 10-4
	p (imag)		1.2 10-5	-
B	p (real)	0.5.0.0	0.499997	6 10-4
	p (imag)		1.2 10-5	-
C	p (real)	– 0.482466	– 0.482352	2.4 10-2
	p (imag)	– 0.131252	– 0.131670	3.2 10-1
D	p (reality)	– 0.482466	– 0.482352	2.4 10-2
	p (imag)	– 0.131252	– 0.131670	3.2 10-1

5 Modelization C

5.1 Characteristic of the modelization

pressure-potential Formulation of displacements elements "AXIS_FLUIDE" (MEAXFLS3 and MEAXFLQ8)



Cutting =	15	meshes QUAD8 according to the axis of y
	the 2	meshes QUAD8 according to the axis of x

the limiting Conditions:

```
ONDE_FLUI:   (GROUP_MA: EntréePRES     : 1.0  )
IMPE_FACE:   (GROUP_MA: SortieIMPE     : 445.9)
```

Name of the nodes $A=No1$ $B=No3$ $C=No151$ $D=No153$

5.2 Characteristic of the mesh

Many nodes: 125
Number of meshes and types: 30 QUAD8 4 SEG3

5.3 Values tested

Localization	Quantities	Reference	*Aster*	% difference
A	p (real)	0.5.0.0	0.499997	6 10-4
	p (imag)		1.2 10-5	-
B	p (real)	0.5.0.0	0.499997	6 10-4
	p (imag)		1.2 10-5	-
C	p (real)	– 0.482466	– 0.482352	2.4 10-2
	p (imag)	– 0.131252	– 0.131670	3.2 10-1
D	p (reality)	– 0.482466	– 0.482352	2.4 10-2
	p (imag)	– 0.131252	– 0.131670	3.2 10-1

6 Summary of the results

the discretization is strong since it is approximately 45 nodes by wave length. This is why we get results of a high accuracy: the pressure calculated by *Code_Aster* at the least favorable point differs from the theoretical value from less than 1%.

It should be also noted that all the modelizations used give identical results.