FDLV100 - Piston coupled to a Summarized incompressible fluid column

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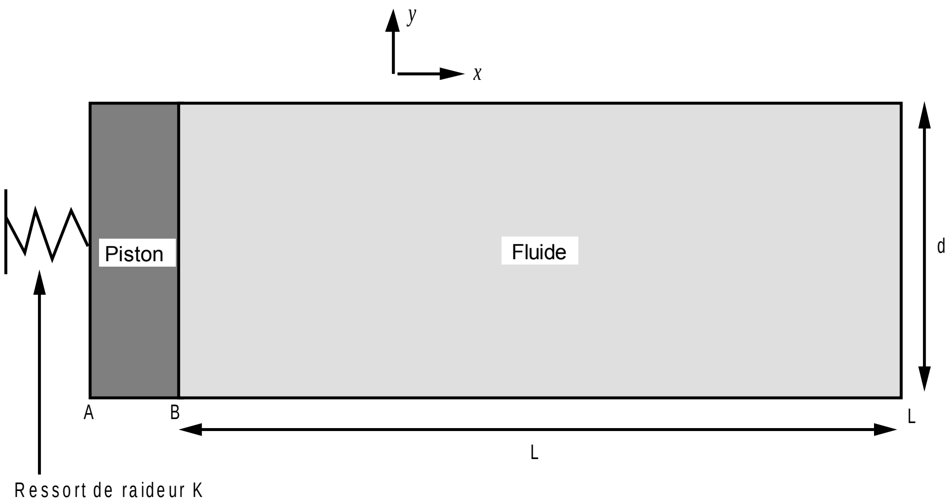
This test of the field of the fluids (fluid-structure coupling) validates the computation of added mass on modal base and carries out, in the frame of a modal analysis, the computation of the eigenfrequency of a system piston-spring coupled to an incompressible fluid column. To model the fluid, plane thermal elements are used; to model the piston, one 2D uses machine elements in plane strain and a discrete element to model a spring. Lastly, the fluid interface/structure is modelled by thermal linear elements modified to introduce a boundary condition of type “acceleration” into the fluid. The benchmark comprises only one modelization, two-dimensional. The eigenfrequency of the coupled system is found to 0.01% of result analytical.

1 Problem of reference

1.1 Geometry



Piston out of steel connected to the solid mass by a spring and coupled to an incompressible fluid column:

length:	$L=1.0\,m$
width:	$d=0.25\,m$
width AB of the piston:	$0.05\,m$

X-coordinates of the points (in m):

	A	B	L
x	– 0.05	0.	1.

1.2 Material properties

Fluid:

Water : $\rho_0=1000.0\,Kg.m^{-3}$

Solid:

Steel: $\rho_S=7800.0\,Kg.m^{-3}$; $E=2.E11\,Pa$; $\nu=0.3$

Arises connecting the piston to the solid mass:

Discrete element of the type `K_T_D_L` : $K=(1.E5, 1.E5, 1.E5)\,N/m$

1.3 Boundary conditions and loading

One imposes a pressure (i.e. by analogy thermal a temperature null [R4.07.03]) in all the nodes of the end of the fluid column.

One imposes the fixed support of spring on the solid mass and one imposes a displacement of the null piston according to Oy .

2 Reference solution

2.1 Method of calculating used for the analytical reference solution

Computation:

When the structure vibrates in the fluid, it modifies the field of pressure which obeys an equation of Laplace with boundary conditions of Von Neuman [R4.07.03].
In our case, taking into account symmetries of the problem, the field of pressure depends only on the variable x and checks:

$$\begin{cases} \dfrac{\partial^2 p}{\partial x^2}=0 & \left(\dfrac{\partial p}{\partial x}\right)_{x=0}=-\rho_f \ddot{x}_S . n \\ p=0 \text{ en } x=L \end{cases}$$

One notes thus that the field of pressure is a function closely connected of the X-coordinate x . The two boundary conditions on the pressure imply: $p=-\rho_f \ddot{x}_S . n(x-L)$

The compressive force which is exerted on the structure writes:

$$F=\int_\Gamma p(0) n \, d\Gamma=\int_\Gamma \rho_f L(\ddot{x}_S . n) n \, d\Gamma$$

As the problem is unidimensional, this force can be expressed in an algebraic way according to the component of acceleration following Ox of structure:

$$F=-\ddot{x}\int_\Gamma \rho_f L \, d\Gamma=-\rho_f L d \ddot{x}=-m_a \ddot{x} \text{ with } m_a=\rho_f L d$$

It is the linear added mass of the fluid on the structure: it is noticed that it corresponds to the mass of fluid in the column, i.e. with the mass of fluid moved by the piston.
The equation of the motion of the piston projected on Ox is written (undamped free vibration taking into account the presence of the fluid):

$$m\ddot{x}+Kx = F = -m_a \ddot{x} \Leftrightarrow (m+m_a)\ddot{x}+Kx=0$$

The eigenfrequency of this immersed system is thus written:

$$f=\frac{1}{2\pi}\sqrt{\frac{K}{m+m_a}}$$

The effect of the fluid is thus to lower the eigenfrequency of the system in air.
Practically, in Aster, the added mass matrix is given on the basis of the structure modal base in the vacuum: To compute: the added mass given above, one is restricted with the computation of the eigen mode of the system piston-spring which corresponds to a translatory movement normalized with the unit: one truncates consequently the modal base of structure to only one mode in air (operator `MODE_ITER_SIMULT` option `PLUS_PETITE`). One determines thanks to this mode the added mass on the piston.

$$K=10^5 N/m \quad m_a=200\,kg/m \quad m=78\,kg/m$$

The eigenfrequency of the system immersed piston-spring is thus $f=3.018\,Hz$

2.2 Results of Analytical

reference

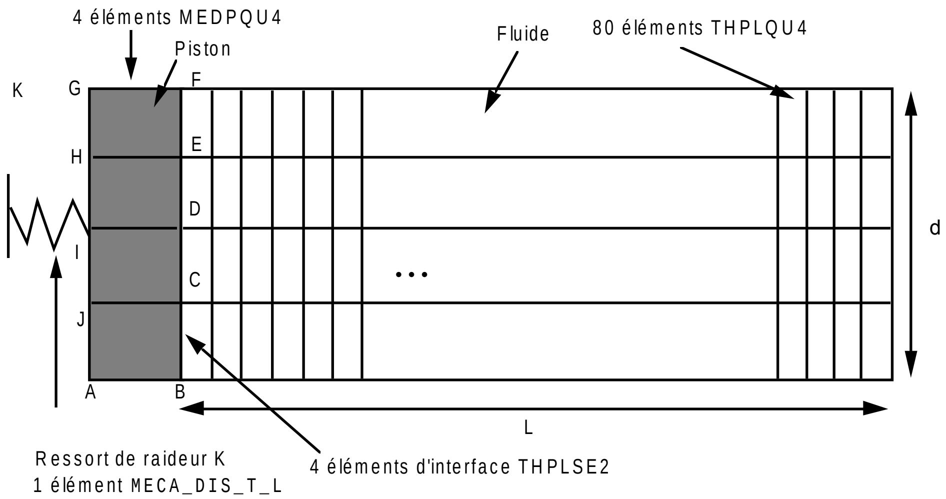
2.3 bibliographical References

• R.J GIBERT - Vibrations of Structures - Interactions with fluids. Eyrolles (1988).

3 Modelization A

3.1 Characteristic of the thermal

modelization Formulation planes for fluid (QUAD4 and SEG2)
plane strain Formulation and discrete for solid (QUAD4 and SEG2)



Cutting = 21 meshes QUAD4 according to the axis x
4 meshes QUAD4 according to the axis y
4 meshes SEG2 on the fluid interface/piston
1 nets SEG2 representing spring binding the piston to the solid mass

Boundary conditions: `DDL_IMPO: (GROUP_NO: noeupist DY: 0. )`
`DDL_IMPO: (GROUP_NO: embed DX: 0. DY: 0. DZ: 0.)`

Name of the nodes: Nodes group `NOEUPIST` is made up by the ten nodes `A, B, C, D, E, F, G, H, I, J`

nodes group `ENCASTRE` is made up by the node `K`

3.2 Characteristic of the mesh

Many nodes: 111 nodes
Number of meshes and types: 84 QUAD4, 5 SEG2

4 Results of the modelization A

4.1 Values tested

Identification	Reference (Hz)	% tolerance
Order of the eigen mode i : 1	3.018	0.1%

4.2 Remarks

Computations of modes carried out by:

```
MODE_ITER_SIMULT     OPTION=' PLUS_PETITE'     NMAX_FREQ=1.
```