WTNA112 – Thermal pressurization of a saturated cylindrical test-tube not drained

Summarized:

It is about a problem of THM saturated and elastic. One increases the temperature of a sample not drained maintained with constant containment (constant total stress with edge). The resulting water pressure varies then linearly with the temperature according to a thermal coefficient of pressurization which one calculates analytically. The solution obtained here is thus to compare with an analytical solution.

1 Problem of reference

1.1 Geometry

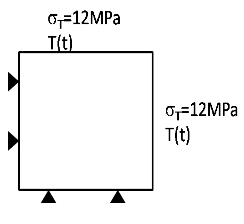
One considers a cylinder of radius $1\,cm$ and height $1\,cm$ (either a mesh corresponding to a square field of $1\,cm\times 1\mathrm{cm}$, the modelization being axisymmetric).

1.2 Properties of the material

One chooses parameters here corresponding to a mudstone so as to obtain a coefficient of realistic thermal pressurization.

Liquid water	Density $(kg.m^{-3})$ Specific heat with pressure constant $(J.K^{-1})$ Dynamic viscosity of liquid water $(Pa.s)$ thermal Coefficient of thermal expansion of the fluid (K^{-1}) Solid (Pa^{-1})	10^3 4180 0.001 1.10^{-4} $K_e=5.10^{-10}$
Compressibility	Young Modulus drained $E(Pa)$ Poisson's ratio thermal Coefficient of thermal expansion of the State (K^{-1})	$3,14\,10^9$ 0.375 10^{-5}
solid of reference	Porosity Temperature (K) liquid Pressure (Pa)	0.18 273 0
homogenized Coefficients	homogenized Density $(kg.m^{-3})$ Coefficient of Biot intrinsic Permeability (m^2) thermal Conductivity	2410 0.6 $K_{\mathrm{int}}=10^{-21}$ $\lambda_T=1.61$

1.3 Boundary conditions and loadings



One imposes:

- On low and left edges: null displacements, hydraulic flux no one, null heat flux. They are conditions of symmetry.
- On edges high and right: Total stress imposed on $12\,MPa$, hydraulic flux no one, temperature imposed function of time $T(t)$ following a linear slope such as:

  $T(t)=T_0+\frac{\Delta T}{t_{sim}}$ where t_{sim} corresponds at the time of simulation (here $t_{sim}=1\mathrm{h}$) and ΔT the temperature variation imposed during this time (here $\Delta T=40\,°C$).

1.4 Initial conditions

$P(x)=4\mathrm{MPa}$ and $T(x)=T_0=20\,°C$ everywhere.

2 Reference solution

One recalls that the contribution of mass of water is written: $m_w=\varphi.\rho_w.(1+\varepsilon_v)$, which one can derive in the following form: $dm_w=d\varphi\,\rho_w(1+\varepsilon_v)+d\rho_w\,\varphi(1+\varepsilon_v)+\rho_w\,\varphi\,d\varepsilon_v$ with φ eulerian porosity.

If one places oneself in assumption of small displacements, one will thus have:

$$dm_w=d\varphi\,\rho_w+d\rho_w\varphi+\rho_w\varphi\,d\varepsilon_v \quad (1)$$

The variation of porosity is written according to the relation:

$$d\varphi=(b-\varphi)\left(d\varepsilon_V-3\alpha_0\,dT+\frac{dp_w}{K_s}\right) \quad (2)$$

with α_0 the linear thermal expansion of the squelette (comparable to the porous environment). One recalls that the coefficient of Biot b and the modulus of compressibility of the solid matter constituents K_s are connected to the “drained” modulus of compressibility of the porous environment K_0 , such as:

$$b=1-\frac{K_0}{K_s}$$

In addition, the variation of the density of water is written:

$$\frac{d\rho_w}{\rho_w}=\frac{dp_w}{K_w}-3\alpha_w\,dT \quad (3)$$

with the modulus of compressibility of water K_w and its modulus of thermal expansion α_w .

Lastly, if the constitutive law is elastic, it is pointed out that the strain is connected to the effective stress such as:

$$d\varepsilon_V=\frac{d\sigma'}{K_0}+3\alpha_0\,dT \quad (4)$$

In addition, the total stress formulation, indicates to us that:

$d\sigma'=d\sigma+b\,dp_w$, considering here that the mediums is with constant containment, one thus has:

$d\sigma' = b\,dp_w$, which gives us with final

$$d\varepsilon_V = \frac{b\,dp_w}{K_0} + 3\alpha_0\,dT \qquad (5)$$

One can now inject (2), (3), (4) and (5) in the equation (1) and one obtains that:

$$\frac{dm_w}{\rho_w} = \left(\frac{b^2}{K_0} + \frac{(b-\varphi)}{K_s} + \frac{\varphi}{K_w}\right) dp_w + \varphi\left(3\alpha_0 - 3\alpha_w\right) dT \qquad (6)$$

Considering that the medium is not drained one thus has:

$$\left(\frac{b^2}{K_0} + \frac{(b-\varphi)}{K_s} + \frac{\varphi}{K_w}\right) dp_w = \varphi\left(3\alpha_w - 3\alpha_0\right) dT \qquad (7)$$

What can be written in the form:

$$dp_w = \Lambda\,dT \qquad (8)$$

With Λ the thermal coefficient of pressurization such as:

$$\Lambda = \frac{\varphi\left(3\alpha_w - 3\alpha_0\right)}{\left(\frac{b^2}{K_0} + \frac{(b-\varphi)}{K_s} + \frac{\varphi}{K_w}\right)}$$

It is noticed that this coefficient revealed the thermal differential $(\alpha_w - \alpha_0)$

numerical Application:

With the data defined previously, one obtains:

$\Lambda = 2{,}25\,10^5\,Pa.\,K^{-1}$

What gives for a temperature variation $\Delta T = 40\,°C$, a variation of pressure of $\Delta p = 9{,}01\,Mpa$.

3 Modelization A

3.1 Characteristic of the modelization A

- plane Modelization "AXIS_THMS". Mechanical model "ELAS". Coupling "LIQU_SATU".
- 20×20 elements $Q4$ of equal width.

3.2 Results of the modelization A

Discretization in time: 10 time step of $180\,s$ each one. The solution calculated by Aster which takes account of motions of the fluid and heat (diffusive phenomena), it is normal not to obtain the reference solution exactly. The differences remain very weak.

Result with final moment $3600\,s$ **:**

$N°$ **NODE**	$COOR\_X$	$COOR\_Y$	**Reference** $PRE1(MPa)$	**Aster** $PRE1(MPa)$	**Differences** $(\%)$	**Tolerance** $(\%)$
1	0	0	13,01	12,98	0,158	1
2	0	0.01	13,01	12,99	0,098	1

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4 Summary of the results

the results are into coherent with the analytical solution.