WTNP122 - Modelization of a bar saturated with compressible gas slightly nonlinear (monophasic flow) subjected to a shock with pressure

Summarized:

This case test aims to validate:

- the diagrams finished volumes developed for the modelization of diphasic flows.
- the hydraulic modelization saturated with finite elements D_PLAN_HS

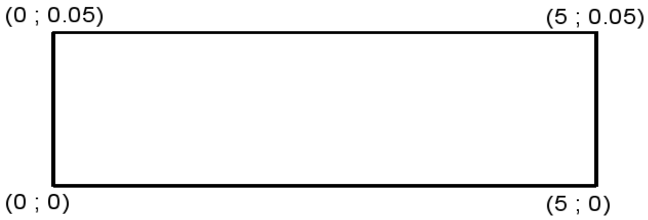
the diphasic problem here will be degenerated in a monophasic problem gas which one knows the analytical solution. It is the monodimensional modelization of a bar saturated with gas subjected to a shock with pressure.

1 Problem of reference

the purpose of this case test is to compare the solution obtained with the various diagrams with an analytical solution.

1.1 Geometry

One considers a bar 1D 5m length. Concretely the mesh area will make $[0\text{m},5\,m]\times[0\text{m}\,;0{,}05\,m]$ (in the case of the modelization in triangle, it is important not to have too "flattened" triangles, the choice height of the field is thus not pain-killer).



The period of simulation is of 100s and the number of time step is of 100.

1.2 Properties of the materials

One gives here only the properties whose solution depends, knowing that the command file contains other data of material which do not play any part in the solution of with the dealt problem.

Gas	Molar mass $(kg\,.\,mol^{-1})$	$0{,}0001$
	Viscosity $(kg\,.\,m^{-1}.\,s^{-1})$	1
	relative Permeability (m^2)	1
dissolved Gas	Coefficient of Henry $(Pa\,.\,mol^{-1}.\,m^3)$	10000000000
Liquidates	relative Permeability (m^2)	1
homogenized Parameters	Permeability $K_{int}(m^2)$	10^{-7}
	Porosity	1
	Fick liquid $(m^2.\,s^{-1})$	0
	gas Fick $(m^2.\,s^{-1})$	0

Table 1.2-1 : Properties of the materials

1.3 Boundary conditions and loadings

the limiting conditions are the following ones:

- conditions of Neumann on the right of field:

$$\frac{\partial(\delta P_g)}{\partial x}(t, x=5, y)=0\,Pa$$

- condition of Dirichlet on the left part of the field:

$$P_g(t, x=0, y)=0\,Pa$$

1.4 Initial conditions

the variation of initial gas pressure compared to the pressure of reference is of $\delta P_g(t=0, x, y)=10^4\,Pa$.

One also has $P_g^{ref}(t=0, x, y)=10^4\,Pa$ what amounts studying a problem slightly nonlinear (to be linear one would have to choose $P_g^{ref}(t=0, x, y)=10^{10}\,Pa$) (because one a: $P_g(t=0, x, y)=P_g^{ref}+\delta P_g(t=0, x, y)$).

2 Analytical solution

2.1 Method of calculating

the non stationary and monodimensional monophasic problem can be written in a general form of the type:

$$N\frac{\partial P}{\partial t}-K_{int.}\Delta P=0$$
$$P(t=0)=P_0$$
$$P(t,x=0)=0$$
$$\frac{\partial P}{\partial x}(t,x,L)=0$$

This problem admits an analytical solution obtained by development in Fourier series.

$$P=\sum_{k=0}^{K}\frac{4\mathrm{P}_0}{(2\mathrm{k}+1)\pi}\exp(\frac{-K_{int}}{N}\omega_k^2 t)\sin(\omega_k x) \text{ with } \omega_k=(k+\frac{1}{2})\frac{\pi}{L}$$

the number of terms K of this series is in the following way given:

That is to say n_x the number of points x_i where the solution is evaluated at one time t.
One poses:

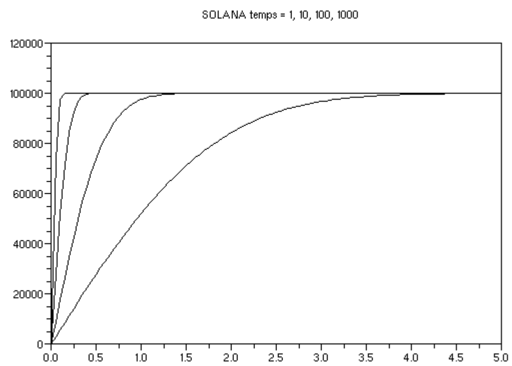
$$a_k^i=\frac{4}{(2\mathrm{k}+1)\pi}\exp(\frac{-K_{int}}{N}\omega_k^2 t)\sin(\omega_k x_i)$$

So that the solution can be written: $P(x_i)=\sum_{k=0}^{K}P_0.a_k^i$

One chooses K such as: $\frac{1}{n_x}\sqrt{\sum_{i=1}^{nx}(a_k^i)^2}<\epsilon$

In practice we took $\varepsilon=10^{-10}$.

The paces of the analytical solution at times 1,10,100,1000 are shown on the figure below:



The following table gives the number of terms according to time:

Time	Many series terms
1.194	
10	64.100
	22
1000	8

Table 2.1-1 : Representation amongst term according to time

2.2 simplifying Assumptions

One considers that the medium is completely saturated with gas and one null imposes in aster a fluid pressure on all the nodes. One imposes an initial pressure of gas P_g^{ref} and one gives boundary conditions which correspond to a variation of this pressure of reference, That is to say then δP_g this variation of pressure. The conservation equation of the mass of gas will be written:

$$\frac{\partial(\varphi \delta P_g)}{\partial t}+div(\frac{K_{int} k_g}{\mu_g}(P_g^{ref}+\delta P_g) div(P_g^{ref}+\delta P_g))=0$$

While supposing δP_g small in front of P_g^{ref}, this equation becomes:

$$\frac{\partial(\varphi \delta P_g)}{\partial t}+\frac{K_{int} k_g}{\mu_g} P_g^{ref} \Delta(\delta P_g)=0$$

It is thus δP_g that one will identify with the solution of the model analytical equation.

In order to find the coefficients of the model problem, one will take:

$$\varphi=1$$
$$k_g=\mu_g=1$$

and it will be made so that

$$K_{int} P_{ref}=10^{-3} .$$

3 Modelization A

3.1 Characteristic of the modelization

Modelization D_PLAN_HH2SUDM. This modelization corresponds to the modelization Volume Finished with decentring on the mesh close for the terms to mobilities (the fickiens terms are realized). The coupling law hydraulic one is LIQU_AD_GAZ_VAPE.

3.2 Characteristics of the mesh

One uses a mesh made up of 100 elements QUAD9.

3.3 Quantities tested and results

One traces the profiles of gas pressure at various times as well as the analytical solution at these same times. We are slightly nonlinear here, it is normal that the curves are slightly far away from the analytical solution.

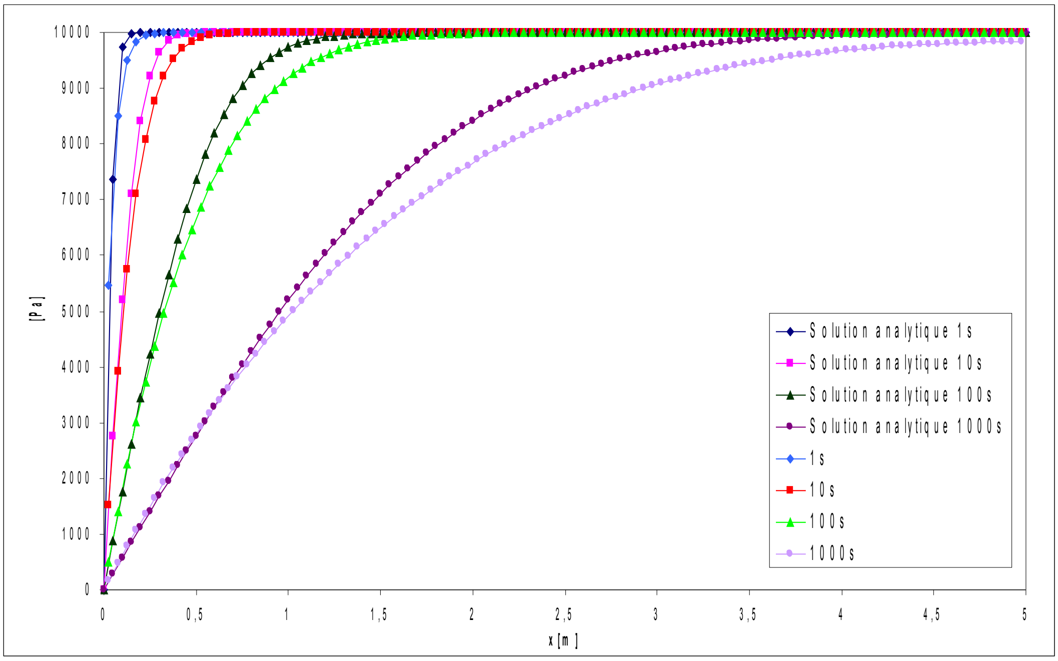


Illustration 1: Profiles of gas pressure

One for the first time carries out tests on 4 nodes with 1 time by comparing the results with the analytical solution and second once by carrying out a test of non regression.

Quantity	Points (x, y)	Time (S)	Standard reference	Reference	analytical
Tolerance	$(0{,}075; 0)$ $N304$	100	100	1331.0	7.0%
non regression	$(0{,}075; 0)$ $N304$	100	100	1412.2251	1.0%
analytical	$(0{,}075; 1)$ $N293$	100	100	1331.0	7.0%
non regression	$(0{,}075; 1)$ $N293$	100	100	1412.2509021789	1.0%
analytical	$(0{,}05; 0{,}05)$ $N469$	100	100	889.3176	7.5%
non regression	$(0{,}05; 0{,}05)$ $N469$	100	100	955.70263804746	1.0%
analytical	$(0{,}075; 0{,}5)$ $NQ95$	100	100	1331.0	9.0%
non regression	$(0{,}075; 0{,}5)$ $NQ95$	100	100	1412.2509021788	1.0%

Table 3.3-1 : Values tested

4 Modelization B

4.1 Characteristic of the modelization

Modelization D_PLAN_HH2SUDA. This modelization corresponds to the modelization Volume Finished eccentric on the edges for mobilities (the fickiens terms are centered). The coupling law hydraulic one is LIQU_AD_GAZ_VAPE.

4.2 Characteristics of the mesh

One uses a mesh made up of 100 elements QUAD9.

4.3 Quantities tested and results

One for the first time carries out tests on 4 nodes with 1 time by comparing the results with the analytical solution and second once by carrying out a test of non regression.

Quantity	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	$(0{,}075; 0)$ *N304*	100	analytical	1331.0	9.0%
non regression	$(0{,}075; 0)$ *N304*	100	100	1444.1	1.0%
analytical	$(0{,}075; 1)$ *N293*	100	100	1331.0	9.0%
non regression	$(0{,}075; 1)$ *N293*	100	100	1444.1	1.0%
analytical	$(0{,}05; 0{,}05)$ *N469*	100	100	889.3176	12.0%
non regression	$(0{,}05; 0{,}05)$ *N469*	100	100	990.9	1.0%
analytical	$(0{,}075; 0{,}5)$ *NQ95*	100	100	1331.0	9.0%
non regression	$(0{,}075; 0{,}5)$ *NQ95*	100	100	1444.1	1.0%

Table 4.3-1 : Values tested

5 Modelization C

5.1 Characteristic of the modelization

Modelization D_PLAN_HH2SUC. This modelization corresponds to the modelization centered Finished Volumes (for the darcéens terms and fickiens). The coupling law hydraulic one is LIQU_AD_GAZ_VAPE.

5.2 Characteristics of the mesh

The mesh consists of 100 elements QUAD9.

5.3 Quantities tested and results

the results are identical that those obtained with the modelization Volumes finished eccentric on the mesh.

One for the first time carries out tests on 4 nodes with 1 time by comparing the results with the analytical solution and second once by carrying out a test of non regression.

Quantity	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	$(0,075; 0)$ *N304*	100	analytical	1331.0	9.0%
non regression	$(0,075; 0)$ *N304*	100	100	1444.0876929334	1.0%
analytical	$(0,075; 1)$ *N293*	100	100	1331.0	9.0%
non regression	$(0,075; 1)$ *N293*	100	100	1444.0876929335	1.0%
analytical	$(0,05; 0,05)$ *N469*	100	100	889.3176	9.0%
non regression	$(0,05; 0,05)$ *N469*	100	100	1444.0876929335	1.0%
analytical	$(0,075; 0,5)$ *NQ95*	100	100	1331.0	12.0%
non regression	$(0,075; 0,5)$ *NQ95*	100	100	990.94043287847	1.0%

Table 5.3-1 : Values tested

6 Modelization D

6.1 Characteristic of the modelization D

Modelization D_PLAN_HH2S. This modelization corresponds to the modelization Conventional finite elements. The coupling law hydraulic one is LIQU_AD_GAZ_VAPE.

6.2 Characteristics of the mesh

The mesh consists of 100 elements QUAD8.

6.3 Quantities tested and results

One for the first time carries out tests on 2 nodes with 1 time by comparing the results with the analytical solution and second once by carrying out a test of non regression.

Quantity	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	$(0{,}05; 0)$ *N104*	100	analytical	889.3176	12.0%
non regression	$(0{,}05; 0)$ *N104*	100	100	992.8591566774	1.0%
analytical	$(0{,}05; 1)$ *N103*	100	100	889.3176	12.0%
non regression	$(0{,}05; 1)$ *N103*	100	100	992.85915667759	1.0%

Table 6.3-1 : Values tested

7 Modelization E

7.1 Characteristic of the modelization

Modelization D_PLAN_HH2SUDM. This modelization corresponds to the modelization Volume Finished with decentring on the mesh close for the terms to mobilities (the fickiens terms are realized). The coupling law hydraulic one is LIQU_AD_GAZ_VAPE. The goal is here to validate this diagram on meshes triangles.

7.2 Characteristics of the mesh

The mesh consists of 200 elements TRIA7.

7.3 Quantities tested and results

One traces the profiles of gas pressure at various times as well as the analytical solution at these same times. We are slightly nonlinear here, it is normal that the curves are slightly far away from the analytical solution.

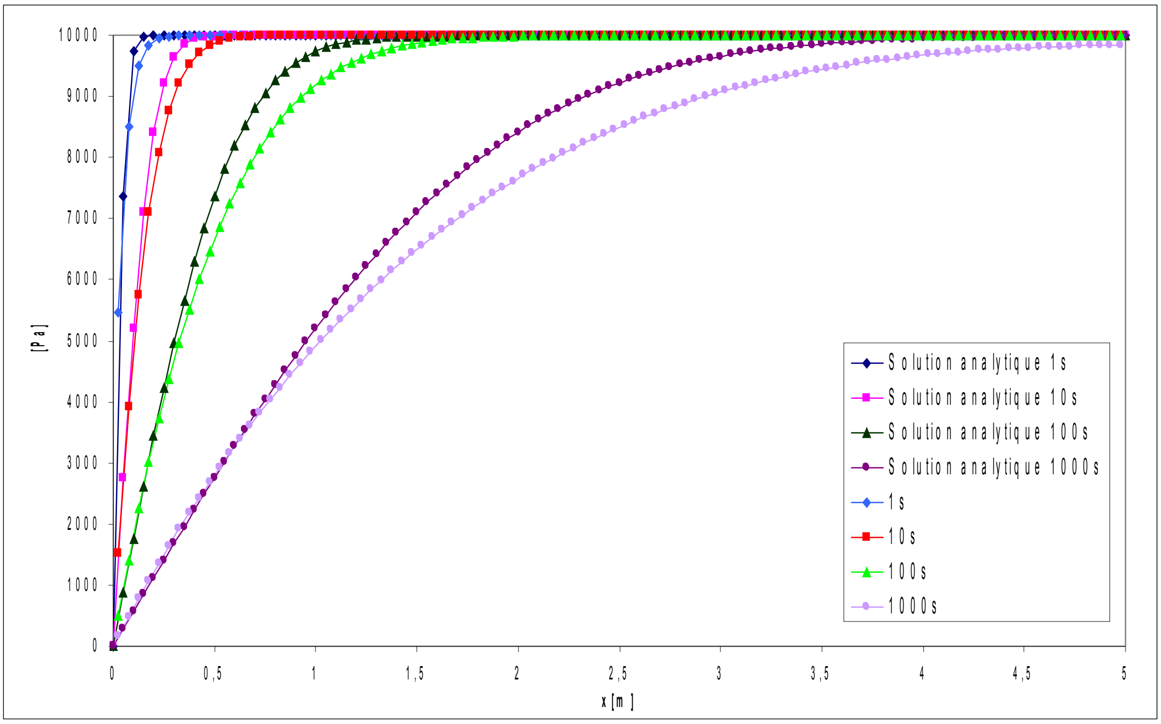


Illustration 2: Profiles of gas pressures

One for the first time carries out tests on 3 nodes with 1 time by comparing the results with the analytical solution and second once by carrying out a test of non regression.

Quantity	Points (x, y)	Time (S)	Standard reference	Reference	analytical
Tolerance	$(0{,}075\,;\,0)$ *N360*	100	100	1331.0	10.0%
non regression	$(0{,}075\,;\,0)$ *N360*	100	100	1454.395434045	1.0%
analytical	$(0{,}075\,;\,0{,}025)$ *N505*	100	100	1331.0	10.0%
non regression	$(0{,}075\,;\,0{,}025)$ *N505*	100	100	1450.3938034156	1.0%
non regression	$(0{,}016\,;\,0{,}0158)$ *NT70*	100	100	353.97760862167	1.0%

Table 7.3-1 : Values tested

8 Modelization F

8.1 Characteristic of the modelization

Modelization D_PLAN_HH2SUDA. This modelization corresponds to the modelization Volumes Finished eccentric on the edges for mobilities (the fickiens terms are centered). The coupling law hydraulic one is LIQU_AD_GAZ_VAPE.

8.2 Characteristics of the mesh

The mesh consists of 200 elements TRIA7.

8.3 Quantities tested and results

One carries out tests on 2 nodes with 1 time by comparing the results with the analytical solution and on 3 nodes with 1 time by carrying out a test of non regression.

Quantity	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	$(0{,}075;0)$ *N360*	100	analytical	1331.0	11.0%
non regression	$(0{,}075;0)$ *N360*	100	100	1454.4	1.0%
analytical	$(0{,}075;0{,}025)$ *N505*	100	100	1331.0	10.0%
non regression	$(0{,}075;0{,}025)$ *N505*	100	100	1450.4	1.0%
non regression	$(0{,}016;0{,}0158)$ *NT70*	100	100	353.978	1.0%

Table 8.3-1: Values tested

9 Modelization G

9.1 Characteristic of the modelization

Modelization D_PLAN_HH2SUC. This modelization corresponds to the modelization centered Volume Finished (for the darcéens terms and fickiens). The coupling law hydraulic one is LIQU_AD_GAZ_VAPE.

9.2 Characteristics of the modelization

The mesh consists of 200 elements TRIA7.

9.3 Quantities tested and results

One carries out tests on 2 nodes with 1 time by comparing the results with the analytical solution and on 3 nodes with 1 time by carrying out a test of non regression.

Quantity	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	$(0{,}075 ; 0)$ *N360*	100	analytical	1473.0	10.0%
non regression	$(0{,}075 ; 0)$ *N360*	100	100	1473.3	1.0%
analytical	$(0{,}075 ; 0{,}025)$ *N505*	100	100	1472.0	10.0%
non regression	$(0{,}075 ; 0{,}025)$ *N505*	100	100	1472.4	1.0%
non regression	$(0{,}016 ; 0{,}0158)$ *NT70*	100	100	353.978	1.0%

Table 9.3-1 : Values tested

10 Modelization H

10.1 Characteristic of the modelization

Modelization D_PLAN_HH2S. This modelization corresponds to the modelization Finite elements. The coupling law hydraulic one is LIQU_AD_GAZ_VAPE.

10.2 Characteristics of the mesh

The mesh consists of 200 elements TRIA6.

10.3 Quantities tested and results

One for the first time carries out tests on 2 nodes with 1 time by comparing the results with the analytical solution and second once by carrying out a test of non regression.

Quantity	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	$(0{,}05\,;0)$ *N103*	100	analytical	889.3176	12.0%
non regression	$(0{,}05\,;0)$ *N103*	100	100	992.699	1.0%
analytical	$(0{,}05\,;0{,}05)$ *N203*	100	100	889.3176	12.0%
non regression	$(0{,}05\,;0{,}05)$ *N203*	100	100	993.06	1.0%

Table 10.3-1 : Values tested

11 Modelization I

11.1 Characteristic of the modelization

Modelization D_PLAN_HS. This modelization corresponds to the modelization Finite elements. The coupling law hydraulic one is GAZ.

11.2 Characteristics of the mesh

The mesh consists of 200 elements TRIA6.

11.3 Quantities tested and results

One for the first time carries out tests on 2 nodes at $t=100\,ans$ time by comparing the results with the analytical solution and second once by carrying out a test of non regression.

Quantity	Points (x, y)	Time (S)	Standard reference	Reference	Tolerance (%)
PRE2	$(0{,}05;0{,}05)$ $N103$	100	analytical	889.3176	12.0%
non regression	$(0{,}05;0{,}05)$ $N103$	100	100	992.699	1.0%
analytical	$(0{,}05;0)$ $N203$	100	100	889.3176	12.0%
non regression	$(0{,}05;0)$ $N203$	100	100	993.06	1.0%

Table 11.3-1 : Values tested

12 Summary of the results

This case test makes it possible to test the diagrams volumes finished in various configurations on a modelization of gas flow:

- the 3 diagrams finished volumes: centered, decentred edge, decentred mesh;
- on various types of meshes (triangles and rectangles).

These same cases are also carried out with the diagrams conventional finite elements. All the results are very close to the analytical solution.

From a performance point of view, one will recommend the use of the eccentric diagram edge.