WTNP115 – Desaturation of a porous environment without air on unit cell

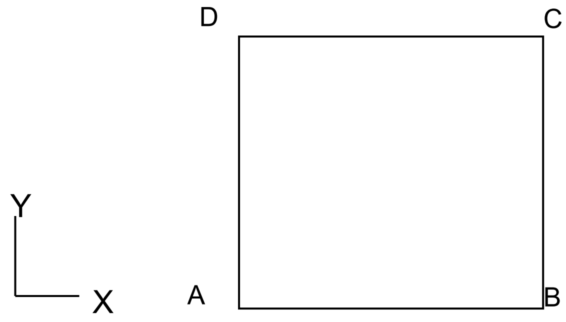
Abstract:

One heats a porous environment whose pores are filled with a mixture of water and steam. Initial fluid saturation is of 50%, the loading is a uniform heat flux on edges of the field. The modelization made by only one element corresponds to the modelization of a homogeneous problem in space.

The reference solution is an approximate analytical solution.

1 Problem of reference

1.1 Geometry



Coordinated points (m):

A	0	0.100.100	C	
B	100	-0	D	0.100

1.2 Properties of the material

One gives here only the properties whose solution depends, knowing that the command file contains other data of material (elasticity moduli, thermal conductivity…) who finally do not play any part in the solution of with the dealt problem.

Liquid water	Density ($kg.m^{-3}$)	103
	Heat with constant pressure ($J.K^{-1}$)	4180
	thermal coefficient of thermal expansion of the fluid (K^{-1})	0.
Vapor	Heat capacity ($J.K^{-1}$)	1900
	initial Enthalpy (latent heat of vaporization)	2,5E6.
	Molar mass ($kg.mol^{-1}$)	0,018
Squelette	Heat capacity with constant stress ($J.K^{-1}$)	1050
initial State	Porosity	fluids
	Temperature (K)	
	Pressure of fluid (Pa)	0,3.300
	Steam pressure (Pa)	1E5
	initial Saturation out of Constant	3700
0,5	Constant of perfect gases	8,315
homogenized Coefficients	homogenized Density ($kg.m^{-3}$)	2200
	Isothermal of sorption	$S(P_c)=0.5-10^{-12}\left(P_c-P_{vp}^0-P_c^0\right)$ With $P_{vp}^0=3700$

$$P_c^0 = -10^5$$

1.3 Boundary conditions and loadings

On all the edges:

Heat flux $\mathbf{q}_{ext}.\mathbf{n}=10^6$

hydraulic Flux no

2 Reference solution

2.1 Method of calculating

2.1.1 Computation of the steam pressure from the temperature

We suppose the linear curve of saturation. She is thus written:

$$S=S_0+S'\Delta P_c \quad \text{éq 2.1.1-1}$$

[R7.01.11 éq 3.2.1-2] gives then:

$$\begin{aligned}\Delta m_{lq}&=\rho_{lq}\varphi S'\Delta P_c-\rho_{lq}^0\varphi^0 S_{lq}^0\\ \Delta m_{vp}&=\left(\rho_{vp}-\rho_{vp}^0\right)\varphi^0\left(1-S_0\right)-S'\rho_{vp}^0\varphi^0\Delta P_c\end{aligned} \quad \text{éq 2.1.1-2}$$

One writes that the total mass of water is preserved (because there is no flux of water to edge) and one obtains:

$$\begin{aligned}&\Delta m_{lq}+\Delta m_{vp}=0 \qquad \Rightarrow\\ &\left(\rho_{lq}-\rho_{vp}\right)S'\Delta P_c+\left(\rho_{vp}-\rho_{vp}^0\right)\left(1-S_0\right)=0\end{aligned} \quad \text{éq 2.1.1-3}$$

[R7.01.11 éq 4.4-1] gives éq

$$\begin{aligned}\ln\left(\frac{p_{vp}}{p_{vp}^0}\right)=&\frac{M_{vp}^{ol}}{RT}\frac{1}{\rho_{lq}}\Delta P_{lq}+\\ &\frac{M_{vp}^{ol}}{R}\left(h_{vp}^0-h_{lq}^0\right)\left(\frac{1}{T^0}-\frac{1}{T}\right)+\frac{M_{vp}^{ol}}{R}\left(C_{vp}^p-C_{lq}^p\right)\left(\ln\left(\frac{T}{T^0}\right)+\frac{T^0}{T}-1\right)\end{aligned} \quad \text{2.1.1-4 in addition}$$

the coupling of the equations [éq 2.1.1-3] and [éq 2.1.1-4], for which it is necessary to add the equation of perfect gases for the vapor, is a strongly nonlinear system which we will solve in small disturbance, which makes it possible to linearize it.
All done calculations, one obtains:

$$\left.\begin{aligned}&\Delta P_{vp}\left(\left(\rho_{lq}-\rho_{vp}^0\right)S'+\frac{\left(1-S_0\right)M_{vp}^{ol}}{RT^0}\right)-\left(\rho_{lq}-\rho_{vp}^0\right)S'\Delta P_{lq}=\left(1-S_0\right)p_{vp}^0\frac{M_{vp}^{ol}}{R}\frac{\Delta T}{{T^0}^2}\\ &\frac{\Delta P_{vp}}{p_{vp}^0}-\frac{M_{vp}^{ol}}{\rho_{lq}RT^0}\Delta P_{lq}=\frac{M_{vp}^{ol}}{R}\left(h_{vp}^0-h_{lq}^0\right)\frac{\Delta T}{{T^0}^2}\end{aligned}\right\} \quad \text{éq 2.1.1-5}$$

2.1.2 Computation of the temperature

[R7.01.11 éq 3.2.4.3 - 1] gives:

$$\Delta Q' = -3\alpha_{gz}^{m} T \Delta p_{vp} + C_{\varepsilon}^{0} \Delta T \quad \text{éq 2.1.2-1}$$

(since the other coefficients of thermal expansion are null).
[éq 3.2.4.3 - 2] gives:

$$\alpha_{gz}^{m} = \frac{\varphi(1-S_{lq})}{3T} \quad \text{éq 2.1.2-2}$$

One thus obtains:

$$\Delta Q' = -\varphi(1-S_{lq})\Delta p_{vp} + C_{\varepsilon}^{0} \Delta T \quad \text{éq 2.1.2-3}$$

In this problem, $\Delta Q'$ is anything else only the heat brought per unit of volume.

By calling Vol the total volume of the part and $Surf$ its side surface and Δt the time of application of flux:

$$\Delta Q' = \Delta t \frac{Surf}{Vol} \mathbf{q}_{ext}.\mathbf{n} \quad \text{éq 2.1.2-4}$$

2.1.3 System to solve

$$\begin{bmatrix} \left((\rho_{lq} - \rho_{vp}^{0}) S' + \dfrac{(1-S_0) M_{vp}^{ol}}{RT^0} \right) & -(\rho_{lq} - \rho_{vp}^{0}) S' & -(1-S_0) p_{vp}^{0} \dfrac{M_{vp}^{ol}}{RT^{0^2}} \\ \dfrac{1}{p_{vp}^{0}} & -\dfrac{M_{vp}^{ol}}{\rho_{lq} RT^0} & -\dfrac{M_{vp}^{ol}}{R} \dfrac{(h_{vp}^{0} - h_{lq}^{0})}{T^{0^2}} \\ 0 & -\varphi(1-S_{lq}) & C_{\varepsilon}^{0} \end{bmatrix} \begin{Bmatrix} \Delta P_{vp} \\ \Delta P_{lq} \\ \Delta T \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \Delta t \dfrac{Surf}{Vol} \mathbf{q}_{ext}.\mathbf{n} \end{Bmatrix}$$

éq 2.1.3-1

2.2 Results of reference

One gives the value of the temperature, the fluid pressure and the steam pressure, solution of the system [éq 2.1.3-1] with the data summarized in the paragraphs [§1.2] and pointed out Ci below. For the computation of heat capacities, one uses the following relations:

$$\left(1-\varphi^0\right)\rho_s = r_0 - \rho_{lq}^0 S_l^0 \varphi^0 - \left(1-S_l^0\right)\varphi^0 \rho_{vp}^0$$
$$C_\sigma^0 = (1-\varphi)\rho_s C_\sigma^s + \rho_{lq} S_l \varphi C_{lq}^p + (1-S_l)\varphi\ \rho_{vp} C_{vp}^p$$

$C_\varepsilon^0 = C_\sigma^0$, this last relation being true because the coefficient of thermal expansion of the grains is null.

S_0	S'	T^0	p_{vp}^0	h_{vp}^0	ρ_{vp}^0 (calculated)	ρ_{lq}
5,00E-01	-1,00E-12	3,00E+02	3,70E+03	2,50E+06	2,67E-02	1,00E+03

r_0	φ^0	ρ_s (calculated)	C_σ^s	C_{lq}^p L	C_{vp}^p	C_ε^0 (calculated)
2,20E+03	3,00E-01	2,93E+03	1,05E+03	4,18E+03	1,90E+03	2,78E+06

$\mathbf{q}_{ext}.\mathbf{n}$	Δt	$Surf$	Vol
1,00E+06	1000	400	1,00E+04

After resolution, one gets the following results:

ΔP_{vp}	3.E+03
P_{lq}	-1E+07
ΔT	14

2.3 Uncertainties

uncertainties are rather large because the analytical solution is a solution approached because of linearization of the equations.

3 Modelization A

3.1 Characteristic of the modelization A

Modelization in plane strains. An element $Q8$
Discretization in time: only one time step: $10^3 s$.

3.2 Values tested

Standard	Node of value	Time (s)	Reference (analytical)	*Aster*	Difference (%)
$NO1$	DEPL/TEMP	10^{3}	14	14,4	2.7%
$NO1$	DEPL/PRE1	103	-1.10^{7}	-1.3 107.30%	
$NO1$	VARI_ELNO/V4		$3.\ 10^{3.3.9}$	$10^{3.30\%}$	

One thus finds results relatively close to the analytical results. Uncertainty remaining rather broad because of linearization of the equations.