WTNP106 - Heating of a porous environment désaturé with dissolved air

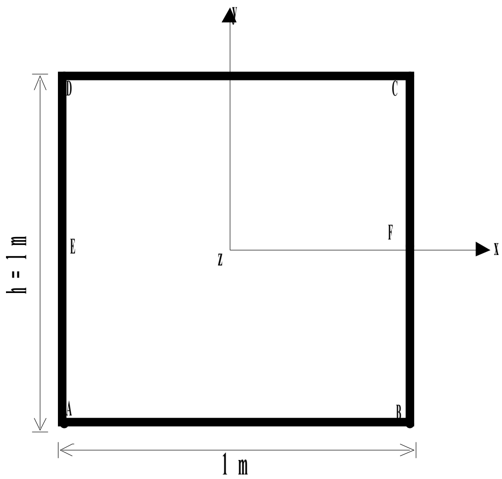
Abstract:

One heats a porous environment of which the pores are filled with a mixture of water (liquidates and vapor) and of air (dry and dissolved in water). Initial fluid saturation is of 50%, the loading is a uniform heat flux on edges of the field. The modelization made by only one element corresponds to the modelization of a homogeneous problem in space.

The reference solution is an approximate analytical solution.

1 Problem of reference

1.1 Geometry



Coordinated of the points (m):

A	-0,5	-0,5	C	0,5. 0,5. 0,5		
B		-0,5	D	-0,5	0,5	

1.2 Properties of the material

One gives here only the properties whose solution depends, knowing that the command file contains other data of material (elasticity moduli, thermal conductivity…) who finally do not play any part in the solution of with the dealt problem.

Liquid water	Density ($kg.m^{-3}$) Heat with constant pressure ($J.K^{-1}$) thermal coefficient of thermal expansion of the fluid (K^{-1}) Dynamic viscosity of liquid water ($Pa.s$) Permeability relating to water	103 4180 0. 0.001 $kr_w(S)=1$
Vapor	Specific heat ($J.K^{-1}$) initial Enthalpy (latent heat of vaporization) J/Kg Molar mass ($kg.mol^{-1}$)	1900 2,5E6. 0,018
Gases	Specific heat ($J.K^{-1}$) Molar mass ($kg.mol^{-1}$)	1900 0,018 $kr_{gz}(S)=1$

	Permeability relating to the gas Viscosity of gas ($kg.m^{-1.s^{-1}}$)	1,8E-5
dissolved Air	Specific heat ($J.K^{-1}$) Constant of Henry ($Pa.m^{3.}mol^{-1}$)	1900 50000
Squelette	Heat capacity to constant stress ($J.K^{-1}$)	1050
initial State	Porosity Temperature (K) Pressure of gas (Pa) Steam pressure (Pa) initial Saturation out of fluid (Pa)	0,3.300 1E5 3700 0,5
Constant	Constant of perfect gases	8,315
homogenized Coefficients	homogenized Density ($kg.m^{-¿3}$) Isothermal of sorption	2200 $S(P_c)=0.5-10^{-12}\left(P_c-P_{vp}^0-P_c^0\right)$ With $P_{vp}^0=3700$ $P_c^0=0$

1.3 Boundary conditions and loadings

On all the edges:

Heat flux $\mathbf{q}_{ext}.\mathbf{n}=10^6$

hydraulic Flux no

2 Reference solution

2.1 Method of calculating

2.1.1 Computation of the steam pressure from the temperature

We suppose the linear curve of saturation. She is thus written:

$$S = S_0 + S'\Delta P_c \quad \text{éq 2.1.1-1}$$

the equation [éq 2.2.3.3 - 2] of the reference document [R7.01.11] gives then:

$$\begin{aligned}
\Delta m_w &= \rho_w \varphi S' \Delta P_c \\
\Delta m_{vp} &= \left(\rho_{vp} - \rho_{vp}^0\right)\varphi^0\left(1 - S_0\right) - S'\rho_{vp}^0\varphi^0\Delta P_c \\
\Delta m_{ad} &= \left(\rho_{ad} - \rho_{ad}^0\right)\varphi^0 S_0 + S'\rho_{ad}^0\varphi^0\Delta P \\
\Delta m_{as} &= \left(\rho_{as} - \rho_{as}^0\right)\varphi^0\left(1 - S_0\right) - S'\rho_{as}^0\varphi^0\Delta P_c
\end{aligned} \quad \text{éq 2.1.1-2}$$

One writes that the total mass of water and the total mass of air are preserved (because there is no water flux nor of gas to edge) and one obtains:

$$\begin{aligned}
&\Delta m_w + \Delta m_{vp} = 0 \quad \Rightarrow \\
&\left(\rho_w - \rho_{vp}\right)S'\Delta P_c + \left(\rho_{vp} - \rho_{vp}^0\right)\left(1 - S_0\right) = 0 \quad \text{éq 2.1.1-3} \\
&\Delta m_{ad} + \Delta m_{as} = 0 \quad \Rightarrow \\
&\left(\rho_{ad} - \rho_{as}\right)S'\Delta P_c + \left(\rho_{as} - \rho_{as}^0\right)\left(1 - S_0\right) + \left(\rho_{ad} - \rho_{ad}^0\right)S_0 = 0 \quad \text{éq 2.1.1-4}
\end{aligned}$$

[R7.01.11] [éq 4.1.4-1] gives in addition:

$$\begin{aligned}
\ln\left(\frac{p_{vp}}{p_{vp}^0}\right) = &\frac{M_{vp}^{ol}}{\rho_w^0}\left(\frac{1}{RT} - \frac{1}{K_H}\right)\left(p_{gz} - p_{gz}^0\right) + \frac{M_{vp}^{ol}}{\rho_w^0 K_H}\left(p_{vp} - p_{vp}^0\right) - \frac{M_{vp}^{ol}}{\rho_w^0 R T^+}\left(p_c - p_c^0\right) + \\
&\frac{M_{vp}^{ol} R}{\rho_w^0 K_H}\left(p_{vp} - p_{gz}\right)\ln\left(\frac{T}{T^0}\right) + \frac{M_{vp}^{ol}}{R}\int_{T^0}^{T}\left(h_{vp}^m - h_w^m\right)\frac{dT}{T^2}
\end{aligned} \quad \text{éq 2.1.1-5}$$

coupling of the equations [éq 2.1.1-3], [éq 2.1.1-4] and [éq 2.1.1-5], for which it the equation of perfect gases for the vapor, dry air and dissolved air are necessary to add as well as the model of Henry is a strongly nonlinear system which we will solve in small disturbances, which makes it possible to linearize it.

All done calculations, one obtains:

$$\left.\begin{aligned}
&\Delta P_{vp}\left(\left(\rho_w-\rho_{vp}^0\right)S'+\frac{\left(1-S_0\right)M_{vp}^{ol}}{RT^0}\right)-\left(\rho_w-\rho_{vp}^0\right)S'\Delta P_w+\Delta P_{as}\left(\left(\rho_w-\rho_{vp}^0\right)S'.\left(1-\frac{RT^0}{K_H}\right)\right)+\\
&\left(-\left(\rho_w-\rho_{vp}^0\right)S'.\frac{RP_{as}}{K_H}-\frac{M_{vp}^{ol}P_{vp}^0}{RT^{0^2}}\left(1-S_0\right)\right)\Delta T=0\\
&\Delta P_{vp}\left(\left(\rho_{ad}^0-\rho_{as}^0\right)S'\right)-\left(\rho_{ad}^0-\rho_{as}^0\right)S'\Delta P_w+\Delta P_{as}\left(\left(\rho_{ad}^0-\rho_{as}^0\right)S'.\left(1-\frac{RT^0}{K_H}\right)+M_{vp}^{ol}.\left(\frac{S_0}{K_H}+\frac{\left(1-S_0\right)}{RT^0}\right)\right)+\\
&\left(-\left(\rho_{ad}^0-\rho_{as}^0\right)S'.\frac{RP_{as}}{K_H}-\frac{M_{vp}^{ol}P_{as}}{RT^{0^2}}\left(1-S_0\right)\right)\Delta T=0\\
&\Delta P_{vp}\left(-\frac{1}{P_{vp}^0}\right)+\frac{M_{vp}^{ol}}{\rho_w^0RT}\Delta P_w+\\
&\left(\frac{M_{vp}^{ol}}{\rho_w^0}\frac{P_{as}}{K_HT^0}\left(1-R\right)+\frac{M_{vp}^{ol}}{R}\frac{h_{vp}^m-h_w^m}{T^{0^2}}\right)\Delta T=0
\end{aligned}\right\}$$

éq 2.1.1-6

2.1.2 Computation of the temperature

the equation [éq 3.2.4.3 - 1] of the reference document [R7.01.11] gives:

$$\Delta Q'=-3\alpha_{gz}^mT\Delta p_{gz}+C_\varepsilon^0\Delta T \qquad \text{éq 2.1.2-1}$$

(since the other coefficients of thermal expansion are null).

The equation [éq 3.2.4.3 - 2] gives:

$$\alpha_{gz}^m=\frac{\varphi\left(1-S_{lq}\right)}{3T} \qquad \text{éq 2.1.2-2}$$

One thus obtains:

$$\Delta Q'=-\varphi\left(1-S_{lq}\right)\left(\Delta p_{vp}+\Delta p_{as}\right)+C_\varepsilon^0\Delta T \qquad \text{éq 2.1.2-3}$$

In this problem, $\Delta Q'$ is anything else only the heat brought per unit of volume.

By calling Vol the total volume of the part and $Surf$ its side surface and Δt the time of application of flux:

$$\Delta Q'=\Delta t\frac{Surf}{Vol}\mathbf{q}_{ext}.\mathbf{n} \qquad \text{éq 2.1.2-4}$$

2.1.3 System to solve

$$\begin{bmatrix} \left((\rho_w-\rho_{vp}^0)S' + \frac{(1-S_0)M_{vp}^{ol}}{RT^0}\right) & -(\rho_w-\rho_{vp}^0)S' & -(1-S_0)p_{vp}^0\frac{M_{vp}^{ol}}{RT^{0^2}}.-(\rho_w-\rho_{vp}^0)S'\frac{RP_{as}^0}{K_H} & (\rho_w-\rho_{vp}^0)S'.(1-\frac{RT^0}{K_H}) \\ (\rho_{ad}^0-\rho_{as}^0)S' & -(\rho_{ad}^0-\rho_{as}^0)S' & -(\rho_{ad}^0-\rho_{as}^0)S'.\frac{RP_{as}^0}{K_H}-\frac{M_{vp}^{ol}P_{as}^0}{RT^{0^2}}(1-S_0) & (\rho_{ad}^0-\rho_{as}^0)S'\left(1-\frac{RT^0}{K_H}\right)+M_{vp}^{ol}\left(\frac{S_0}{K_H}+\frac{(1-S_0)}{RT^0}\right) \\ -\frac{1}{P_{vp}^0} & \frac{M_{vp}^{ol}}{\rho_w^0 RT} & \frac{M_{vp}^{ol}P_{as}^0}{\rho_w^0 K_H T^0}(1-R)+\frac{M_{vp}^{ol}}{R}\frac{h_{vp}^m-h_w^m}{T^{0^2}} & 0 \\ -\varphi(1-S_{lq}) & 0 & C_\varepsilon^0 & -\varphi(1-S_{lq}) \end{bmatrix}$$

$$\times \begin{Bmatrix} \Delta P_{vp} \\ \Delta P_w \\ \Delta T \\ \Delta P_{as} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ \Delta t\frac{Surf}{Vol}\mathbf{q}_{ext}.\mathbf{n} \\ 0 \end{Bmatrix}$$

éq 2.1.2-5

S_0	S'	T^0	p_{vp}^0	h_{vp}^0	ρ_{vp}^0 (calculated)	ρ_{lq}
5,00E-01	-1,00E-12	3,00E+02	3,70E+03	2,50E+06	2,67E-02	1,00E+03

r_0	φ^0	ρ_s (calculated)	C_σ^s	C_{lq}^p L	C_{vp}^p	C_ε^0 (calculated)
2,20E+03	3,00E-01	2,93E+03	1,05E+03	4,18E+03	1,90E+03	2,78E+06

$\mathbf{q}_{ext}.\mathbf{n}$	Δt	$Surf$	Vol
1,00E+06	10.400		1,00E+04

One gets the following results:

After resolution of this system, one obtains:

$$\begin{Bmatrix} \Delta P_{vp} \\ \Delta P_w \\ \Delta T \\ \Delta P_{as} \end{Bmatrix} = \begin{Bmatrix} 29.4 \\ -99500 \\ 0.144 \\ 45.7 \end{Bmatrix}$$

What gives in term of result Aster (increment):

$PRE1$	$PRE2$	DT	$PVP(V3)$
9.95 E4	7.5E1	1.44E-1	2.94E1

2.2 Uncertainties

uncertainties are rather large because the analytical solution is a solution approached because of linearization of the equations.

3 Modelization A

3.1 Characteristic of the modelization A

Modelization in plane strains. An element $Q8$.
Discretization in time: only one time step: $10\,s$.

3.2 Values tested

Node	Urgent	Field	Component (s)	Reference (analytical)
$NO1$	DEPL	$TEMP$	10	0.1440
$NO1$	DEPL	$PRE1$	10	$9.95\,10^4$
$NO1$	DEPL	$PRE2$	10	75
$NO1$	VARI_ELNO	$V3$	10	29.4

4 Summary of the results

the solution is in very good agreement with the analytical solution except for the gas pressure. The weak differences are due to the linearization.