WTNV132 - Construction of a column of soil with the model of Summarized

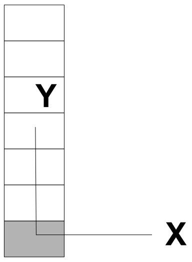
Hujeux

One wishes to numerically model the successive installation of the layers of a unidimensional column of soil, in taking into account at the same time the hydraulic coupling and the plasticization of the soil due to its nonlinear behavior (by the model of Hujeux [R7.01.23]). The calculated solutions are compared with results resulting from the code finite elements `GEFDYN` of the Central School Paris.

1 Problem of reference

1.1 Geometry

The model consists of $N+1$ elements on the whole: it is a question of posing N soil horizons ($N=10$ in the benchmark) on a porous elastic substratum infinitely rigid represented by $0^{ème}$ a layer. Each layer consists of an element of mesh (quadratic), a height of $2\mathrm{m}$ each one. The column of soil once built measurement thus $20\mathrm{m}$ on the whole. By principle, the problem is two-dimensional (the plane strains occur in a vertical plan): indeed, one places oneself on the assumption of an invariance of the soil by horizontal adjustment, which imposes that the mechanical strains and the hydraulic flux are null in the horizontal direction (models œdometric).



1.2 Properties of the materials

the elastic, anelastic properties and hydraulics of the layers are given hereafter:

	Parameters		**ELASTIC**
Values PROPERTIES	E	Young Modulus (for the substratum, one takes $100\times E$)	$100\,MPa$
	ν	Poisson's ratio	0.3
	ρ_h	homogenized density	$2105\,kg/m^3$
PROPERTIES HUJEUX	n	exposing of the elastic model in power	0.89
	d		1.7
	b		1
	α	coefficient of dilatancy	1
	φ	friction angle	21°
	ψ	angle of critical	dilatancy
	P_{co}	21° pressure or consolidation	$25\,kPa$
	$P_{réf}$	pressure of reference	$1\,MPa$
	a_{mon}		0,005
	a_{cyc}		0,005
	c_{mon}		0,18
	c_{cyc}		0,18
	$r^m_{dév}$	elastic radius déviatoire monotonous	0,025
	r^m_{iso}	monotonous isotropic elastic radius	0,01
	$r^c_{dév}$	elastic radius déviatoire cyclic	0,025
	r^c_{iso}	cyclic isotropic elastic radius	0,01
	r_{hys}		0,1.0,5

	r_{mob}		
	x_m		2
HYDRAULIC PROPERTIES	ϕ	porosity	0,35
	ρ_e	density of water	$1000\,kg/m^3$
	B	opposite coefficient of	Biot
	K^{-1}	1 of the compressibility of intrinsic	$9{,}35\times10^{-8}\,Pa^{-1}$
	K_{int}	water permeability of water	10^{-12}
	ν	viscosity of water	$0{,}001\,Pa.s$
	$D\nu/DT$	derived from viscosity by the temperature	$0\,Pa.s.K^{-1}$

1.3 Boundary conditions and loadings

In the model considered, limiting conditions apply to $n+1$ the layers present at the stage n of computation. They are the same ones as for a oedometer (the column of soil is a sample of an infinite space by horizontal adjustment):

Conditions of horizontal invariance:

- $u_x=0$ on the meshes side ones;

A condition of blocking of the 0ème layer (presumedly rigid):

- $u_y=0$ on the mesh of bottom;

A condition of water pressure null at the free surface of the column:

- $PRE_1=0$ on the mesh of the top of $n+1^{ème}$ the layer (the last posed);

An isotropic *and* non-zero *initial* stress state *effective* in each layer posed, because of the aversion of the model of Hujeux for stress states close to zero:

- $\sigma_{xx}'=\sigma_{yy}'=\sigma_{zz}'=\sigma_0'=-20.10^{+3}\,Pa$ in $n+1^{ème}$ the layer (the last posed);

Conditions of loading:

- the group of the column is subjected to gravity (acceleration $g=9{,}81\,m/s^2$ and directed according to $-\vec{e}_y$);

The construction of the column is carried out by respecting a period of time $\Delta t=10^{+6}$ seconds between the beginning of the stage n and that of the stage $n+1$. During this period of time, there is diffusion of the fluid and consolidation of the column under the effect of its own weight (compressing). It is important to take care that this period of time is sufficient, by putting it in keeping with the value of permeability of the porous material [1]. In particular, the product of Δt with this permeability gives a distance from diffusion of the fluid which must be sufficient (here about $10\,m$) compared to the dimension of the column ($20\,m$ height).

	Elements of the model	**Values**
LOW	`Boundary conditions`	$DY=0$; **hydraulic flux no**
	`SIDE SIDES`	$DX=0$; **hydraulic flux no one**
	`HIGH`	$PRE1=0$
Initial conditions	`COUCHE` $n+1$ (at the stage n)	$SIXX=SIYY=SIZZ=20\,kPa$
Loading	`TOUT`	$PESANTEUR=9{,}81\,m/s^2$

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[1] *calculates initially the conductivity of the fluid from intrinsic conductivity:* $\lambda=\frac{K^{\mathrm{int}}}{\upsilon}=10^{-9}\,kg^{-1}.m^3.s$ *; what gives finally for the permeability of the porous material:* $K=\rho_e\times g\times\lambda=9{,}81\times10^{-6}\,m.s^{-1}$

2 Reference solutions

2.1 Méthode de calcul

One uses the method **multi-models** to carry out computation. A each stage $n+1$ of the installation, one associates a model strictly containing $n+1$ the layers posed. The stress states, from displacement and the local variables at the conclusion of the preceding stage are transferred at the following stage by transfer operations from fields. The field of initial displacement of the soil horizon posed must vary linearly upwards, It varies indeed between the value of the compressing of the sub-bases and the geometrical coast to respect, associated with an initial displacement no one.

2.2 Quantities and results of reference

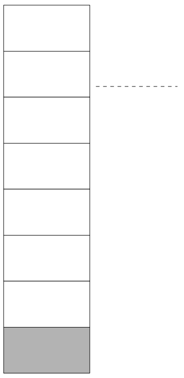
One post-draft solutions in terms of compressing. However, the use of the method multi-models does not give access directly to the compressing of the column: the vertical displacement of the last layer posed is the sum of really sudden compressing by it and compressing already carried out when it was not there: it is this last component which it is necessary to remove.
Let us consider for example layer 4 (cf appears hereafter). This one is posed at time $n=4$. Compressing has meaning there only for $n\geq 4$. That is to say δu_4^n the increment of compressing enters times n and $n+1$ above layer 4. One defines the compressing of layer 4 in time n : Δu_4^n, as the accumulation of the increments of compressing undergone by the layer during the total process of construction of the column of soil, i.e. by the installation of the successive layers located above it ($n\geq 5$).
As follows: $\Delta u^{n\geq 4}=\sum_{i=4}^{n}\delta u_4^i$ with $\delta u_4^n=u_4^n-u_4^{n-1}$

Thus, by decomposition of the iterative process, one rewrites easily compressing as being:

$$\Delta u_4^{n\geq 4}=u_4^n-u_4^0 .$$

i.e. displacement above the 4th layer with time n (under the action of the layers located above it), *less its displacement during its installation* ($n=4$).

The validation is carried out by comparison with solutions GEFDYN provided by the Central School Paris.



At time $n\geq 4$:

$$\Delta \boldsymbol{u}_4^{n\geq 4}=\sum_{i=4}^{n}\delta \boldsymbol{u}_4^i$$

$$=\sum_{i=4}^{n}\boldsymbol{u}_4^i-\boldsymbol{u}_4^{i-1}$$

$$=\boldsymbol{u}_4^n-\boldsymbol{u}_4^4=\boldsymbol{u}_4^n-\boldsymbol{u}_{4,0}$$

For the modelization C, one also carries out the elementary computation of option `PDIL_ELGA` with an aim of validating the numerical developments for modelizations `HM`. The values obtained are tested in NON-regression.

2.3 Uncertainties on the solution

the results established at the time of the modelization with the software GEFDyn Finite elements of the Central School Paris are precise according to the levels of the convergence criteria used in this software. The definition of the convergence criteria is specified in the instruction manual of the software [1]. The value of the criteria relating to displacements and the water pressure is equal to 10^{-3} and the value of the criteria relating to the mechanical unbalances (forces) and hydraulics (flux) is equal to 10^{-2}.

2.4 Bibliographical references

[1] D. Aubry, A. Modaressi. *GEFDyn, Scientific Maneul*. Central school Paris, LMSS-Mat, 1996.

3 Modelization A

3.1 Characteristic of the modelization

The modelization **A** is *three-dimensional* and *quasi-static*. One uses *a modelization with selective integration* (`MODELISATION = "HMS"`).

3.2 Characteristics of the mesh

The mesh used is composed of 11 `HEXA20`, that is to say an element for each layer of the column of soil (10 elements), and the rock on which puts back the column of soil.

3.3 Quantities tested and results

compressing is calculated at the top of each layer and compared to solutions given GEFDYN:

Compressing (in millimetres) of the layer n°1

Number of stage	Code_Aster	GEFDYN	relative error
2	-4.573	-4.648	-1.622%
3	-8.297	-8.391	-1.116%
4	-11.828	-11.940	-0.935%
5	-15.356	-15.440	-0.543%
6	-18.931	-18.980	-0.260%
7	-22.587	-22.650	-0.279%
8	-26.368	-26.470	-0.384%
9	-30.307	-30.490	-0.600%
10	-34.419	-34.730	-0.895%

Compressing (in millimetres) of the layer n°2

Number of stage	Code_Aster	GEFDYN	relative error
3	-9.093	-8.409	8.136%
4	-16.386	-15.720	4.236%
5	-23.453	-22.780	2.956%
6	-30.555	-29.830	2.430%
7	-37.785	-37.030	2.039%
8	-45.222	-44.510	1.600%
9	-52.942	-52.360	1.112%
10	-60.992	-60.610	0.630%

Compressing (in millimetres) of the layer n°3

Number of stage	Code_Aster	GEFDYN	relative error
4	-12.010	-11.980	0.251%
5	-22.813	-22.800	0.055%
6	-33.445	-33.410	0.105%
7	-44.202	-44.120	0.186%
8	-55.213	-55.150	0.115%
9	-66.589	-66.650	-0.091%
10	-78.419	-78.730	-0.395%

Compressing (in millimetres) of the layer n°4

Number of stage	Code_Aster	GEFDYN	relative error
5	-16.280	-15.470	5.238%
6	-30.682	-29.840	2.821%
7	-44.984	-44.110	1.981%
8	-59.522	-58.650	1.486%
9	-74.471	-73.690	1.060%
10	-89.956	-89.430	0.588%

Compressing (in millimetres) of the layer n°5

Number of stage	Code_Aster	GEFDYN	relative error
6	-20.109	-19.020	5.725%
7	-38.202	-37.050	3.109%
8	-56.300	-55.150	2.085%
9	-74.776	-73.700	1.460%
10	-93.833	-92.980	0.917%

Compressing (in millimetres) of the layer n°6

Number of stage	Code_Aster	GEFDYN	relative error
7	-23.832	-22.680	5.080%
8	-45.723	-44.540	2.657%
9	-67.763	-66.650	1.669%
10	-90.345	-89.440	1.012%

Compressing (in millimetres) of the layer n°7

Number of stage	Code_Aster	GEFDYN	relative error
8	-27.632	-26.500	4.271%
9	-53.465	-52.380	2.072%
10	-79.610	-78.720	1.131%

Compressing (in millimetres) of the layer n°8

Number of stage	Code_Aster	GEFDYN	relative error
9	-31.369	-30.520	2.783%
10	-61.288	-60.630	1.085%

Compressing (in millimetres) of the layer n°9

Number of stage	Code_Aster	GEFDYN	relative error
10	-35.435	-34.750	1.973%

3.4 Comments

the relative error is of to the maximum 9% , which is relatively satisfactory.

4 Modelization B

4.1 Characteristic of the modelization

The modelization **B** is *two-dimensional* and *quasi-static*. One uses *a modelization with classical integration* (`MODELISATION = "HM"`). One superimposes on the hydro-mechanical problem "macroscopic" thus defined a model of *second gradient of thermal expansion*, said "microscopic", the parameters constitutive of the model of second gradient are determined in order not to modify the basic solution of the macroscopic problem. In particular, the small coefficient of penalization is chosen: `PENA_LAGR` = 1,

The model of second gradient is used as a patch that one superimposes on the macroscopic problem: for the initial mesh (here `QUAD8`), it is necessary to superimpose a mesh (made up of `QUAD9`) on which will be defined the model of second gradient. It is what is made in this modelization where as starter a double mesh `QUAD8` is given, of which one of both is transformed into `QUAD9` using the command: `CREA_MAILLAGE → MODI_MAILLAGE → OPTION=' QUAD8_9'`,

The model of second gradient makes it possible to treat the phenomena of instability material related to the loss of ellipticity of the tensor stress-strain, and which result in a localization of the strains, Here, one really does not seek to regularize the problem (since there is no material instability), but the model to implement of second gradient in a relatively representative benchmark,

4.2 Characteristic of the mesh

For modelization `DPLAN_HM`, each soil horizon is represented by an element `QUAD8`, that is to say 11 elements on the whole. The mesh of superposition consists of elements `QUAD9`, that is to say also 11 elements on the whole.

One superimposes on the problem of hydraulic coupling macroscopic a model of second microscopic gradient of thermal expansion, In the integration of the balance equations, one thus asks for a reactualization of the tangent matrix, which is provided by the routines of the model of Hujeux and accelerates convergence appreciably. One also asks for the subdivision of time step (command `DEFI_LIST_INST`) to treat the situations of failure of the local integration of with increments of too large loading. *This functionality is largely recommended*.

4.3 Quantities tested and Unchanged

results compared to the modelization **A.**

4.4 Commentaires

These results validate the capacity of the modelization of second gradient of thermal expansion to being used for computations with a loading of gravity.

5 Modelization C

5.1 Characteristic of the modelization

The modelization **C** is *two-dimensional* and *quasi-static*. One uses *a modelization with classical integration* (`MODELISATION = "HM"`). The first difference with the modelization **A** is related to the transition of a modelization `3D_HM` to a modelization `DPLAN_HM`. The second difference and the holds with the use of a secant matrix of stiffness for the total resolution of the equilibrium of structure between the internal forces external forces applied.

5.2 Characteristics of the mesh

Each layer is represented by an element `QUAD8`. The mesh complete thus consists of 11 elements `QUAD8`.

5.3 Quantities tested and results

compressing is calculated at the top of each layer and compared to solutions given GEFDYN:

Compressing (in millimetres) of the layer n°1

Number of stage	GEFDYN	Tolerance (%)
2	-4.648	4.00
3	-8.391	3.00
4	-11.940	2.00
5	-15.440	2.00
6	-18.980	2.00
7	-22.650	2.00
8	-26.470	2.00
9	-30.490	2.00
10	-34.730	2.00

Compressing (in millimetres) of the layer n°2

Number of stage	GEFDYN	Tolerance (%)
3	-8.409	3.00
4	-15.720	2.00
5	-22.780	2.00
6	-29.830	2.00
7	-37.030	2.00
8	-44.510	2.00
9	-52.360	2.00
10	-60.610	2.00

Compressing (in millimetres) of the layer n°3

Number of stage	GEFDYN	Tolerance (%)
4	-11.980	2.00
5	-22.800	2.00
6	-33.410	2.00
7	-44.120	2.00
8	-55.150	2.00
9	-66.650	2.00

10	-78.730	2.00

Compressing (in millimetres) of the layer n°4

Number of stage	GEFDYN	Tolerance (%)
5	-15.470	2.00
6	-29.840	2.00
7	-44.110	2.00
8	-58.650	2.00
9	-73.690	2.00
10	-89.430	2.00

Compressing (in millimetres) of the layer n°5

Number of stage	GEFDYN	Tolerance (%)
6	-19.020	2.00
7	-37.050	2.00
8	-55.150	2.00
9	-73.700	2.00
10	-92.980	2.00

Compressing (in millimetres) of the layer n°6

Number of stage	GEFDYN	Tolerance (%)
7	-22.680	2.00
8	-44.540	2.00
9	-66.650	2.00
10	-89.440	2.00

Compressing (in millimetres) of the layer n°7

Number of stage	GEFDYN	Tolerance (%)
8	-26.500	2.00
9	-52.380	2.00
10	-78.720	2.00

Compressing (in millimetres) of the layer n°8

Number of stage	GEFDYN	Tolerance (%)
9	-30.520	2.00
10	-60.630	2.00

Compressing (in millimetres) of the layer n°9

Number of stage	GEFDYN	Tolerance (%)
10	-34.750	2.00

elementary computation option `INDL_ELGA` is also tested in NON-regression to validate its development in modelization `DPLAN_HM`. All the components are tested on the mesh $M2$, located at the base of the column, just above elastic rock.

Number of Component	stage `INDL_ELGA`	Standard of reference	Reference	Tolerance (absolute)
10	`INDEX`	`NON_REGRESSION`	0.0	0.001
10	`DIR1`	`NON_REGRESSION`	0.0	0.001

10	`DIR2`	`NON_REGRESSION`	0.0	0.001
10	`DIR3`	`NON_REGRESSION`	0.0	0.001
10	`DIR4`	`NON_REGRESSION`	0.0	0.001

elementary computation option `PDIL_ELGA` is tested in NON-regression to validate its development in modelization `DPLAN_HM`. Component `A1_LC2` is tested on the mesh $M2$, located at the base of the column, just above elastic rock.

Number of Component	stage `INDL_ELGA`	Standard of reference	Reference	Tolerance (absolute)
10	`A1_LC2`	`NON_REGRESSION`	0.0	0.001

5.4 Comments

the relative error is of to the maximum 4% , which is relatively satisfactory. The results are overall closer to the results resulting from the reference, in comparison with modelizations **A** and **B**. One can add that the algorithm used in this modelization to solve the equilibrium of structure is identical to that of the reference, which can explain these results.

6 Modelization D

6.1 Characteristic of the modelization

The modelization **D** is *two-dimensional* and *quasi-static*. It is identical to the modelization **C**, with three differences near:

- one uses *a modelization under-integrated* (`MODELISATION = "D_PLAN_HM_SI"`) instead of the classical modelization (`MODELISATION = "D_PLAN_HM"`);
- one carries out a redimensioning of the hydro-mechanical problem saturated by applying the factors with redimensioning $P_0=10^{+6}$ and $K_0=10^{-5}$;
- The convergence criterion used is `RESI_REFE_RELA` = 10-4, with `SIGM_REFE` = 1 and `FLUX_HYD1_REFE` = 1;

6.2 Unchanged characteristics of

the mesh compared to the modelization **C**.

6.3 Quantities tested and results

Unchanged compared to the modelization **C**. postprocessing must take account of the reverse transformation towards the units of origin.

6.4 Comments

These results validate the under-integrated modelization and the method of redimensioning.

7 Modelization E

7.1 Characteristic of the modelization

The modelization **E** is *two-dimensional* and *quasi-static*. It is identical to the modelization **D**, except that the convergence criterion used is `RESI_GLOB_RELA` = 10-8.

7.2 Unchanged characteristics of

the mesh compared to the modelization **D.**

7.3 Grandeurs tested and results

Unchanged compared to the modelization **D.**

7.4 Commentaires

These results validate the under-integrated modelization and the method of redimensioning.

8 Summary of the results

One represents in **Figure 1** a comparison of compressings calculated along the column of soil by *Code_Aster* and *GEFDYN* (using the modelization **A)**, As one can note it, the coincidence of the results is rather satisfactory.

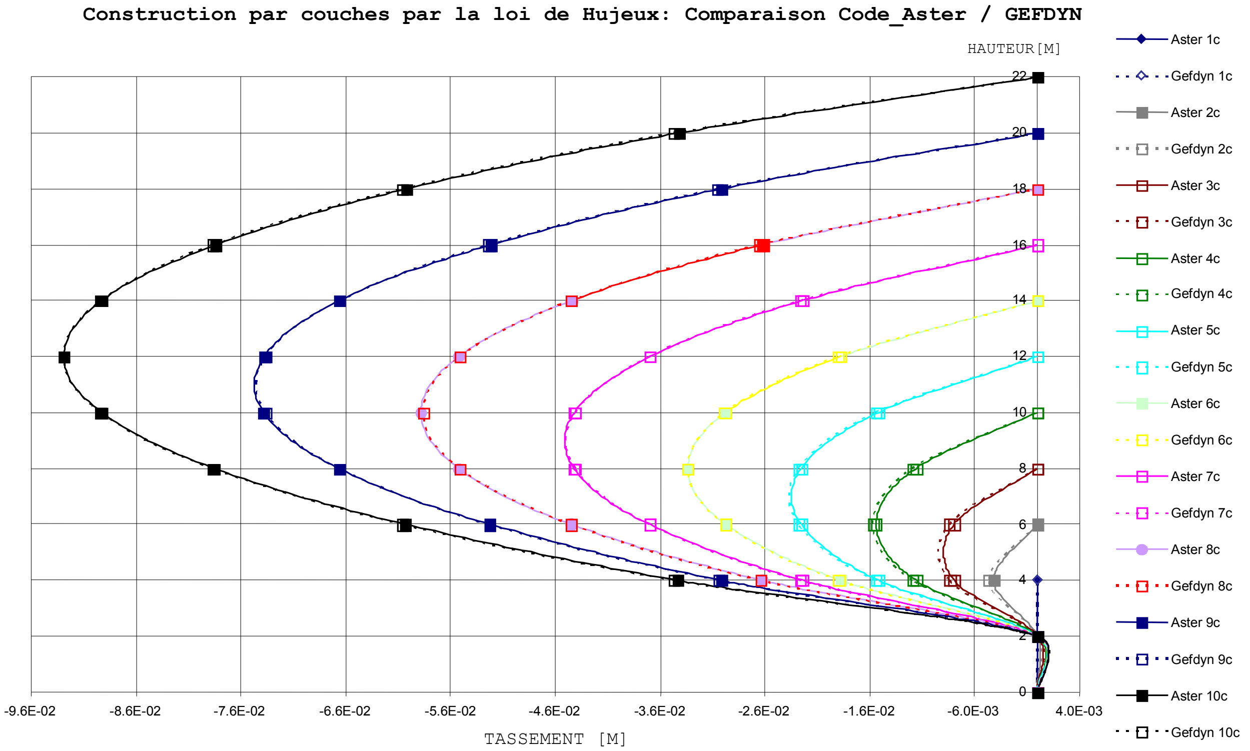


Figure 1 : *Compressing in the column of soil to each stage of computation: comparison of the solutions Code_Aster and GEFDYN.*