WTNV112 – Gravitating flow in a porous environment unsaturated

Summarized:

One studies the hydraulic behavior of a porous environment unsaturated. Five modelizations are carried out: one is three-dimensional (modelization B) and the four others are two-dimensional (modelizations A, C, D, E)

This test consists in studying the influence of a gravitating flow on the distribution of the pressure of the fluids (liquidates and gas) medium unsaturated.

The studied models are $2D$ plane (DPQ8 and DPTR6) and $3D$ voluminal HEXA20 with a linear behavior, it acts of an evolutionary problem.

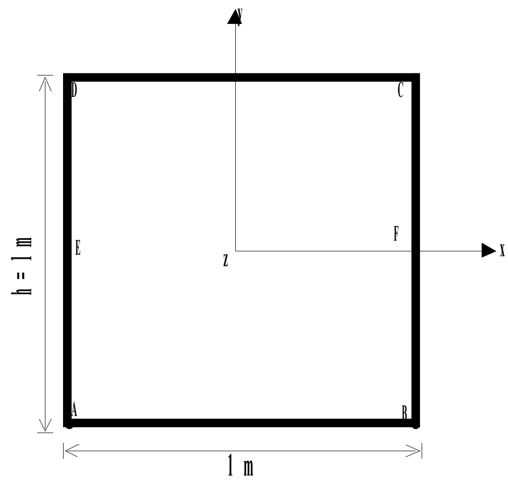
The reference solution is unidimensional because it depends only on the vertical coordinate.

1 Problem of reference

1.1 Presentation

One studies in this case test the hydraulic behavior of a porous environment unsaturated made up by two fluids: water in its liquid phase and dry air. It acts in *Code_Aster* of a modelization HHM. The associated constitutive law of the fluids is of type LIQU_GAZ .

1.2 Geometry



Coordinated of the points (m) :

$A: -0{,}5\ -0{,}5$ $\quad C: \ 0{,}5\ 0{,}5$

$B: \ 0{,}5\ -0{,}5$ $\quad D: -0{,}5\ 0{,}5$

1.3 Properties of the Fluid

material (liquid water)	Density $(kg.m^{-3})$	10^3
	Compressibility of the fluid (Pa)	10^7
	Dynamic viscosity of the liquid water $(Pa.s)$	10^{-3}
	Derived from the viscosity of the fluid compared to the temperature	$0.$
Gas (dry air)	Molar mass $(kg.Pa.K^{-1})$	1.8×10^{-3}
	Viscosity of gas $(Pa.s)$	10^{-5}
	Derived from viscosity from gas compared to the temperature	$0.$
Coefficients from homogenization	Coefficient from $Biot$	$1.$
	Constant	0.14
Porosity homogenized	Coefficients from perfect gases	8.315
	Density homogenized $(kg.m^{-3})$	1.6×10^3
	Saturation	0.5
	Derived from saturation compared to the pressure	$0.$
	Gravity according to X	$0.$
	Gravity according to Y	-10 in 2D, 0 in 3D
	Gravity according to Z	-10 in 3D, 0 in 2D
	intrinsic Permeability (m^2)	10^{-18}
	Permeability relating to the fluid (m^2)	$1.$
	Permeability relating to the gas (m^2)	$1.$

1.4 Boundary conditions and loadings

•complete Element:
•displacements $u_x=0.0\,m, u_y=0.0\,m, u_z=0.0\,m$.

1.5 Initial conditions

the fields of displacement, of capillary pressure are initially null, the air pressure dryness is equal to the atmospheric pressure and the reference temperature is worth $T_0=273\,°K$

2 Reference solution

2.1 Method of calculating used for the reference solution

the conservation equation of the fluid mass is given by the following statement:

$$\frac{dm_i}{dt}+DivM_i=0 \quad i \text{ varying } 1 \text{ with the number of components} \tag{1}$$

In our example, the model consists of two fluids: liquid water (e) and dry air (a). The equation (1) is thus divided into two:

$$\begin{cases} \dfrac{dm_e}{dt}+DivM_e=0 \\ \dfrac{dm_a}{dt}+DivM_a=0 \end{cases} \tag{2}$$

the fluid flux have as a statement:

$$\begin{cases} M_e=\rho_e\lambda_e(-\nabla p_e+\rho_e g) \\ M_a=\rho_a\lambda_a(-\nabla p_a+\rho_a g) \end{cases} \tag{3}$$

But the mass fluid contribution is defined by the equations (4) where $N=\begin{bmatrix} N_{ee} & N_{ea} \\ N_{ae} & N_{aa} \end{bmatrix}$ is a symmetric matrix of which the terms (equations (5)) depend on the degree on saturation S, porosity ϕ, coefficient of $Biot\, b$, permeability of the fluid and K_e elasticity of the solid matrix K_s.

$$\begin{cases} \dfrac{dm_e}{dt}=\rho_e N_{ee}\dfrac{dp_e}{dt}+\rho_e N_{ea}\dfrac{dp_a}{dt} \\ \dfrac{dm_a}{dt}=\rho_a N_{ae}\dfrac{dp_e}{dt}+\rho_a N_{aa}\dfrac{dp_a}{dt} \end{cases} \tag{4}$$

$$\begin{cases} N_{ee}=-\varphi\dfrac{\partial S}{\partial p_c}+S\left(\dfrac{\varphi}{K_e}+\dfrac{b-\varphi}{K_s}S\right) \\ N_{aa}=-\varphi\dfrac{\partial S}{\partial p_c}+(1-S)\left(\dfrac{\varphi}{p_a}+\dfrac{b-\varphi}{K_s}(1-S)\right) \\ N_{ea}=N_{ae}=\varphi\dfrac{\partial S}{\partial p_c}+(1-S)\left(\dfrac{b-\varphi}{K_s}S\right) \end{cases} \tag{5}$$

the variational formulation of the equations (2), by taking account of (3) and (4) is:

$$\begin{cases} \int_\Omega N_{ee}\frac{dp_e}{dt}p_e^* + \int_\Omega N_{ea}\frac{dp_a}{dt}p_e^* + \int_\Omega \lambda_e \nabla p_e . \nabla p_e^* = \int_\Omega \lambda_e \rho_e g . \nabla p_e^* - \int_{\partial\Omega}\frac{M_e^{ext}}{\rho_e}p_e^* \\ \int_\Omega N_{ea}\frac{dp_e}{dt}p_a^* + \int_\Omega N_{aa}\frac{dp_a}{dt}p_a^* + \int_\Omega \lambda_a \nabla p_a . \nabla p_a^* = \int_\Omega \lambda_a \rho_a g . \nabla p_a^* - \int_{\partial\Omega}\frac{M_a^{ext}}{\rho_a}p_a^* \end{cases} \quad (6)$$

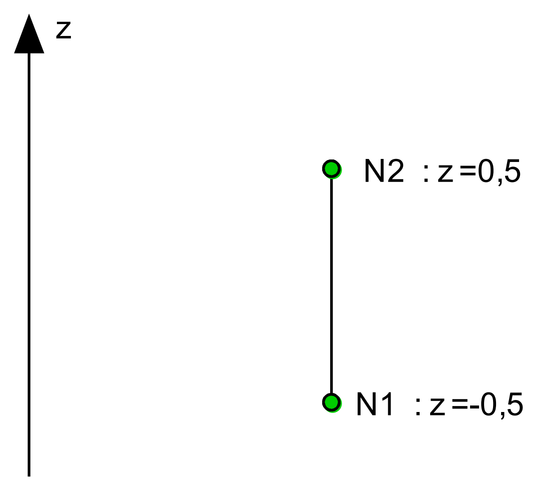
Discretization

For the computation of the analytical solution, one is placed in a unidimensional case with only one element of degree 1 .

Note:
In THM , all the meshes must be quadratic, but in the hydraulic case integration is always linear (the nodes mediums are ignored).

One supposes in both cases that gravity is directed according to z the negative ones.
It is supposed in addition that non-linearities are low and that the coefficients N, λ, ρ are constant. It is necessary thus that the variations of pressure are sufficiently weak so that N and ρ can be presumedly constant.
In hydraulics, the discretization will be always linear.

Linear discretization:



One will write:

$$p(z,t) = \sum_{i=1}^{2} p^i(t)\lambda_i(z) \quad (7)$$

With:

$$\begin{cases} \lambda_1 = \frac{1}{2} - z \\ \lambda_2 = \frac{1}{2} + z \end{cases} \quad (8)$$

By introducing the matrixes and vectors then:

$$\begin{cases} [A] = [A_{ij}] \quad ; \quad A_{ij} = \int_{-1/2}^{1/2} \lambda_i \lambda_j \, dz \\ [B] = [B_{ij}] \quad ; \quad B_{ij} = \int_{-1/2}^{1/2} \frac{d\lambda_i}{dz} \frac{d\lambda_j}{dz} dz \\ \{F_g\} = \{F_{gi}\} \quad ; \quad F_{gj} = \int_{-1/2}^{1/2} \frac{d\lambda_i}{dz} dz \end{cases} \quad (9)$$

And while noting:

$$\{p_e\} = \begin{Bmatrix} P_e^1 \\ p_e^2 \end{Bmatrix} \quad ; \quad \{p_a\} = \begin{Bmatrix} P_a^1 \\ p_a^2 \end{Bmatrix} \quad (10)$$

$$\{M_e^{ext}\} = \begin{Bmatrix} M_{e\ 1}^{ext} \\ M_{e\ 2}^{ext} \end{Bmatrix} \quad ; \quad \{M_a^{ext}\} = \begin{Bmatrix} M_{a\ 1}^{ext} \\ M_{a\ 2}^{ext} \end{Bmatrix} \quad (11)$$

the equations (6) become:

$$\begin{cases} \frac{N_{ee}}{\lambda_e}[A]\begin{Bmatrix} \frac{dp_e}{dt} \end{Bmatrix} + \frac{N_{ea}}{\lambda_e}[A]\begin{Bmatrix} \frac{dp_a}{dt} \end{Bmatrix} + [B]\{p_e\} = \rho_e\{F_g\} - \frac{1}{\lambda_e \rho_e}\{M_e^{ext}\} \\ \frac{N_{ae}}{\lambda_a}[A]\begin{Bmatrix} \frac{dp_e}{dt} \end{Bmatrix} + \frac{N_{aa}}{\lambda_a}[A]\begin{Bmatrix} \frac{dp_a}{dt} \end{Bmatrix} + [B]\{p_a\} = \rho_a\{F_g\} - \frac{1}{\lambda_a \rho_a}\{M_a^{ext}\} \end{cases} \quad (12)$$

The computation of the matrixes $[A]$ and $[B]$ the vector $\{F\}$ gives:

$$[A] = \frac{1}{3}\begin{bmatrix} 1 & 1/2 \\ 1/2 & 1 \end{bmatrix} \quad ; \quad [B] = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad ; \quad \{F_g\} = \begin{Bmatrix} -1 \\ 1 \end{Bmatrix} \quad (13)$$

One defines $\{v_1\}, \{v_2\}$ the eigenvectors then of $[A]^{-1}[B]$.

There are the properties:

$$\{v_i\}^T [A]\{v_j\} = \{v_i\}^T [B]\{v_j\} = 0 \quad si \quad i \neq j \quad (14)$$

And one poses:

$$a_i=\{v_i\}^T[A]\{v_i\} \ , \ b_i=\{v_i\}^T[B]\{v_i\} \ , \ f_i=\{v_i\}^T\{F_g\} et\ M^i=\{v_i\}^T\{M^{ext}\} \quad (15)$$

One finds:

$$\{v_1\}=\left\{\begin{matrix}1\\1\end{matrix}\right\} \quad ; \quad \{v_2\}=\left\{\begin{matrix}-1\\1\end{matrix}\right\} \quad (16)$$

$$\begin{cases} a_1=1 \quad ; \quad b_1=0 \quad ; \quad f_1=0 \\ a_2=\dfrac{1}{3} \quad ; \quad b_2=4 \quad ; \quad f_2=-2g \end{cases} \quad (17)$$

One breaks up then $\{p_e\}$ and $\{p_a\}$ on the basis of $\{v_i\}$

$$\{p_e\}=\sum_{i=1}^{2}\alpha_e^i\{v_i\} \quad ; \quad \{p_a\}=\sum_{i=1}^{2}\alpha_a^i\{v_i\} \quad (18)$$

Taking into account the properties of orthogonality (14), the system of equations (12) is written:

$$\begin{cases} \dfrac{N_{ee}}{\lambda_e}a_i\dfrac{d\alpha_e^i}{dt}+\dfrac{N_{ea}}{\lambda_e}a_i\dfrac{d\alpha_a^i}{dt}+b_i\alpha_e^i=\rho_e f_i-\dfrac{1}{\lambda_e\rho_e}M_e^i \\ \dfrac{N_{ae}}{\lambda_a}a_i\dfrac{d\alpha_e^i}{dt}+\dfrac{N_{aa}}{\lambda_a}a_i\dfrac{d\alpha_a^i}{dt}+b_i\alpha_a^i=\rho_a f_i-\dfrac{1}{\lambda_a\rho_a}M_a^i \end{cases} \quad (19)$$

Posing:

$$\{\alpha^i\}=\left\{\begin{matrix}\alpha_e^i\\ \alpha_a^i\end{matrix}\right\} \quad ; \quad [N]=\begin{bmatrix}N_{ee} & N_{ea}\\ N_{ae} & N_{aa}\end{bmatrix} \quad ; \quad [L]=\begin{bmatrix}\lambda_e & 0\\ 0 & \lambda_a\end{bmatrix} \quad (20)$$

the equation (19) is written:

$$[N]=\left[\frac{d\alpha^i}{dt}\right]+\frac{b_i}{a_i}[L]\{\alpha^i\}=\frac{f_i}{a_i}\begin{Bmatrix}\rho_e\lambda_e\\ \rho_a\lambda_a\end{Bmatrix}-\begin{Bmatrix}M_e^i/\rho_e a_i\\ M_a^i/\rho_a a_i\end{Bmatrix} \quad (21)$$

Initial conditions

It is supposed that:

$$\begin{matrix} p_a(x,t=0)=p_a^0 \\ p_e(x,t=0)=p_a^0-p_c^0 \end{matrix} \text{ uniforms in space;}$$

Taking into account the values of the vectors $\{v_1\},\{v_2\}$ (equations (16)), it is seen easily that:

$$\begin{matrix} \alpha_a^1(t=0)=p_a^0 \quad ; \quad \alpha_e^1(t=0)=p_a^0-p_c^0 \\ \alpha_a^2(t=0)=\alpha_e^2(t=0)=0 \end{matrix} \quad (22)$$

One places oneself in a case where the equations of the hydraulics are decoupled $\left(N_{ea}=N_{ae}=0\right)$ and in which the fluid flux are null $\left(\{M_e^{ext}\}=\{M_a^{ext}\}=0\right)$.

Taking into account (21), of $f_1=f_3=0$ (equations (17)), the system of equations (21) has as a solution:

$$\begin{cases} \alpha_e^1=P_a^0-p_c^0 \\ \alpha_e^2=\frac{f_2}{b_2}\rho_e\left(1-\exp\left(-\frac{b_2}{a_2}\frac{\lambda_e}{N_{ee}}t\right)\right) \\ \alpha_a^1=P_a^0 \\ \alpha_a^2=\frac{f_2}{b_2}\rho_a\left(1-\exp\left(-\frac{b_2}{a_2}\frac{\lambda_a}{N_{aa}}t\right)\right) \end{cases} \quad (23)$$

One finds while returning to the nodal variables:

$$\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}=\begin{Bmatrix} \alpha_1-\alpha_2 \\ \alpha_1+\alpha_2 \end{Bmatrix}$$

$$\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}_{eau}=\begin{Bmatrix} P_a^0-p_c^0+\frac{\rho_e g}{2}\left(1-\exp\left(-12\frac{\lambda_e}{N_{ee}}t\right)\right) \\ P_a^0-p_c^0-\frac{\rho_e g}{2}\left(1-\exp\left(-12\frac{\lambda_e}{N_{ee}}t\right)\right) \end{Bmatrix} \quad (24)$$

$$\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}_{air}=\begin{Bmatrix} P_a^0+\frac{\rho_a g}{2}\exp\left(-12\frac{\lambda_a}{N_{aa}}t\right) \\ P_a^0-\frac{\rho_a g}{2}\exp\left(-12\frac{\lambda_a}{N_{aa}}t\right) \end{Bmatrix} \quad (25)$$

and the definite capillary pressure like the difference between the air pressure and the pressure of water has as a value:

$$\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}_{capillaire}=\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}_{air}-\begin{Bmatrix} P_1 \\ p_2 \end{Bmatrix}_{eau}$$

We considered the following calculation case:

$$S\neq 1 \quad ; \quad \frac{\partial S}{\partial p_c}=0 \quad ; \quad K_s=\infty$$

$$N_{ee}=S\frac{\varphi}{K_e} \quad ; \quad N_{aa}=(1-S)\frac{\varphi}{p_a}$$

2.2 Variable reference

1) Evolution of the capillary pressure and the air pressure dryness according to time to the points
   - $C, D(z=h)$
   - $A, B(z=0)$
1) For the quadratic discretization, checking of the constant value of the pressure to the nodes $E, F(z=\frac{h}{2})$.

2.3 Uncertainties

analytical Solution on the equations of negligible hydraulics thus uncertainties for the modelizations A, B, C.
Attention these solutions analytical do not apply to the selective or lumped modelizations (D and E). Indeed, in this last case, integrations are made with the nodes and either with Gauss points. Indeed integration by Gauss point is exact in 1D for polynomial of degree lower or equal to 3 and thus for all the integrals presented in the equation (9). On the other hand the integration method at the top is not exact that for the polynomials of degree 1. It is thus seen that the terms of the matrix $[A]$ under will be integrated. It is thus logical that on a unit mesh as here the results got here are not exact. One however preserves these tests but with result in “non regression”.

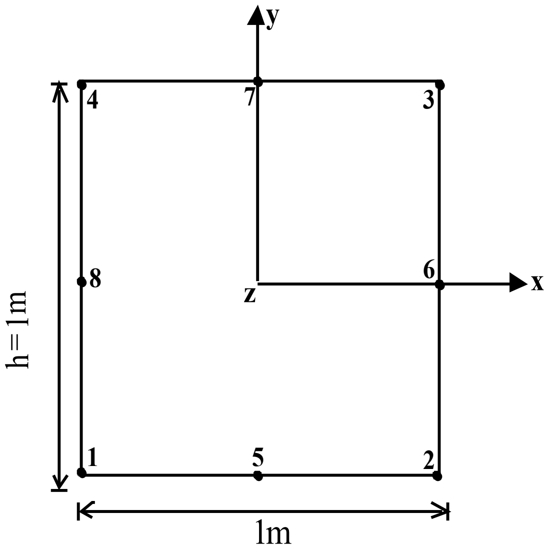
2.4 Bibliographical references

1 Thermo-hydro-mechanics of the porous environments in Code_Aster – Note EDF, HI-74/99/011/A

3 Modelization A

3.1 Characteristic of the modelization A

plane Modelization: D_PLAN_HHM



1 nets DPQ8 of modelization D_PLAN_HHM : HHM_ DPQ8

3.2 Result of the modelization A

Discretization in time: Several time step (16) to study the evolution of the pressure during the transitional stage until stabilizing itself. The time scheme is implicit $(\theta=1)$.

List times of computation in seconds:
$1, 5, 10, 50, 100, 500, 10^3, 5.\times10^3, 10^4, 5.\times10^4, 10^5, 5.\times10^5, 10^6, 5.\times10^6, 10^7, 10^{10}$.

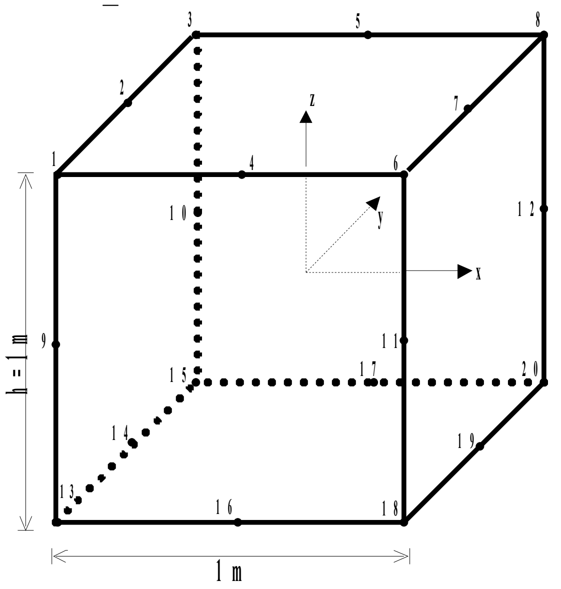
Nodal unknowns: fluid pressures evaluated in *Code_Aster* are variations compared to the initial pressures, this is why this table present of the variations of pressure in our comparison between computation *Code_Aster* and the reference solution. Moreover variables of pressure used in *Code_Aster* to evaluate the constitutive laws are the stagnation pressure of gas and the capillary pressure.

Node/not	urgent Sequence number/ (s)	Value	Pressure (Pa)	Tolerance
$1,2,5/A,B$	$1(t=1s)$	$PRE1$	-8,565 .10 $^{-3}$	10^{-4}
	$2(t=5s)$	$PRE1$	-4,282.10 $^{-2}$	10^{-4}
	$3(t=10s)$	$PRE1$	-8,565.10 $^{-2}$	10^{-4}
	$4(t=50s)$	$PRE1$	-4,282.10 $^{-1}$	1%
	$8(t=5.10^3 s)$	$PRE1$	-4,26.10 $^{+1}$	1%
	$16(t=10^{10} s)$	$PRE1$	-4,996.10 $^{+3}$	1%
	$1(t=1s)$	$PRE2$	6,796.10 $^{-6}$	10^{-4}
	$2(t=5s)$	$PRE2$	3,398.10 $^{-5}$	10^{-4}
	$3(t=10s)$	$PRE2$	6,796.10 $^{-5}$	10^{-4}
	$4(t=50s)$	$PRE2$	3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^3 s)$	$PRE2$	3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10} s)$	$PRE2$	3,964	10^{-3}
$3,4,7/C,D$	$1(t=1s)$	$PRE1$	8,565 .10 $^{-3}$	10^{-4}
	$2(t=5s)$	$PRE1$	4,282.10 $^{-2}$	10^{-4}
	$3(t=10s)$	$PRE1$	8,565.10 $^{-2}$	10^{-4}
	$4(t=50s)$	$PRE1$	4,282.10 $^{-1}$	1%
	$8(t=5.10^3 s)$	$PRE1$	4,26.10 $^{+1}$	1%
	$16(t=10^{10} s)$	$PRE1$	4,996.10 $^{+3}$	1%
	$1(t=1s)$	$PRE2$	-6,796.10 $^{-6}$	10^{-4}
	$2(t=5s)$	$PRE2$	-3,398.10 $^{-5}$	10^{-4}
	$3(t=10s)$	$PRE2$	-6,796.10 $^{-5}$	10^{-4}
	$4(t=50s)$	$PRE2$	-3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^3 s)$	$PRE2$	-3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10} s)$	$PRE2$	-3,964	10^{-3}

4 Modelization B

4.1 Characteristic of the voluminal modelization

B Modelization: 3D_HHM



1 nets HEXA20 of modelization 3D_HHM : HHM_HEXA20

4.2 Result of the modelization B

Discretization in time: Several time step (16) to study the evolution of the pressure during the transitional stage until stabilizing itself. The time scheme is implicit $(\theta=1)$.

List times of computation in seconds:
$1, 5, 10, 50, 100, 500, 10^3, 5.\times10^3, 10^4, 5.\times10^4, 10^5, 5.\times10^5, 10^6, 5.\times10^6, 10^7, 10^{10}$.

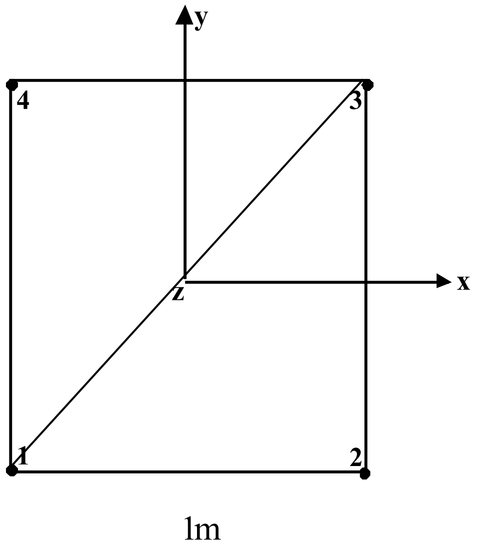
The nodal unknowns of fluid pressure evaluated in *Code_Aster* are variations compared to the initial pressures, this is why this table present of the variations of pressure in our comparison between computation *Code_Aster* and the reference solution. Moreover variables of pressure used in *Code_Aster* to evaluate the constitutive laws are the stagnation pressure of gas and the capillary pressure.

Node/not	urgent Sequence number/ (s)	Value	Pressure (Pa)	Tolerance
13 to 20 /et A B	$1(t=1s)$	$PRE1$	-8,565 .10 $^{-3}$	10^{-4}
	$2(t=5s)$	$PRE1$	-4,282.10 $^{-2}$	10^{-4}
	$3(t=10s)$	$PRE1$	-8,565.10 $^{-2}$	10^{-4}
	$4(t=50s)$	$PRE1$	-4,282.10 $^{-1}$	1%
	$8(t=5.10^3 s)$	$PRE1$	-4,26.10 $^{+1}$	1%
	$16(t=10^{10} s)$	$PRE1$	-4,996.10 $^{+3}$	1%
	$1(t=1s)$	$PRE2$	6,796.10 $^{-6}$	10^{-4}
	$2(t=5s)$	$PRE2$	3,398.10 $^{-5}$	10^{-4}
	$3(t=10s)$	$PRE2$	6,796.10 $^{-5}$	10^{-4}
	$4(t=50s)$	$PRE2$	3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^3 s)$	$PRE2$	3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10} s)$	$PRE2$	3,964	10^{-3}
1 to 8/ C and D	$1(t=1s)$	$PRE1$	8,565 .10 $^{-3}$	10^{-4}
	$2(t=5s)$	$PRE1$	4,288.10 $^{-2}$	10^{-4}
	$3(t=10s)$	$PRE1$	8,565.10 $^{-2}$	10^{-4}
	$4(t=50s)$	$PRE1$	4,282.10 $^{-1}$	1%
	$8(t=5.10^3 s)$	$PRE1$	4,26.10 $^{+1}$	1%
	$16(t=10^{10} s)$	$PRE1$	4,996.10 $^{+3}$	1%
	$1(t=1s)$	$PRE2$	-6,796.10 $^{-6}$	10^{-4}
	$2(t=5s)$	$PRE2$	-3,398.10 $^{-5}$	10^{-4}
	$3(t=10s)$	$PRE2$	-6,796.10 $^{-5}$	10^{-4}
	$4(t=50s)$	$PRE2$	-3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^3 s)$	$PRE2$	-3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10} s)$	$PRE2$	-3,964	10^{-3}

5 Modelization C

5.1 Characteristic of the modelization C

plane Modelization: `D_PLAN_HHM`



2 meshes `DPTR6` of modelization `D_PLAN_HHM` : `HHM_ DPTR6`

5.2 Result of the modelization C

Discretization in time: Several time step (16) to study the evolution of the pressure during the transitional stage until stabilizing itself. The time scheme is implicit $(\theta=1)$.

List times of computation in seconds:
$1, 5, 10, 50, 100, 500, 10^3, 5.\times 10^3, 10^4, 5.\times 10^4, 10^5, 5.\times 10^5, 10^6, 5.\times 10^6, 10^7, 10^{10}$.

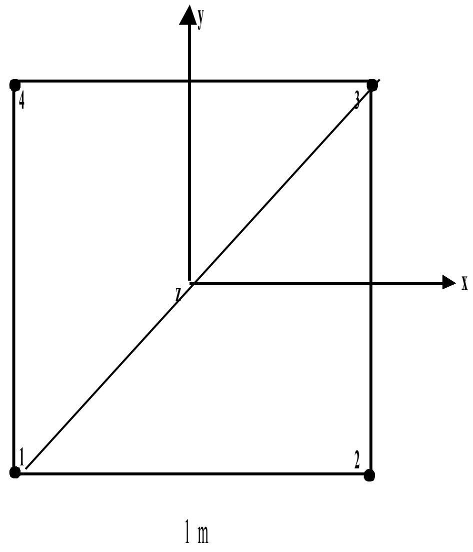
Nodal unknowns: fluid pressures evaluated in *Code_Aster* are variations compared to the initial pressures, this is why this table present of the variations of pressure in our comparison between computation *Code_Aster* and the reference solution. Moreover variables of pressure used in *Code_Aster* to evaluate the constitutive laws are the stagnation pressure of gas and the capillary pressure.

Node/not	**urgent Sequence number/** (s)	**Value**	**Pressure** (Pa)	**Tolerance**
1,2/ A and B	$1(t=1s)$	*PRE1*	-8,565 .10 $^{-3}$	10^{-4}
	$2(t=5s)$	*PRE1*	-4,282.10 $^{-2}$	10^{-4}
	$3(t=10s)$	*PRE1*	-8,565.10 $^{-2}$	10^{-4}
	$4(t=50s)$	*PRE1*	-4,282.10 $^{-1}$	1%
	$8(t=5.10^3 s)$	*PRE1*	-4,26.10 $^{+1}$	1%
	$16(t=10^{10} s)$	*PRE1*	-4,996.10 $^{+3}$	1%
	$1(t=1s)$	*PRE2*	6,796.10 $^{-6}$	10^{-4}
	$2(t=5s)$	*PRE2*	3,398.10 $^{-5}$	10^{-4}
	$3(t=10s)$	*PRE2*	6,796.10 $^{-5}$	10^{-4}
	$4(t=50s)$	*PRE2*	3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^3 s)$	*PRE2*	3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10} s)$	*PRE2*	3,964	10^{-3}
3,4/ C and D	$1(t=1s)$	*PRE1*	8,565 .10 $^{-3}$	10^{-4}
	$2(t=5s)$	*PRE1*	4,282.10 $^{-2}$	10^{-4}
	$3(t=10s)$	*PRE1*	8,565.10 $^{-2}$	10^{-4}
	$4(t=50s)$	*PRE1*	4,282.10 $^{-1}$	1%
	$8(t=5.10^3 s)$	*PRE1*	4,26.10 $^{+1}$	1%
	$16(t=10^{10} s)$	*PRE1*	4,996.10 $^{+3}$	1%
	$1(t=1s)$	*PRE2*	-6,796.10 $^{-6}$	10^{-4}
	$2(t=5s)$	*PRE2*	-3,398.10 $^{-5}$	10^{-4}
	$3(t=10s)$	*PRE2*	-6,796.10 $^{-5}$	10^{-4}
	$4(t=50s)$	*PRE2*	-3,398.10 $^{-4}$	10^{-4}
	$8(t=5.10^3 s)$	*PRE2*	-3,384.10 $^{-2}$	10^{-4}
	$16(t=10^{10} s)$	*PRE2*	-3,964	10^{-3}

6 Modelization D

6.1 Characteristic of the modelization D

plane Modelization: D_PLAN_HHMS



1 nets DPQ8 of modelization D_PLAN_HHMS : HHM_DPQ8S

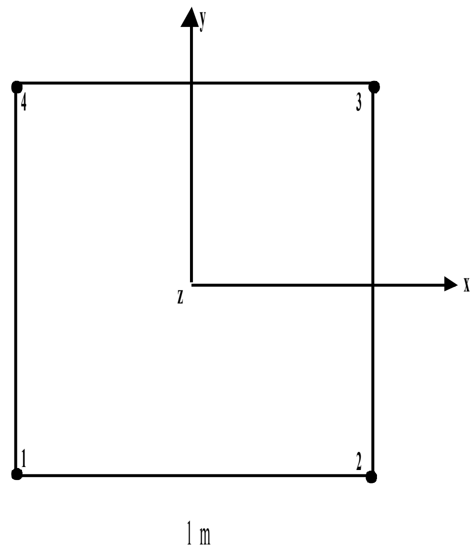
6.2 Result of the modelization D

Node/not	**urgent Sequence number/** (s)	**Value**	**Pressure** (Pa)	**Tolerance**
1,2/ A and B	$1(t=1s)$	*PRE1*	$-2,8549\times10^{-3}$	10^{-4}
	$2(t=5s)$	*PRE1*	$-0,01427$	10^{-4}
	$3(t=10s)$	*PRE1*	$-0,028549$	10^{-4}
	$4(t=50s)$	*PRE1*	$-0,1427$	10^{-2}
	$8(t=5.10^{3}s)$	*PRE1*	$-14,24$	10^{-2}
	$16(t=10^{10}s)$	*PRE1*	$-4995,0$	10^{-2}
	$1(t=1s)$	*PRE2*	$2,2656\times10^{-6}$	10^{-4}
	$2(t=5s)$	*PRE2*	$1,1328\times10^{-5}$	10^{-4}
	$3(t=10s)$	*PRE2*	$2,2656\times10^{-5}$	10^{-4}
	$4(t=50s)$	*PRE2*	$1,133\times10^{-4}$	10^{-4}
	$8(t=5.10^{3}s)$	*PRE2*	$0,011301$	10^{-4}
	$16(t=10^{10}s)$	*PRE2*	$3,9647$	10^{-3}
3,4/ C and D	$1(t=1s)$	*PRE1*	$2,8549\times10^{-3}$	10^{-4}
	$2(t=5s)$	*PRE1*	$0,01427$	10^{-4}
	$3(t=10s)$	*PRE1*	$0,028549$	10^{-4}
	$4(t=50s)$	*PRE1*	$0,1427$	10^{-2}
	$8(t=5.10^{3}s)$	*PRE1*	$14,24$	10^{-2}
	$16(t=10^{10}s)$	*PRE1*	$4997,0$	10^{-2}
	$1(t=1s)$	*PRE2*	$-2,2656\times10^{-6}$	10^{-4}
	$2(t=5s)$	*PRE2*	$-1,1328\times10^{-5}$	10^{-4}
	$3(t=10s)$	*PRE2*	$-2,2656\times10^{-5}$	10^{-4}
	$4(t=50s)$	*PRE2*	$-1,133\times10^{-4}$	10^{-4}
	$8(t=5.10^{3}s)$	*PRE2*	$-0,0113$	10^{-4}
	$16(t=10^{10}s)$	*PRE2*	$-3,9647$	10^{-3}

7 Modelization E

7.1 Characteristic of the modelization E

plane Modelization: D_PLAN_HHMD



1 nets DPQ8 of modelization D_PLAN_HHM : HHM_DPQ8D

7.2 Result of the modelization E

Node/not	**urgent Sequence number/** (s)	**Value**	**Pressure** (Pa)	**Tolerance**
1,2/ A and B	$1(t=1\,s)$	*PRE1*	$-2{,}85486\times10^{-3}$	10^{-4}
	$2(t=5\,s)$	*PRE1*	$-0{,}0142743$	10^{-4}
	$3(t=10\,s)$	*PRE1*	$-0{,}0285487$	10^{-4}
	$4(t=50\,s)$	*PRE1*	$-0{,}14274$	10^{-4}
	$8(t=5.10^{3}\,s)$	*PRE1*	$-14{,}2406$	10^{-4}
	$16(t=10^{10}\,s)$	*PRE1*	$-4995{,}06$	10^{-4}
	$1(t=1\,s)$	*PRE2*	$-2{,}26558\times10^{-6}$	10^{-4}
	$2(t=5\,s)$	*PRE2*	$1{,}13279\times10^{-5}$	10^{-4}
	$3(t=10\,s)$	*PRE2*	$2{,}26557\times10^{-5}$	10^{-4}
	$4(t=50\,s)$	*PRE2*	$1{,}132764\times10^{-4}$	10^{-4}
	$8(t=5.10^{3}\,s)$	*PRE2*	$0{,}0113012$	10^{-4}
	$16(t=10^{10}\,s)$	*PRE2*	$3{,}96734$	10^{-4}
3,4/ C and D	$1(t=1\,s)$	*PRE1*	$2{,}85488\times10^{-3}$	10^{-4}
	$2(t=5\,s)$	*PRE1*	$-0{,}0142743$	10^{-4}
	$3(t=10\,s)$	*PRE1*	$0{,}0285487$	10^{-4}
	$4(t=50\,s)$	*PRE1*	$0{,}14274$	10^{-3}
	$8(t=5.10^{3}\,s)$	*PRE1*	$14{,}2407$	10^{-4}
	$16(t=10^{10}\,s)$	*PRE1*	$4996{,}93$	10^{-4}
	$1(t=1\,s)$	*PRE2*	$-2{,}26557\times10^{-6}$	10^{-4}
	$2(t=5\,s)$	*PRE2*	$-1{,}13279\times10^{-5}$	10^{-4}
	$3(t=10\,s)$	*PRE2*	$-2{,}26557\times10^{-5}$	10^{-4}
	$4(t=50\,s)$	*PRE2*	$-1{,}13276\times10^{-4}$	10^{-4}
	$8(t=5.10^{3}\,s)$	*PRE2*	$-0{,}0113012$	10^{-4}
	$16(t=10^{10}\,s)$	*PRE2*	$-3{,}96734$	10^{-4}

8 Summary of the results

the values of *Code_Aster* are in concord with the values of reference.