WTNV100 - Triaxial compression test not drained with the model CJS (level 1)

Summarized

This test makes it possible to validate level 1 of model CJS. It is about a triaxial compression test in not drained condition.
In the first two modelizations, computations are carried out only on the solid part of the soil, the aspect not drained being modelled by a voluminal strain null squelette, in fact modelizations $3D$ differ one from the other only by the mesh.

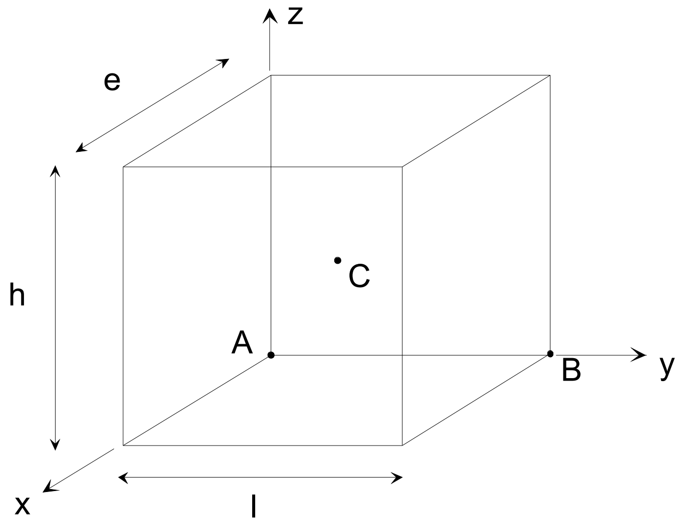
In the third modelization, the hydraulic coupling is taken into account, the sample is completely saturated, the squelette and the fluid is supposed to be incompressible.

By reason of symmetry, one is interested only in the eighth of a sample subjected to a triaxial compression test.
The level of containment is of $100\,kPa$.

The results got with the model CJS1 are compared with an analytical solution.

1 Problem of reference

1.1 Geometry



hauteur : h = 1 m
largeur : l = 1 m
épaisseur : e = 1 m

Coordinated of the points (in meters):

	A	B	C
x	0.	0.	0.5
y	0.	1.	0.5
z	0.	0.	0.5

1.2 Material property

$E=22,4 10^3 kPa$
$\nu=0,3$
Coefficient of biot $b=1$
water is supposed to be incompressible: $\text{UN\_SUR\_K}=0$
Parameters CJS1: $\beta=-0,03$ $\gamma=0,82$ $R_m=0,289$ $P_a=-100 kPa$

1.3 Initial conditions, boundary conditions, and mechanical

1.3.1 loading Modelization pure

Phase 1:

One brings the sample in a homogeneous state: $\sigma_{xx}^0=\sigma_{yy}^0=\sigma_{zz}^0$, by imposing the corresponding confining pressure on the front, side right and higher sides. Displacements are blocked on the sides postpones ($u_x=0$), side left ($u_y=0$) and lower ($u_z=0$).

Phase 2:

One maintains displacements blocked on the sides postpones ($u_x=0$), side left ($u_y=0$) and lower ($u_z=0$). One applies a displacement imposed to the upper face: $u_z(t)$, in order to 2) obtain $\varepsilon_{zz}=-20\,\%$ a strain (counted starting from the beginning of the phase. On the front sides and side right, one imposes displacements respectively $u_x(t)$ and $u_y(t)$, in order to have a voluminal strain null for the sample, i.e. finally that one imposes $\varepsilon_{xx}=\varepsilon_{yy}=-\frac{\varepsilon_{zz}}{2}$. It is the way reproduce the behavior of the solid phase during a triaxial compression test not drained.

1.3.2 Modelization coupled with hydraulics

Phase 1:

One brings the sample in a homogeneous state of effective stresses: $\sigma_{xx}^0=\sigma_{yy}^0=\sigma_{zz}^0$, by imposing the corresponding stagnation pressure on the front, side right and higher sides and by imposing null water pressures everywhere. Displacements are blocked on the sides postpones ($u_x=0$), side left ($u_y=0$) and lower ($u_z=0$).

Phase 2:

One maintains displacements blocked on the sides postpones ($u_x=0$), side left ($u_y=0$) and lower ($u_z=0$).
On all the sides, the hydraulic flux are null.
One applies a displacement forced to the upper face in order to 2) obtain $\varepsilon_{zz}=-20\,\%$ a strain (counted starting from the beginning of the phase. On the front sides and side right, one imposes boundary conditions in total stress:

$$\sigma.n=\sigma^0\ (=100kPa)$$

2 Reference solution

2.1 Reference solution for the water pressure into linear

σ_0, ε_0 p_0 indicating the stresses, strains and pressures of water obtained in the phase one a:

$$\sigma-\sigma_0=\lambda tr\left(\varepsilon-\varepsilon_0\right)+2\mu\left(\varepsilon-\varepsilon_0\right)-b\left(p-p_0\right)$$

$$\frac{m}{\rho^{fl}}=\frac{p-p_0}{M}+btr\left(\varepsilon-\varepsilon_0\right)$$

In this writing, M indicates the modulus of biot and $N=\frac{1}{M}$.

The null flux boundary conditions and the conservation of the mass of water give $m=0$
the boundary conditions on the side walls and the fact that the stress state is homogeneous give:

$$\sigma_{xx}-\sigma_{xx0}=0$$

One has thus finally to solve the two equations:

$$\begin{cases}\lambda\left(2\Delta\varepsilon_{xx}+\Delta\varepsilon_{zz}\right)+2\mu\Delta\varepsilon_{xx}=bp\\ b\left(2\Delta\varepsilon_{xx}+\Delta\varepsilon_{zz}\right)=-\frac{p}{M}=-Np\end{cases}$$

And one obtains:

$$\begin{cases}\Delta\varepsilon_{xx}=-\frac{\Delta\varepsilon_{zz}}{2}\frac{b^2+\lambda N}{b^2+\left(\lambda+\mu\right)N}\\ p=-\frac{\mu b\Delta\varepsilon_{zz}}{b^2+\left(\lambda+\mu\right)N}\end{cases}$$

In our case,

$$\Delta\varepsilon_{xx}=-\frac{\Delta\varepsilon_{zz}}{2} \quad ; \quad p=-\mu\Delta\varepsilon_{zz}$$

2.2 Development of analytical solution CJS

One has permanently:

for the strains: $\varepsilon_{xx}=\varepsilon_{yy}=-\frac{\varepsilon_{zz}}{2}$

for the stresses: $\sigma_{xx}=\sigma_{yy}$

Elastic phase:
While writing the elastic model simply, it comes:

$$\sigma_{xx}=\sigma_{xx}^0-\mu\,\varepsilon_{zz}$$

$$\sigma_{zz}=\sigma_{zz}^0+2\,\mu\,\varepsilon_{zz}$$

In addition, it is also known that during this phase I_1 ($=trace(\sigma)$) remains constant because $\varepsilon_v = 0$. One from of deduced for the components from the deviator:

$$s_{xx} = \sigma_{xx} - \frac{I_1}{3} = \sigma^0_{xx} - \frac{I^0_1}{3} - \mu\,\varepsilon_{zz} = -\mu\,\varepsilon_{zz} \text{ and } s_{zz} = \sigma_{zz} - \frac{I_1}{3} = \sigma^0_{zz} - \frac{I^0_1}{3} + 2\,\mu\,\varepsilon_{zz} = 2\,\mu\,\varepsilon_{zz}$$

is: $s_{II} = -\sqrt{6}\,\mu\,\varepsilon_{zz}$ and $\det\left(\underline{\underline{s}}\right) = 2\,\mu^3\,\varepsilon^3_{zz}$

Consequently: $h\left(\theta_s\right) = \left(1-\gamma\right)^{1/6}$

Thus when the criterion is reached $f^d = 0$, one a:

$$s_{II}\,\left(1-\gamma\right)^{1/6} + R_m\,I^0_1 = -\sqrt{6}\,\mu\,\varepsilon_{zz}\,\left(1-\gamma\right)^{1/6} + R_m\,I^0_1 = 0$$

I.e. the transition enters the states elastic and perfectly plastic makes for an axial strain equalizes with:

$$\varepsilon^{trans}_{zz} = \frac{R_m\,I^0_1}{\sqrt{6}\,\mu\left(1-\gamma\right)^{1/6}}$$

The stress state corresponding is noted:

$$\sigma^{trans}_{xx} = \sigma^0_{xx} - \mu\,\frac{R_m\,I^0_1}{\sqrt{6}\,\mu\left(1-\gamma\right)^{1/6}} \quad \text{and} \quad \sigma^{trans}_{zz} = \sigma^0_{zz} + 2\,\mu\,\frac{R_m\,I^0_1}{\sqrt{6}\,\mu\left(1-\gamma\right)^{1/6}}$$

plastic Phase:

One Generally s^{-d} notes the deviator of the reverse of the tensor
S, one has the following quantities:

$$s_{xx} = -\frac{1}{3}\left(\sigma_{zz} - \sigma_{xx}\right) = s_{yy} \quad s^{-1}_{xx} = \frac{-3}{\sigma_{zz} - \sigma_{xx}} \quad s^{-d}_{xx} = \frac{-3}{2\left(\sigma_{zz} - \sigma_{xx}\right)}$$

$$s_{zz} = \frac{2}{3}\left(\sigma_{zz} - \sigma_{xx}\right) \quad s^{-1}_{zz} = \frac{3}{2\left(\sigma_{zz} - \sigma_{xx}\right)} \quad s^{-d}_{zz} = \frac{3}{\sigma_{zz} - \sigma_{xx}}$$

that is to say: $s_{II} = -\sqrt{\frac{2}{3}}\left(\sigma_{zz} - \sigma_{xx}\right)$ and $\det\left(\underline{\underline{s}}\right) = \frac{2}{3^3}\left(\sigma_{zz} - \sigma_{xx}\right)^3$

Consequently: $h\left(\theta_s\right) = \left(1-\gamma\right)^{1/6}$

One from of deduced:

$$Q_{xx} = \frac{1}{\sqrt{6}}\left(1-\gamma\right)^{1/6} \text{ and } Q_{zz} = -\sqrt{\frac{2}{3}}\left(1-\gamma\right)^{1/6}$$

moreover: $\frac{\partial f^d}{\partial \sigma_{xx}} = \frac{1}{\sqrt{6}}\left(1-\gamma\right)^{1/6} + R_m$ and $\frac{\partial f^d}{\partial \sigma_{zz}} = -\sqrt{\frac{2}{3}}\left(1-\gamma\right)^{1/6} + R_m$

Like one a: $\beta' = signe\left(s_{ij}\;\dot{\varepsilon}_{ij}\right)\,\beta\left(\frac{s_{II}}{s^c_{II}} - 1\right) = \beta\left(\frac{R_m}{R_c} - 1\right) = \beta$

then the tensor $\underline{\underline{n}}$ is written:

$$n_{xx}=\frac{1}{\sqrt{\beta^2+3}}\left(\frac{1}{\sqrt{6}}\beta+1\right) \text{ and } n_{zz}=\frac{1}{\sqrt{\beta^2+3}}\left(-\sqrt{\frac{2}{3}}\beta+1\right)$$

It comes then for $\underline{\underline{G}}^d$:

$$G_{xx}^d=\frac{1}{\sqrt{6}}(1-\gamma)^{1/6}+R_m-\frac{\beta(1-\gamma)^{1/6}+3R_m}{\beta^2+3}\left(\frac{1}{\sqrt{6}}\beta+1\right)$$
$$G_{zz}^d=-\sqrt{\frac{2}{3}}(1-\gamma)^{1/6}+R_m-\frac{\beta(1-\gamma)^{1/6}+3R_m}{\beta^2+3}\left(-\sqrt{\frac{2}{3}}\beta+1\right)$$

One also has according to the elastic model:

$$\sigma_{xx}=\sigma_{xx}^{trans}+\Delta\sigma_{xx} \text{ and } \sigma_{zz}=\sigma_{zz}^{trans}+\Delta\sigma_{zz}$$

where:

$$\Delta\sigma_{xx}=2\mu\left(\Delta\varepsilon_{xx}-\Delta\lambda^d\ G_{xx}^d\right)+\lambda\left(\Delta\varepsilon_v-\Delta\lambda^d\ tr\left(\underline{\underline{G}}^d\right)\right)=-\mu\,\Delta\varepsilon_{zz}-2\mu\,\Delta\lambda^d\ G_{xx}^d-\lambda\,\Delta\lambda^d\left(2G_{xx}^d+G_{zz}^d\right)$$
$$\Delta\sigma_{zz}=2\mu\left(\Delta\varepsilon_{zz}-\Delta\lambda^d\ G_{zz}^d\right)+\lambda\left(\Delta\varepsilon_v-\Delta\lambda^d\ tr\left(\underline{\underline{G}}^d\right)\right)=2\mu\,\Delta\varepsilon_{zz}-2\mu\,\Delta\lambda^d\ G_{zz}^d-\lambda\,\Delta\lambda^d\left(2G_{xx}^d+G_{zz}^d\right)$$

and with: $\Delta\varepsilon_{xx}=\varepsilon_{xx}-\varepsilon_{xx}^{trans}$ and $\Delta\varepsilon_{zz}=\varepsilon_{zz}-\varepsilon_{zz}^{trans}$

is, according to what precedes, one has for s_{II} :

$$s_{II}=-\sqrt{\frac{2}{3}}\left[\left(\sigma_{zz}^{trans}-\sigma_{xx}^{trans}\right)+3\mu\left(\varepsilon_{zz}-\varepsilon_{zz}^{trans}\right)-2\mu\,\Delta\lambda^d\left(G_{zz}^d-G_{xx}^d\right)\right]$$
$$=s_{II}^{trans}-\sqrt{\frac{2}{3}}\left[3\mu\left(\varepsilon_{zz}-\varepsilon_{zz}^{trans}\right)-2\mu\,\Delta\lambda^d\left(G_{zz}^d-G_{xx}^d\right)\right]$$

and for I_1 :

$$I_1=I_1^{trans}-(3\lambda+2\mu)\,\Delta\lambda^d\left(2G_{xx}^d+G_{zz}^d\right)$$

One from of deduced that the loading function déviatoire is written:

$$f^d=s_{II}^{trans}(1-\gamma)^{1/6}-\sqrt{\frac{2}{3}}\left[3\mu\left(\varepsilon_{zz}-\varepsilon_{zz}^{trans}\right)-2\mu\,\Delta\lambda^d\left(G_{zz}^d-G_{xx}^d\right)\right](1-\gamma)^{1/6}$$
$$+R_m\,I_1^{trans}-R_m(3\lambda+2\mu)\,\Delta\lambda^d\left(2G_{xx}^d+G_{zz}^d\right)$$

By taking account owing to the fact that $f^d\left(\underline{\underline{\sigma}}^{trans}\right)=0$, one finds then for the plastic multiplier:

$$\Delta\lambda^d=\frac{3\mu(1-\gamma)^{1/6}}{2\mu\left(G_{zz}^d-G_{xx}^d\right)-\sqrt{\frac{3}{2}}R_m(3\lambda+2\mu)\left(2G_{xx}^d+G_{zz}^d\right)}\left(\varepsilon_{zz}-\varepsilon_{zz}^{trans}\right)$$

what gives with the formulas of G_{xx}^{d} and G_{zz}^{d} the preceding ones:

$$\Delta\lambda^{d} = \frac{\sqrt{\frac{2}{3}}\,\mu\left(1-\gamma\right)^{1/6}\left(\beta^{2}+3\right)}{\left(R_m\ \beta-\left(1-\gamma\right)^{1/6}\right)\left(2\ \mu\left(1-\gamma\right)^{1/6}-\left(3\ \lambda+2\ \mu\right)\ R_m\ \beta\right)}\left(\varepsilon_{zz}-\varepsilon_{zz}^{trans}\right)$$

One concludes from it finally the analytical statement from the stresses:

While posing:

$$a=\left(1-\gamma\right)^{1/6}\quad;\qquad b=\left(\beta^{2}+3\right)$$

One a:

$$\sigma_{xx}-\sigma_{xx}^{trans}=$$

$$-\left[\mu+\frac{\sqrt{\frac{2}{3}}\,\mu\,a\,b\left(2\ \mu\left(\frac{1}{\sqrt{6}}\,a+R_m-\frac{\beta\,a+3\ R_m}{b}\left(\frac{1}{\sqrt{6}}\,\beta+1\right)\right)+3\ \lambda\ \beta\frac{\left(R_m\ \beta-a\right)}{b}\right)}{\left(R_m\ \beta-a\right)\left(2\ \mu a-\left(3\ \lambda+2\ \mu\right)\ R_m\ \beta\right)}\right]\left(\varepsilon_{zz}-\varepsilon_{zz}^{trans}\right)$$

$$\sigma_{zz}-\sigma_{zz}^{trans}=$$

$$\left[2\ \mu-\frac{\sqrt{\frac{2}{3}}\,\mu\,a\,b\left(2\ \mu\left(-\sqrt{\frac{2}{3}}\,a+R_m-\frac{\beta a+3\ R_m}{b}\left(-\sqrt{\frac{2}{3}}\,\beta+1\right)\right)+3\ \lambda\ \beta\frac{\left(R_m\ \beta-a\right)}{b}\right)}{\left(R_m\ \beta-a\right)\left(2\ \mu a-\left(3\ \lambda+2\ \mu\right)\ R_m\ \beta\right)}\right]\left(\varepsilon_{zz}-\varepsilon_{zz}^{trans}\right)$$

2.3 Forced Results of

reference σ_{xx}, σ_{yy} and σ_{zz} at the points A , B and C .

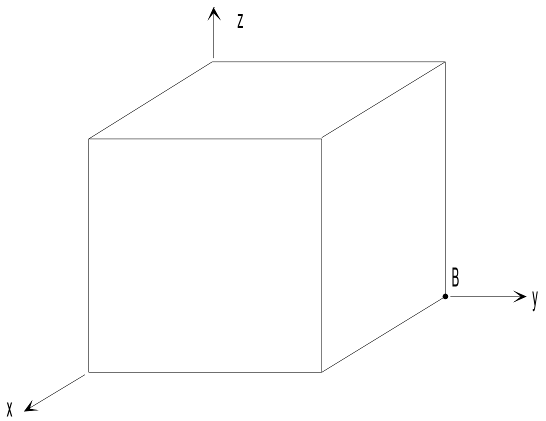
2.4 Uncertainty on the solution

exact analytical Solution for CJS1.

3 Modelization A

3.1 Characteristic of the modelization

3D :



Cutting: 1 in height, in width and thickness.

Loading of phase 1:
Confining pressure: $\sigma_{xx}^0 = \sigma_{yy}^0 = \sigma_{zz}^0 : -100\,kPa$.
Level 1 of model CJS

3.2 Characteristic of the mesh

Many nodes: 8
Number of meshes and types: 1 HEXA8 and 6 QUA4

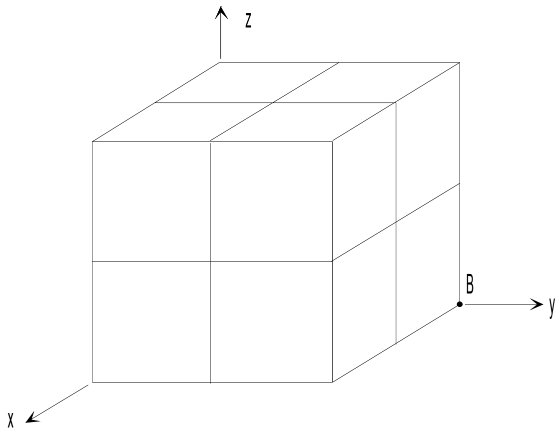
3.3 Quantities tested and results

Localization	Sequence number	axial Strain ε_{zz} ($\%$)	Forced (kPa)	Reference
Points A and B	1	– 0.25	σ_{xx}	– 78.461538
	2	– 0.50	σ_{xx}	– 56.923077
	3	– 0.75	σ_{xx}	– 53.606
	4	– 1.0	σ_{xx}	– 54.480
	8	– 5.0	σ_{xx}	– 68.467
	23	– 20.0	σ_{xx}	– 120.918
	1	– 0.25	σ_{yy}	– 78.461538
	2	– 0.50	σ_{yy}	– 56.923077
	3	– 0.75	σ_{yy}	– 53.606
	4	– 1.0	σ_{yy}	– 54.480
	8	– 5.0	σ_{yy}	– 68.467
	23	– 20.0	σ_{yy}	– 120.918
	1	– 0.25	σ_{zz}	– 143,07692
	2	– 0.50	σ_{zz}	– 186.153846
	3	– 0.75	σ_{zz}	– 196.818
	4	– 1.0	σ_{zz}	– 200.028
	8	– 5.0	σ_{zz}	– 251.383
	23	– 20.0	σ_{zz}	– 443.961

4 Modelization B

4.1 Characteristic of the modelization

This modelization differ from the preceding one by the smoothness of the mesh 3D :



Cutting: 2 in height, in width and thickness.

Loading of phase 1:

Confining pressure: $\sigma_{xx}^0 = \sigma_{yy}^0 = \sigma_{zz}^0 \ : \ -100 \, kPa$.

Level 1 of model CJS

4.2 Characteristic of the mesh

Many nodes: 27

Number of meshes and types: 8 HEXA8 and 24 QUA4

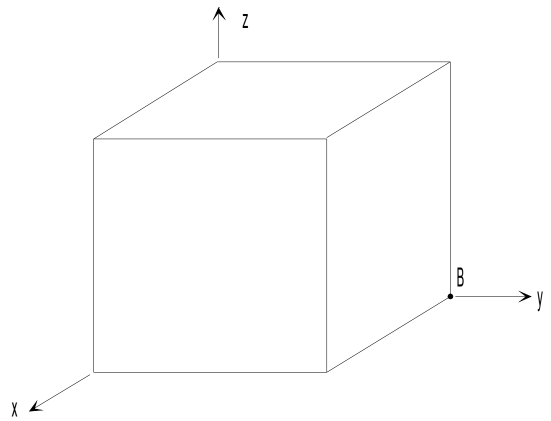
4.3 Quantities tested and results

Localization	Sequence number	axial Strain ε_{zz} ($\%$)	Forced (kPa)	Reference
Points A , B and C	5	– 0.2	σ_{xx}	– 82.76923
	10	– 0.4	σ_{xx}	– 65.53846
	20	– 0.8	σ_{xx}	– 53.78079
	40	– 1.6	σ_{xx}	– 56.578176
	60	– 5.6	σ_{xx}	– 70.565109
	100	– 20.0	σ_{xx}	– 120.918065
	5	– 0.2	σ_{yy}	– 82.76923
	10	– 0.4	σ_{yy}	– 65.53846
	20	– 0.8	σ_{yy}	– 53.78079
	40	– 1.6	σ_{yy}	– 56.578176
	60	– 5.6	σ_{yy}	– 70.565109
	100	– 20.0	σ_{yy}	– 120.918065
	5	– 0.2	σ_{zz}	– 134.46154
	10	– 0.4	σ_{zz}	– 168.92308
	20	– 0.8	σ_{zz}	– 197.460849
	40	– 1.6	σ_{zz}	– 207.731697
	60	– 5.6	σ_{zz}	– 259.085935
	100	– 20.0	σ_{zz}	– 443.961194

5 Modelization C

5.1 Characteristic of the modelization

3D_HM :



Cutting: 1 in height, in width and thickness.

Loading of phase 1:

Confining pressure: $\sigma_{xx}^{0} = \sigma_{yy}^{0} = \sigma_{zz}^{0}$: $-100\, kPa$.

Level 1 of model CJS
Coefficient of biot: 1
UN_SUR_K of water: 0

5.2 Characteristic of the mesh

Many nodes: 20
Number of meshes and types: 1 HEXA20 and 6 QUA8

5.3 Quantities tested and results

Localization	Sequence number	axial Strain ε_{zz} (%)	Forced (kPa)	Reference
Points A and B	1	– 0.25	σ_{xx}	– 78.461538
	2	– 0.50	σ_{xx}	– 56.923077
	3	– 0.75	σ_{xx}	– 53.606
	4	– 1.0	σ_{xx}	– 54.480
	8	– 5.0	σ_{xx}	– 68.467
	23	– 20.0	σ_{xx}	– 120.918
	1	– 0.25	σ_{yy}	– 78.461538
	2	– 0.50	σ_{yy}	– 56.923077
	3	– 0.75	σ_{yy}	– 53.606
	4	– 1.0	σ_{yy}	– 54.480
	8	– 5.0	σ_{yy}	– 68.467
	23	– 20.0	σ_{yy}	– 120.918
	1	– 0.25	σ_{zz}	– 143,07692
	2	– 0.50	σ_{zz}	– 186.153846
	3	– 0.75	σ_{zz}	– 196.818
	4	– 1.0	σ_{zz}	– 200.028
	8	– 5.0	σ_{zz}	– 251.383
	23	– 20.0	σ_{zz}	– 443.961
	1	– 0.25	pressure water	2,1538E+04
	2	– 0.50	pressure water	4,3077E+04

For the water pressure, one has the reference as long as the behavior is elastic linear

6 Summary of the results

the values of *Code_Aster* are in perfect agreement with the values of reference. Concerning the coupling with the hydraulics, this test proves that by means of computer, coupling CJS/THM functions and that the equations of hydraulics are at least able to give again the variation of volume null when water is incompressible.