WTNL102 - Dimensional mono problem of forced convection

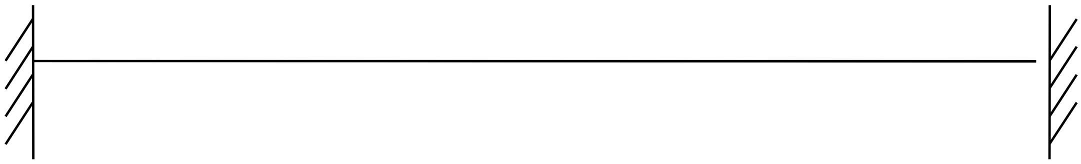
Summarized:

It is the dimensional mono transport of heat by a flux constant velocity. The hydraulic mode is characterized by a linear pressure in space. The reference solution is analytical.

1 Problem of reference

1.1 Geometry

One is placed in the frame of a dimensional mono problem in Cartesian coordinates. The “structure” considered, is finally a segment length 1



$x=0\begin{cases} p=P \\ T=0 \end{cases}$ $x=1\quad\begin{cases} p=0 \\ T=1 \end{cases}$

1.2 Boundary conditions and loadings

One imposes a pressure varying linearly P in $x=0$ on 0 in $x=1$: $p(x)=P(1-x)$

In $x=0$: the temperature is imposed null
In $x=1$: the temperature is imposed on 1.

1.3 Initial conditions

$T(x)=0$ everywhere
One is interested in the permanent mode

2 Reference solution

2.1 Method of calculating

One leaves the equation of energy [éq 3.1.3-1] the document [R7.01.11], which in this case gives:

$$h\dot{m} + \dot{Q}' + Div(h\mathbf{M}) + Div(\mathbf{q}) = 0 \quad \text{éq 2.1-1}$$

In which h the enthalpy of water, its $\mathbf{M}$ mass flux indicates, m the mass water contribution and $\mathbf{q}$ heat flux.

Taking into account the made assumptions, one sees easily that:

$$\mathbf{M} = M_x = \rho_w \lambda_h P \quad \text{éq 2.1-2}$$

$$h = C_w^p T \quad \text{éq 2.1-3}$$

$$\mathbf{q} = q_x = -\lambda_T \frac{\partial T}{\partial x} \quad \text{éq 2.1-4}$$

$$\dot{Q}' = \rho_w C_w^p \dot{T} \quad \text{éq 2.1-5}$$

λ_T is the thermal coefficient of diffusion process, $\lambda_h = \frac{K_{\text{int}}}{\mu_w}$ is the hydraulic coefficient of diffusion, K_{int} the intrinsic permeability ρ_w μ_w, C_w^p are respectively the density, viscosity and calorific heat with constant pressure of water.

While deferring [éq 2.1-2], [éq 2.1-3], [éq 2.1-4] and [éq 2.1-5] in [éq 2.1-1] one finds:

$$\frac{\rho_w C_w^p}{\lambda_T} \dot{T} + \rho_w C_w^p \frac{\lambda_h}{\lambda_T} P \frac{\partial T}{\partial x} - \frac{\partial^2 T}{\partial x^2} = 0 \quad \text{éq 2.1-6}$$

One poses:

$$R = \rho_w C_w^p \frac{\lambda_h}{\lambda_T} P$$

and

$$S = \frac{\rho_w C_w^p}{\lambda_T}$$

One obtains

$$S\dot{T} + R \frac{\partial T}{\partial x} - \frac{\partial^2 T}{\partial x^2} = 0 \quad \text{éq 2.1-7}$$

2.2 Results of reference

In order to obtain the permanent mode more quickly, one chooses coefficients such as:

$$\frac{S}{R} = \frac{1}{\lambda_h P} << 1$$

The solution of [éq 2.1-7] is then:

$$T = \frac{e^{Rx} - 1}{e^R - 1}$$

3 Modelization A

3.1 Characteristic of the modelization A

One makes a modelization with 500 elements, each element thus has a length $h=\frac{1}{500}$.

The coefficients are chosen:

ρ_w	1
C_w^p	1
μ_w	1
K_{int}	100
λ_T	10
P	1

These values lead to a number of Peclet $R=10$ and a Peclet number local $Rh=\frac{1}{50}$.

3.2 Results

X	Reference temperature	Temperature *Aster*	relative Error ($\%$)
6,00E-01	0,0182710686	0,0182567	0,079%
7,00E-01	0,0497439270	0,0497269	0,034%
8,00E-01	0,1352960260	0,1352760	0,015%
9,00E-01	0,3678507400	0,3678309	0,005%
1,00E+00	1,0000000000	1,0000000	0,0%

4 Summary of the results

a good agreement is obtained between the temperatures calculated by *Code_Aster* and the values of reference.