WTNL100 - Consolidation of a column of soil saturated poro-elastic (Terzaghi)

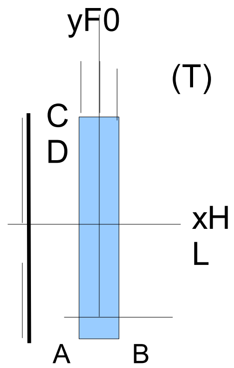
Summarized:

This benchmark relates to the consolidation of a column of soil saturated poro-elastic, seals laterally and at the base, subjected to a level of force at the head. The purpose is to test displacements (compressing of the soil), the pressures and to compare all the results with an analytical solution, whose broad outlines are presented.

1 Problem of reference

1.1 Geometry

This benchmark unidimensional is resulting from the literature and has an analytical solution [1]. It is inspired by the problem of consolidation of a porous soil saturated with water to room temperature (problem of Terzaghi). Water is supposed to be incompressible ($1/K_{lq}=0,0$). The temperature is uniform. The action of gravity in this case is neglected. The problem coupled poro-mechanics saturated is described by the variables of displacements u_y (compressing of the soil) and of fluid pressure P_{lq} (the hydraulic head being P_{lq}/ρ_{eau}). In order to obtain a purely unidimensional solution, the Poisson's ratio is selected equal to $\nu_0=0,0$. Dimensions are the following ones: $L=1,00\,m$ $H=10,00\,m$ ·



drained Young's modulus: $E_0=10\,MPa$	Intrinsic permeability: $K_{intr}=10\text{x}10^{-8}$
Poisson's ratio: $\nu_0=0,0$	Density of the fluid: $\rho_{lq}=1000\,kg/m^3$
Density: $r_0=2800\,kg/m^3$	Porosity: $\phi^0=0,5$
Coefficient of Biot: $b=1,0$	Dynamic viscosity of water: $\mu_{lq}(T)=1$
Saturation $S_{lq}(p_c)=1,0$	Permeability relating to the fluid: $k_{lq}^{rel}(S_{lq})=1$

The characteristics of behavior and thermal coupling are not significant.

The hydraulic permeability of the medium with water is then: $\lambda_{lq}^H=\frac{K_{intr}(\varphi).k_{lq}^{rel}(S_{lq})}{\mu_{lq}(T)}$ in ($m^3\,s/kg$).

In soil mechanics, it is often noted the permeability by $k=\lambda_{lq}^H\rho_{lq}\,g$, that is to say here: $k\approx10^{-14}\,m/s$.

The coefficient of consolidation $c_v=\lambda_{lq}^H E_0/b^2$ is worth here: $c_v=0,1\,m^2/s$.

1.2 Boundary conditions and loadings

1.2.1 Boundary conditions

the interstitial fluid pressure remains null on all the upper face CD : $P_{lq}=0$. The side sides have the displacements blocked in x . The lower face AB has the displacements blocked in x and in y , and it is tight: $P_{lq,y}=0$

1.2.2 Initial conditions

the column is initially at rest in a virgin state: $P_{lq}=0$ $\sigma_{yy}=0$.

1.2.3 Loading

It is exerted a level of pressure $F_0=-1{,}0\,Pa$ on the upper face CD with $t=0\mathrm{s}$.
Gravity is neglected here.

2 Reference solution

the total stresses poro-elastics are: $\sigma=E_0\varepsilon(u)-bP_{lq}$. Only the vertical component is present: $\sigma_{yy}(y,t)=E_0u_{y,y}(y,t)-bP_{lq}(y,t)$.

The hydro-mechanical equilibrium poro-elastic $1\mathrm{D}$ is thus written, in the absence of force of gravity, for $t\geq 0$:

$$\begin{cases} E_0 u_{y,yy}(y,t)-bP_{lq,y}(y,t)=0\,,\, \textit{équilibre mécanique} \\ \lambda_{lq}^H P_{lq,yy}(y,t)-b\dot{u}_{y,y}(y,t)=0\,,\, \textit{équilibre hydraulique} \end{cases}$$

with the initial conditions: $u_y(y,0)=0$, $P_{lq}(y,0)=0$ and the boundary conditions for $t>0$: $u_y(0,t)=0$ $P_{lq}(0,t)=0$, $\sigma_{yy}(H,t)=F_0\eta(t)=E_0u_{y,y}(H,t)-bP_{lq}(H,t)$ where $\eta(t)$ is the function level in $t=0$ (Heaviside).

The mechanical equilibrium directly gives the uniformity of the total stresses for $t>0$ on $|0,H|$, from where: $\sigma_{yy}(y,t)=F_0=E_0u_{y,y}(y,t)-bP_{lq}(y,t)$, that is to say $u_{y,y}(y,t)=\frac{1}{E_0}(F_0+bP_{lq}(y,t))$, for $t>0$ on $|0,H|$.

The hydraulic equilibrium leads then to:
$\frac{b^2}{E_0}\dot{P}_{lq}(y,t)-\lambda_{lq}^H P_{lq,yy}(y,t)=0$, for $t>0$ on $|0,H|$

with like initial conditions $P_{lq}(y,0)=-\frac{F_0}{b}$ on $|0,H|$, and two boundary conditions $P_{lq}(H,t)=0$ and $P_{lq,y}(0,t)=0$ for $t>0$, i.e. a problem of the standard thermal shock on $|0,H|$.
The coefficient of consolidation $c_v=\lambda_{lq}^H E_0/b^2$ is worth here: $c_v=0{,}1\,m^2/s$. It controls the period of the process of consolidation.

It results from this thus a characteristic time $\tau_c=H^2/c_v$ ($=1000\mathrm{s}$ here) being used to identify the step of the temporal discretization for the numerical method of integration. A this value τ_c , it was reached a little more than 90% of the consolidation.

The solution is, cf [2]:

$$P_{lq}(y,t)=\frac{-4F_0}{\pi b}\sum_{m=1,2,3..}^{+\infty}\frac{(-1)^{m-1}}{2\mathrm{m}-1}e^{-\lambda_{lq}^H E_0\pi^2(2\mathrm{m}-1)^2\frac{t}{(4\mathrm{b}^2H^2)}}.\cos(\frac{\pi y(2\mathrm{m}-1)}{2\mathrm{H}})$$

and: $u_y(y,t)=\frac{F_0 y}{E_0}+\frac{b}{E_0}\int_0^y P_{lq}(\xi,t)d\xi$

that is to say: $$u_y(y,t)=\frac{F_0 y}{E_0}-\frac{8HF_0}{\pi^2 E_0}\sum_{m=1,2,3..}^{+\infty}\frac{(-1)^{m-1}}{(2\mathrm{m}-1)^2}e^{-\lambda_{lq}^H E_0\pi^2(2\mathrm{m}-1)^2\frac{t}{(4\mathrm{b}^2H^2)}}.\sin(\frac{\pi y(2\mathrm{m}-1)}{2\mathrm{H}})$$

The effective stresses (acting on the squelette) are: $\sigma_{yy}^{eff}(y,t)=E_0 u_{y,y}(y,t)$. For times $t\rightarrow\infty$, we obtain: $P_{lq}(0,\infty)=0$ and $u_y(H,\infty)=\frac{F_0 H}{E_0}$ (either here $-10^{-6} m$).

2.1 Uncertainties on the solution

the reference solution is analytical.

2.2 Bibliographical references

1.L.MEIROVITCH: *Analytical methods in vibrations*. McMillan ED., 1967.

2.J.J.MARIGO, E.PLANCHAIS: Introduction *to the asymptotic methods. Application to linear thermal problems. Note EDF* DER/IMA/MMN HI-70/7563, 8/31/1992. Modelization

3 A Characteristic

3.1 of the modelization the characteristics

are identical to the reference solution. Quantities

3.2 tested and Value results tested

Urgent Node	(	S) Standard Reference		Displacement
PRE1 No29 0. ANALYTIQUE			1.0 Displacement	
PRE1 No31 0. ANALYTIQUE			1.0 Displacement	
PRE1 No1 250.	ANALYTIQUE		0.68544576689	Displacement
PRE1 NO3 250.	ANALYTIQUE		0.682208147164	Displacement
PRE1 NO5 250.	ANALYTIQUE		0.67252104433	Displacement
PRE1 No7 250.	ANALYTIQUE		0.656461946263	Displacement
PRE1 No9 250.	ANALYTIQUE		0.634160686593	Displacement
PRE1 No11 250.	ANALYTIQUE		0.605800331394	Displacement
PRE1 No13 250.	ANALYTIQUE		0.571618145927	Displacement
PRE1 No15 250.	ANALYTIQUE		0.531906397249	Displacement
PRE1 No17 250.	ANALYTIQUE		0.487012719208	Displacement
PRE1 No19 250.	ANALYTIQUE		0.437339762565	Displacement
PRE1 No21 250.	ANALYTIQUE		0.38334387542	Displacement
PRE1 No23 250.	ANALYTIQUE		0.32553260623	Displacement
PRE1 No25 250.	ANALYTIQUE		0.264460889851	Displacement
PRE1 No27 250.	ANALYTIQUE		0.200725860656	Displacement
PRE1 No29 250.	ANALYTIQUE		0.134960328921	Displacement
PRE1 No31 250.	ANALYTIQUE		0.0678250497631	Displacement
PRE1 No33 250.	ANALYTIQUE		0.00 Stress	
SIYY No48 0.00	ANALYTIQUE		0.00 Stress	
SIYY No34 250.	ANALYTIQUE		-0.31455423311	Stress
SIYY No35 250.	ANALYTIQUE		-0.317791852836	Stress
SIYY No36 250.	ANALYTIQUE		-0.32747895567	Stress
SIYY No37 250.	ANALYTIQUE		-0.343538053737	Stress
SIYY No38 250.	ANALYTIQUE		-0.365839313407	Stress
SIYY No39 250.	ANALYTIQUE		-0.394199668606	Stress
SIYY No40 250.	ANALYTIQUE		-0.428381854073	Stress
SIYY No41 250.	ANALYTIQUE		-0.468093602751	Stress
SIYY No42 250.	ANALYTIQUE		-0.512987280792	Stress
SIYY No43 250.	ANALYTIQUE		-0.562660237435	Stress
SIYY No44 250.	ANALYTIQUE		-0.61665612458	Stress
SIYY No45 250.	ANALYTIQUE		-0.67446739377	Stress
SIYY No46 250.	ANALYTIQUE		-0.735539110149	Stress
SIYY No47 250.	ANALYTIQUE		-0.799274139344	Stress
SIYY No48 250.	ANALYTIQUE		-0.865039671079	Stress
SIYY No49 250.	ANALYTIQUE		-0.932174950237	Stress
SIYY No50 250.	ANALYTIQUE		-1,0 Stress	
VMIS No50 250.	NOT	REGRESSION	1.0 Stress	
VMIS_SG No 50.250.	NOT	REGRESSION	-1.0 Stress	
PRIN_1 No 50.250.	NOT	REGRESSION	-1.0 Stress	
PRIN_2 No 50.250.	NOT	REGRESSION	0,00 Stress	
PRIN_3 No 50.250.	NOT	REGRESSION	0.00 Stress	
TRESCA No 50.250.	NOT	REGRESSION	the 1.0 values of reference	

in NON_REGRESSION are obtained for version 10.01.21. Modelization

4 B Characteristic

4.1 of the modelization The modelization

is generalized in the case of 3D. Quantities

4.2 tested and Value results tested

Urgent Node	(	S) Standard Reference		Displacement
PRE1 No168 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No170 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No172 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No174 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No176 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No178 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No180 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No182 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No184 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No186 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No188 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No190 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No192 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No194 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No196 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No198 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No200 0.	ANALYTIQUE		0,0 Stress	
VMIS No83 0. NON	- REGRESSION		1.64519502	Stress
VMIS_SG No 83 0. NON	- REGRESSION		1.8098495947	Stress
PRIN_1 No 83 0. NON	- REGRESSION		-1,4049 Stress	
PRIN_2 No 83 0. NON	- REGRESSION		0,0 Stress	
PRIN_3 No 83 0. NON	- REGRESSION		0,4049 Stress	
TRESCA No 83 0. NON	- REGRESSION		-0,1645	the values of reference

in NON_REGRESSION are obtained for version 10.01.21. Modelization

5 C Characteristic

5.1 of the modelization The modelization

is identical to modelization A. Grandeurs

5.2 tested and Value results tested

Urgent Node	(	S) Standard Reference		Displacement
PRE1 No29 0. ANALYTIQUE			1.0 Displacement	
PRE1 No31 0. ANALYTIQUE			1.0 Displacement	
PRE1 No1 250.	ANALYTIQUE		0.68544576689	Displacement
PRE1 NO3 250.	ANALYTIQUE		0.682208147164	Displacement
PRE1 NO5 250.	ANALYTIQUE		0.67252104433	Displacement
PRE1 No7 250.	ANALYTIQUE		0.656461946263	Displacement
PRE1 No9 250.	ANALYTIQUE		0.634160686593	Displacement
PRE1 No11 250.	ANALYTIQUE		0.605800331394	Displacement
PRE1 No13 250.	ANALYTIQUE		0.571618145927	Displacement
PRE1 No15 250.	ANALYTIQUE		0.531906397249	Displacement
PRE1 No17 250.	ANALYTIQUE		0.487012719208	Displacement
PRE1 No19 250.	ANALYTIQUE		0.437339762565	Displacement
PRE1 No21 250.	ANALYTIQUE		0.38334387542	Displacement
PRE1 No23 250.	ANALYTIQUE		0.32553260623	Displacement
PRE1 No25 250.	ANALYTIQUE		0.264460889851	Displacement
PRE1 No27 250.	ANALYTIQUE		0.200725860656	Displacement
PRE1 No29 250.	ANALYTIQUE		0.134960328921	Displacement
PRE1 No31 250.	ANALYTIQUE		0.0678250497631	Displacement
PRE1 No33 250.	ANALYTIQUE		0.00 Stress	
SIYY No48 0.00	ANALYTIQUE		0.00 Stress	
SIYY No34 250.	ANALYTIQUE		-0.31455423311	Stress
SIYY No35 250.	ANALYTIQUE		-0.317791852836	Stress
SIYY No36 250.	ANALYTIQUE		-0.32747895567	Stress
SIYY No37 250.	ANALYTIQUE		-0.343538053737	Stress
SIYY No38 250.	ANALYTIQUE		-0.365839313407	Stress
SIYY No39 250.	ANALYTIQUE		-0.394199668606	Stress
SIYY No40 250.	ANALYTIQUE		-0.428381854073	Stress
SIYY No41 250.	ANALYTIQUE		-0.468093602751	Stress
SIYY No42 250.	ANALYTIQUE		-0.512987280792	Stress
SIYY No43 250.	ANALYTIQUE		-0.562660237435	Stress
SIYY No44 250.	ANALYTIQUE		-0.61665612458	Stress
SIYY No45 250.	ANALYTIQUE		-0.67446739377	Stress
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SIYY No47 250.	ANALYTIQUE		-0.799274139344	Stress
SIYY No48 250.	ANALYTIQUE		-0.865039671079	Stress
SIYY No49 250.	ANALYTIQUE		-0.932174950237	Stress
SIYY No50 250.	ANALYTIQUE		-1,0 Stress	
VMIS No50 250.	NOT	REGRESSION	0.4915089111	Stress
VMIS_SG No 50.250.	NOT	REGRESSION	-0.4915089111	Stress
PRIN_1 No 50.250.	NOT	REGRESSION	-0.4915089111	Stress
PRIN_2 No 50.250.	NOT	REGRESSION	-1.826682526	E-17 Forced
PRIN_3 No 50.250.	NOT	REGRESSION	1.60461944	E-17 Forced
TRESCA No 50.250.	NOT	REGRESSION	the 0.4915089111	values of reference

in NON_REGRESSION are obtained for version 10.01.21. Modelization

6 D Characteristic

6.1 of the modelization The modelization

is identical to the modelization B. Quantities

6.2 tested and Value results tested

Urgent Node	(	S) Standard Reference		Displacement
PRE1 No168 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No170 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No172 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No174 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No176 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No178 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No180 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No182 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No184 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No186 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No188 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No190 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No192 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No194 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No196 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No198 0.	ANALYTIQUE		1,0 Displacement	
PRE1 No200 0.	ANALYTIQUE		0,0 Stress	
VMIS No83 0. NON	- REGRESSION		0.375001823900	Stress
VMIS_SG No 83 0. NON	- REGRESSION		-0.375001823900	Stress
PRIN_1 No 83 0. NON	- REGRESSION		-0.375001823900	Stress
PRIN_2 No 83 0. NON	- REGRESSION		0.O Forced	
PRIN_3 No 83 0. NON	- REGRESSION		0.O Forced	
TRESCA No 83 0. NON	- REGRESSION		the -0.375001823900	values of reference

in NON_REGRESSION are obtained for version 10.01.21. Modelization

7 E Characteristic

7.1 of the modelization The modelization

is identical to the modelization C but with a successive adaptation of the mesh via MACR_ADAP _MAIL. Quantities

7.2 tested and Value results tested

Standard	Sequence number Reference		9.02.13 ESTERG1 component
of field ERRE_NOEU _ELEM 25 NON-REGRESSION		5.1838946544447	E-03 ERRE_TPS_
GLOB 25 NON-REGRESSION		0.0902526795091860	Modelization

8 F Characteristic

8.1 of the modelization The modelization

is identical to the modelization E but without the error indicator in time. Quantities

8.2 tested and Value results tested

Standard	Sequence number Reference		9.02.13 ESTERG1 component
of field ERRE_NOEU _ELEM 25 NON-REGRESSION		5.1838946544447	E-03 Modelization

9 G Characteristic

9.1 of the modelization the characteristics

are identical to modelization A. The mesh is composed of 83 nodes and 16 QUAD8 . Quantities

9.2 tested and Value results tested

Urgent Node	(	S) Standard Reference		Displacement
PRE1 No29 1.E-	4 AUTRE_ASTER		1.0 Displacement	
PRE1 No31 1.E-	4 AUTRE_ASTER		1.0 Displacement	
PRE1 No1 250.	AUTRE_ASTER		0.68544576689	Displacement
PRE1 NO3 250.	AUTRE_ASTER		0.682208147164	Displacement
PRE1 NO5 250.	AUTRE_ASTER		0.67252104433	Displacement
PRE1 No7 250.	AUTRE_ASTER		0.656461946263	Displacement
PRE1 No9 250.	AUTRE_ASTER		0.634160686593	Displacement
PRE1 No11 250.	AUTRE_ASTER		0.605800331394	Displacement
PRE1 No13 250.	AUTRE_ASTER		0.571618145927	Displacement
PRE1 No15 250.	AUTRE_ASTER		0.531906397249	Displacement
PRE1 No17 250.	AUTRE_ASTER		0.487012719208	Displacement
PRE1 No19 250.	AUTRE_ASTER		0.437339762565	Displacement
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PRE1 No25 250.	AUTRE_ASTER		0.264460889851	Displacement
PRE1 No27 250.	AUTRE_ASTER		0.200725860656	Displacement
PRE1 No29 250.	AUTRE_ASTER		0.134960328921	Displacement
PRE1 No31 250.	AUTRE_ASTER		0.0678250497631	Displacement
PRE1 No33 250.	AUTRE_ASTER		the 0.00	values

of pressure tested are compared with the values of pressure obtained with modelization A. the differences observed between the two modelizations are lower than 6.E-3%. Modelization

10 H Characteristic

10.1 of the modelization The modelization

is identical to the modelization E but with under-integrated modelization D_PLAN_HM _SI. Quantities

10.2 tested and Value results tested

Standard	Sequence number Reference		11.3.7 ESTERG1 component
of field ERRE_NOEU _ELEM 25 NON-REGRESSION		5.2034169568591	E-03 ERRE_TPS_
GLOB 25 NON-REGRESSION		0.095460688688661	Summary

11 of the results In conclusion

, the Code_Aster results *are in agreement* with the analytical reference solutions.