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## HSNS101 - Square plate in tension and temperature variables. Plane stresses method OF Summarized

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### **BORST:**

This test, inspired by that proposed by the IPSI for the Phi2As day of March 30th, 2000 (test HSNV124) makes it possible to validate the method of Borst to treat the constraint plane for an unspecified nonlinear behavior. In particular, one tests here the method for behaviors `VMIS_CINE_LINE`, `VMIS_CIN1_CHAB` and `VMIS_CIN2_CHAB`, by comparison with `VMIS_ECMI_LINE`, which uses a direct integration method of the plane stresses. All the parameters of these various models are adjusted in order to reproduce in fact the same behavior.

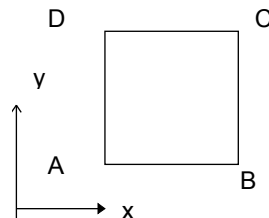
Four modelizations make it possible to validate the plane stresses (method of Borst).

The results are compared with those of test HSNV124 (analytical reference).

## 1 Problem of reference

### 1.1 Geometry

Volume element materialized by a square plate  $ABCD$  on unit side ( 1 mm ):



### 1.2 Properties of the materials

$$E = 2.10^5 \text{ MPa} \quad \nu = 0.3, \quad \alpha = 2.10^5 \text{ } ^\circ\text{C}^{-1}$$

the material is elastoplastic with a linear kinematic hardening:

$$\sigma = \pm \sigma_y(T) + C(T) \varepsilon^p$$

$$SIGY = 200. - 1.7.T \text{ (in MPa)}$$

$$\text{Hardening modulus } D\_SIGM\_EPSI = C(T) = 1000 + 2990.T \text{ (in MPa)}$$

### 1.3 Boundary conditions and loadings

Such as the stress state and of strain are uniform in the volume element:

$A$  Not blocked in  $x$  and  $y$ .  
 $DY = 0$  on  $AB$

Distributed force on  $CD$ :  $Fy$   
Uniform temperature  $T$  on  $ABCD$ . The reference temperature is worth  $0^\circ\text{C}$ .

$Fy$  and  $T$  vary according to time in the following way:

time $t$	0	1s	2s
$Fy(t)$	0	210 MPa	210 MPa
$T(t)$	0	$0^\circ\text{C}$	$100^\circ\text{C}$

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution

Solution analytical for the plastic strain, identical to test HSNV124B.

For displacements (structural elements): Comparison with the displacements obtained with Code\_Aster for modelization A.

### 2.2 Résultats of reference

Evolution of the plastic strain according to  $y$  and from following displacement  $y$  (plastic strain not being accessible for the structural elements).

Plastic strain:

$$t = 1s \ (T = 0^\circ C) : \varepsilon^p = 1\%$$

**Heating:** Constant plastic strain until  $t = 356/316 = 1.12658s$  ( $T = 12.658^\circ C$ ):

Then, the plastic strain decreases to reach with  $t = 2s$  :  $\varepsilon^p = 0.08\%$

One tests also components  $SIYY$  (constant) and  $SIZZ$  (null, if computation is well carried out in plane stresses).

$T(s)$	Plastic strain EPYY	DY ( mm )	SIYY ( Mpa )	SIZZ ( Mpa )
1	0.01	$1.105 \cdot 10^{-2}$	210	0.1.1
	0.01	$1.115 \cdot 10^{-2}$	210	0
2.210	$8.10^{-4}$	$2.85 \cdot 10^{-3}$		0

### 2.3 Accuracy on the results of Analytical

reference

### 2.4 References bibliographical

- 1) IPSI: day of Phi2AS study on the nonlinear behaviors of the materials of March 30th, 2000

## 3 Modelization A

### 3.1 Characteristic of the modelization

Modelization C\_PLAN. Behavior with linear kinematic hardening is modelled in four ways:

- maybe using behavior VMIS\_CINE\_LINE, while taking:  
 $D\_SIGM\_EPSI = E.C(T)/(E+C(T))$  with  $C(T)=(1000+2990.T)$
- is using behavior VMIS\_ECMI\_LINE, while taking:  
 $D\_SIGM\_EPSI = E.C(T)/(E+C(T))$  and the constant of Prager  $PRAG=2/3 C(T)$
- are using behavior VMIS\_CIN1\_CHAB, by keeping only linear kinematic hardening: It is enough to take then:  $R_0=R_I=SIGY$   $b=0$   $C_I=C(T)$ ,  $G_0=0$
- that is to say using behavior VMIS\_CIN2\_CHAB, by choosing the parameters in such way that the two kinematical variables are identical: It is enough to take then:  
 $R_0=R_I=SIGY$   $b=0$   $CI_I=C2_I=C(T)/2$ ,  $G1_0=G2_0=0$

temporal Discretization: 1 time step enters  $t=0s$  and  $t=1s$  40 time step enters  $t=1s$  and  $t=2s$ .

### 3.2 Characteristics of the mesh

The mesh comprises Behavior a mesh

### 3.3 QUAD4 Quantities tested and

results	Urgent	Strain and forced	Reference	Aster	% difference
VMIS_CINE_LINE	1.1	EPYY	0.01	0.01	0.
	2	EPYY	8.E-4	8.E-4	0
	1.	SIYY	210.210		0
	1	SIZZ	0.	5.e-7	5.e-7
VMIS_ECMI_LINE	T1 = 1.1	EPYY	0.01	0.01	0.
	2	EPYY	8.E-4	8.E-4	0
	1.	SIYY	210.210		0
	1	SIZZ	0.	0.	0.
VMIS_CIN1_CHAB	T1 = 1.1	EPYY	0.01	0.01	0.
	2	EPYY	8.E-4	8.E-4	0
	1.	SIYY	210.210		0
	1	SIZZ	0.	5.e-7	5.e-7
VMIS_CIN2_CHAB	T1 = 1.1	EPYY	0.01	0.01	0.
	2	EPYY	8.E-4	8.E-4	0
	1.	SIYY	210.210		0
	1	SIZZ	0.	2.8.e-5	2.8.e-5

## 4 Modelization B

### 4.1 Characteristic of the modelization

Modelization DKT. Thickness unit (to find the same reference solutions as case C\_PLAN). Behavior with linear kinematic hardening is modelled in four ways:

- maybe using behavior VMIS\_CINE\_LINE, while taking:  
 $D\_SIGM\_EPSI = E.C(T)/(E+C(T))$  with  $C(T)=(1000+2990.T)$
- is using behavior VMIS\_ECMI\_LINE, while taking:  
 $D\_SIGM\_EPSI = E.C(T)/(E+C(T))$  and the constant of Prager  $PRAG=2/3 C(T)$
- are using behavior VMIS\_CIN1\_CHAB, by keeping only linear kinematic hardening: It is enough to take then:  $R_0=R_I=SIGY$   $b=0$   $C_I=C(T)$ ,  $G_0=0$
- that is to say using behavior VMIS\_CIN2\_CHAB, by choosing the parameters in such way that the two kinematical variables are identical: It is enough to take then:  
 $R_0=R_I=SIGY$   $b=0$   $C_I=C2_I=C(T)/2$ ,  $G1_0=G2_0=0$

temporal Discretization: 1 time step enters  $t=0s$  and  $t=1s$  40 time step enters  $t=1s$  and  $t=2s$ .

### 4.2 Characteristics of the mesh

The mesh comprises a mesh QUAD4 and two meshes TRIA3

### 4.3 Quantities tested and results

Behavior	Urgent	Displacement and force	Reference	Aster	% difference
VMIS_CINE_LINE	1	NY Y	210.210		0
	1	DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
		DY	$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0
		DY	$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0
	2	DY	$2.85 \cdot 10^{-3}$	$2.85 \cdot 10^{-3}$	0
	VMIS_ECMI_LINE	1	NY Y	210.210	
1		DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
		DY	$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0
		DY	$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0
2		DY	$2.85 \cdot 10^{-3}$	$2.85 \cdot 10^{-3}$	0
VMIS_CIN1_CHAB		1	NY Y	210.210	
	1	DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
		DY	$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0
		DY	$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0
	2	DY	$2.85 \cdot 10^{-3}$	$2.85 \cdot 10^{-3}$	0
	VMIS_CIN2_CHAB	1	NY Y	210.210	
1		DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
		DY	$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0
		DY	$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0
2		DY	$2.85 \cdot 10^{-3}$	$2.85 \cdot 10^{-3}$	0

## 5 Modelization C

### 5.1 Characteristic of the modelization

Modelization COQUE\_3D. Thickness unit (to find the same reference solutions as case C\_PLAN). Behavior with linear kinematic hardening is modelled in four ways:

- maybe using behavior VMIS\_CINE\_LINE, while taking:  
 $D\_SIGM\_EPSI = E.C(T)/(E+C(T))$  with  $C(T)=(1000+2990.T)$
- is using behavior VMIS\_ECMI\_LINE, while taking:  
 $D\_SIGM\_EPSI = E.C(T)/(E+C(T))$  and the constant of Prager  $PRAG=2/3 C(T)$
- is using behavior VMIS\_CIN1\_CHAB, by keeping only linear kinematic hardening: It is enough to take then:  $R_0=R_T=SIGY$   $b=0$   $C_T=C(T)$ ,  $G_0=0$
- that is to say using behavior VMIS\_CIN2\_CHAB, by choosing the parameters in such way that the two kinematical variables are identical: It is enough to take then:  
 $R_0=R_T=SIGY$   $b=0$   $C1_T=C2_T=C(T)/2$ ,  $G1_0=G2_0=0$

temporal Discretization: 1 time step enters  $t=0s$  and  $t=1s$  40 time step enters  $t=1s$  and  $t=2s$ .

### 5.2 Characteristics of the mesh

The mesh comprises a mesh QUAD8

### 5.3 Quantities tested and results

Behavior	Urgent	Displacement and force	Reference	Aster	% difference
VMIS_CINE_LINE	1	NY Y	210.210		0
	1	DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
		DY	$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0
VMIS_ECMI_LINE	2	DY	$2.85 \cdot 10^{-3}$	$2.85 \cdot 10^{-3}$	0
	1	NY Y	210.210		0
		DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
DY		$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0	
VMIS_CIN1_CHAB	2	DY	$2.85 \cdot 10^{-3}$	$2.85 \cdot 10^{-3}$	0
	1	NY Y	210.210		0
		DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
DY		$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0	
VMIS_CIN2_CHAB	2	DY	$2.85 \cdot 10^{-3}$	$2.85 \cdot 10^{-3}$	0
	1	NY Y	210.210		0
		DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
DY		$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0	

## 6 Modelization D

### 6.1 Characteristic of the modelization

Modelization PIPE. Section unit (to find the same reference solutions as case C\_PLAN). Behavior with linear kinematic hardening is modelled in four ways:

- maybe using behavior VMIS\_CINE\_LINE, while taking:  
 $D\_SIGM\_EPSI = E.C(T)/(E + \bar{C}(T))$  with  $C(T) = (1000 + 2990.T)$
- is using behavior VMIS\_ECMI\_LINE, while taking:  
 $D\_SIGM\_EPSI = E.C(T)/(E + \bar{C}(T))$  and the constant of Prager  $PRAG = 2/3 C(T)$
- are using behavior VMIS\_CIN1\_CHAB, by keeping only linear kinematic hardening: It is enough to take then:  $R_0 = R_f = SIGY$   $b = 0$   $C_f = C(T)$ ,  $G_0 = 0$
- that is to say using behavior VMIS\_CIN2\_CHAB, by choosing the parameters in such way that the two kinematical variables are identical: It is enough to take then:  
 $R_0 = R_f = SIGY$   $b = 0$   $C1_f = C2_f = C(T)/2$ ,  $G1_0 = G2_0 = 0$

temporal Discretization: 1 time step enters  $t = 0s$  and  $t = 1s$  40 time step enters  $t = 1s$  and  $t = 2s$ .

### 6.2 Characteristics of the mesh

The mesh comprises a mesh SEG3

### 6.3 Quantities tested and results

Behavior	Urgent	Displacement and forced	Reference	Aster	% difference
VMIS_CINE_LINE	1	SIXX	210.210		0
	1	DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
		DY	$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0
VMIS_ECMI_LINE	2	DY	$2.85 \cdot 10^{-3}$	$2.85 \cdot 10^{-3}$	0
	1	SIXX	210.210		0
		DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
DY		$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0	
VMIS_CIN1_CHAB	2	DY	$2.85 \cdot 10^{-3}$	$2.85 \cdot 10^{-3}$	0
	1	SIXX	210.210		0
		DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
DY		$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0	
VMIS_CIN2_CHAB	2	DY	$2.85 \cdot 10^{-3}$	$2.85 \cdot 10^{-3}$	0
	1	SIXX	210.210		0
		DY	$1.105 \cdot 10^{-2}$	$1.105 \cdot 10^{-2}$	0.1.1
DY		$1.115 \cdot 10^{-2}$	$1.115 \cdot 10^{-2}$	0	
	2	DY	$2.85 \cdot 10^{-3}$	$2.85 \cdot 10^{-3}$	0

## 7 Summary of the results

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This test makes it possible to highlight the good taking into account of the plane stresses for elastoplastic behaviors VMIS\_ISOT\_TRAC, VMIS\_CINE\_LINE, VMIS\_CIN1\_CHAB, and VMIS\_CIN2\_CHAB.

The results are identical to the analytical solution. The stresses  $SIZZ$  which must be null if the assumption of the plane stresses is checked it are indeed with convergence of the iterations of Newton with a good accuracy (  $SIZZ = 2.8.E-5 MPa$  to the maximum, in comparison with  $SIYY = 210 MPa$  ), and this for a TEMPS CPU and a nearby nombre of iterations.