HSNV120 - Tension hyper elastic of a bar under thermal loading

Abstract:

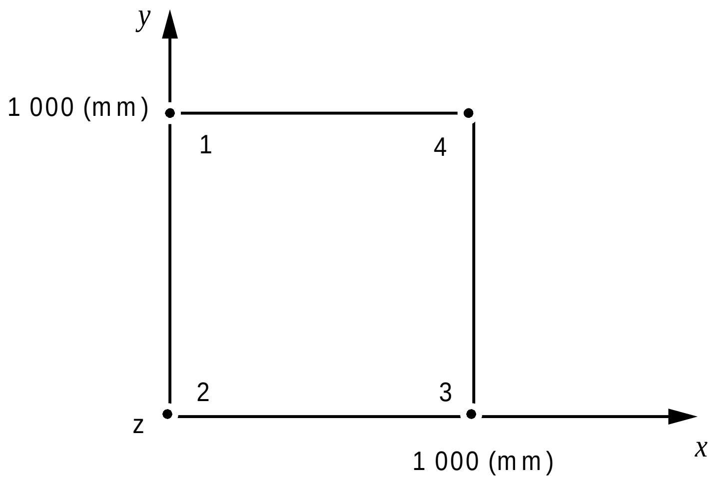
This quasi-static thermomechanical test consists in heating a parallelepipedic bar uniformly, to subject it to an important tension for finally letting it return in a discharged state. One validates thus the kinematics of the large deformations hyper elastics (command STAT_NON_LINE, key word COMP_ELAS) for a nonlinear elastic behavior model (ELAS_VMIS_LINE and ELAS_VMIS_TRAC) with thermal loading.

The bar is modelled by an element voluminal (HEXA20, modelization A) or quadrangular (QUAD8, assumption of the plane stresses, modelization B).

The results got by *Aster* do not differ from the theoretical solution.

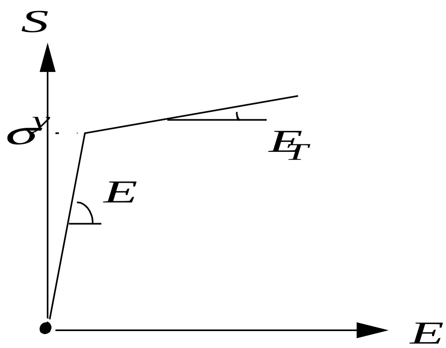
1 Problem of reference

1.1 Geometry



1.2 Material properties

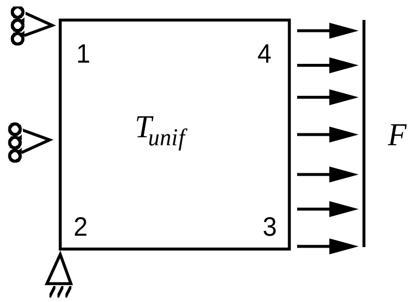
the material obeys a constitutive law isotropic nonlinear hyper elastic with isotropic linear hardening.

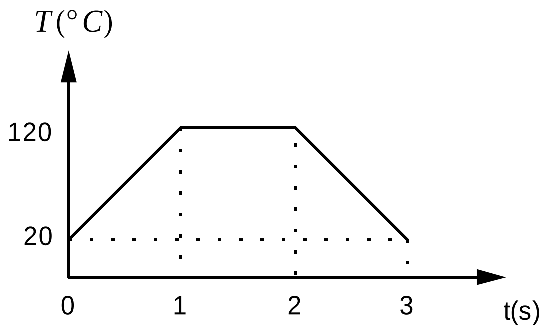


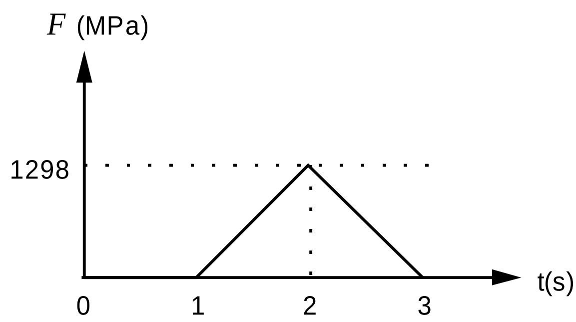
$$E = 2\,10^{5}\,\mathrm{MPa}$$
$$E_T = 2\,10^{3}\,\mathrm{MPa}$$
$$\sigma^{y} = 10^{3}\,\mathrm{MPa}$$
$$\nu = 0.3$$
$$\alpha = 10^{-4}\,\mathrm{K}^{-1}$$

1.3 Boundary conditions and loadings

the bar blocked in the direction $O\mathbf{x}$ on the face [1,2] is subjected to a uniform temperature T and a tractive effort F distributed on the face [3,4]. The sequences of loading are the following ones:







Reference temperature: $T_{réf}=20°C$.

2 Reference solution

2.1 Method of calculating used for the reference solution

One seeks the field of displacement U in the form:

$$\mathbf{U}(x,y,z) = \begin{bmatrix} ux \\ vy \\ vz \end{bmatrix}$$

The gradient of the transformation, the strain and its mechanical share are then:

$$\mathbf{F} = \begin{bmatrix} 1+u & 0 & 0 \\ 0 & 1+v & 0 \\ 0 & 0 & 1+v \end{bmatrix}$$

$$\mathbf{E} = \frac{1}{2}\left(\mathbf{F}^T\,\mathbf{F}-\mathbf{1}\right) = \begin{bmatrix} \frac{u(u+2)}{2} & 0 & 0 \\ 0 & \frac{v(v+2)}{2} & 0 \\ 0 & 0 & \frac{v(v+2)}{2} \end{bmatrix}$$

$$\mathbf{E^m} = \mathbf{E}-\alpha\,\Delta T\,\mathbf{1} = \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & b \end{bmatrix}$$

with:

$$\begin{cases} a = \frac{u(u+2)}{2}-\alpha\,\Delta T \\ b = \frac{v(v+2)}{2}-\alpha\,\Delta T \end{cases}$$

Note:

$\left(\mathbf{E^m}\right)_{eq} = |a-b| = a-b$ (on suppose que $a>b$)

The behavior model is written:

$$\begin{cases} S_{xx} = K(a+2b)+\frac{2}{3}G(a-b) \\ S_{yy} = S_{zz} = K(a+2b)-\frac{1}{3}G(a-b) \end{cases}$$

with:

$$3K = \frac{E}{1-2\nu} \quad \text{module de compressibilité}$$

To determine G by taking account of linear hardening, one introduces:

- the shear modulus: $2\mu = \frac{E}{1+\nu}$
- the hardening modulus: $R' = \frac{E\,E_T}{E-E_T}$,

The "pseudonym local variable" p is worth then:

$$p = \frac{2\mu\left(\mathbf{E^m}\right)_{eq}-\sigma^y}{R'+3\mu} = \frac{2\mu(a-b)-\sigma^y}{R'+3\mu}$$

Finally, G is written:

$$G = \frac{\sigma^y+R'\,p}{a-b}$$

By taking account of the boundary conditions:

$$\begin{cases} S_{xx} = \frac{F}{1+u} & \text{(charge morte)} \\ S_{yy} = 0 & \text{(bord libre)} \end{cases}$$

The system to be solved is written:

$$\begin{cases} K(a+2b)+\frac{2}{3}\left[\sigma^y+R'\,\frac{2\mu(a-b)-\sigma^y}{R'+3\mu}\right] = \frac{F}{1+u} \\ K(a+2b)-\frac{1}{3}\left[\sigma^y+R'\,\frac{2\mu(a-b)-\sigma^y}{R'+3\mu}\right] = 0 \end{cases}$$

He is also written:

$$\begin{cases} 3K(a+2b) = \dfrac{F}{1+u} \\ 2\mu(a-b) = \dfrac{F}{1+u}\left(1+\dfrac{3\mu}{R'}\right)-\sigma^y \dfrac{3\mu}{R'} \end{cases}$$

A F fixed, it is thus about a nonlinear system in u and v, since a is quadratic in u and b quadratic in v.

Nevertheless, one can choose to fix u (thus a) and to solve a linear system in F and b (from which one deduces p and v):

- $a = \dfrac{u(u+2)}{2} - \alpha\,\Delta T$
- $\begin{cases} \dfrac{1}{1+u}F - 6\,K\,b = 3\,K\,a \\ \left(1+\dfrac{3\mu}{R'}\right)\dfrac{1}{1+u}F + 2\mu\,b = 2\mu\,a + \sigma^y \dfrac{3\mu}{R'} \end{cases}$
- $p = \dfrac{2\mu\,(a-b)-\sigma^y}{R'+3\mu}$
- $v = \sqrt{1+2(b+a\,\Delta T)} - 1$

It then remains to express the stress of Cauchy:

$$\boldsymbol{\sigma} = \frac{1}{\mathrm{Det}(\mathbf{F})}\,\mathbf{F}\cdot\mathbf{S}\cdot\mathbf{F}^T$$

That is to say here:

$$\begin{cases} \sigma_{xx} = \dfrac{1+u}{(1+v)^2} S_{xx} \\ \sigma_{yy} = \sigma_{zz} = 0 \end{cases}$$

As for the force exerted on the face [3,4], because of assumption of died loads, she is written simply:

$$\begin{cases} F_x = F\ S_o \quad \text{où } S_o : \text{surface initiale de la face } [3{,}4] \\ F_y = 0 \\ F_z = 0 \end{cases}$$

2.2 Results of reference

One will adopt like results of reference displacements, the stress of Cauchy and the force exerted on the face $[3,4]$ (in $3D$ only):

At time $t=2\ s$ ($\Delta T=100\,°C$, tension F)

makes some, one seeks F such as lengthening:

$$u = 0{,}1$$

- $K=166\,666\ MPa$ $\quad\mu=76\,923\ MPa$ $\quad R'=2\,020\ MPa$
- $a=0.095$
- $$\begin{cases} 0.90909\ F-10^6\ b = 47\,500 \\ 104.76\ F+153.85\ 10^3\ b = 128.85\ 10^3 \end{cases}$$
  $$\Rightarrow \begin{cases} F = 1\,298\ \text{MPa} \\ b = -0.046 \end{cases}$$
- $p = 8.91\ 10^{-2}$
- $v = -3.70\ 10^{-2}$
- $$\begin{array}{lll} \sigma_{xx} = 1\,399.66\ \text{MPa} & \sigma_{xy} = 0 & F_x = 1\,298\ 10^9\ \text{N} \\ \sigma_{yy} = 0 & \sigma_{xz} = 0 & F_y = 0 \\ \sigma_{zz} = 0 & \sigma_{yz} = 0 & F_z = 0 \end{array}$$

At time $t=3\ s$ $(\Delta T=0,\ F=0)$

the bar returned in its initial state:

$$\begin{cases} \mathbf{U} = 0 \\ \boldsymbol{\sigma} = 0 \\ p = 0 \end{cases}$$

2.3 Uncertainty on the solution

the solution is analytical. With the rounding errors near, one can consider it exact.

2.4 Bibliographical references

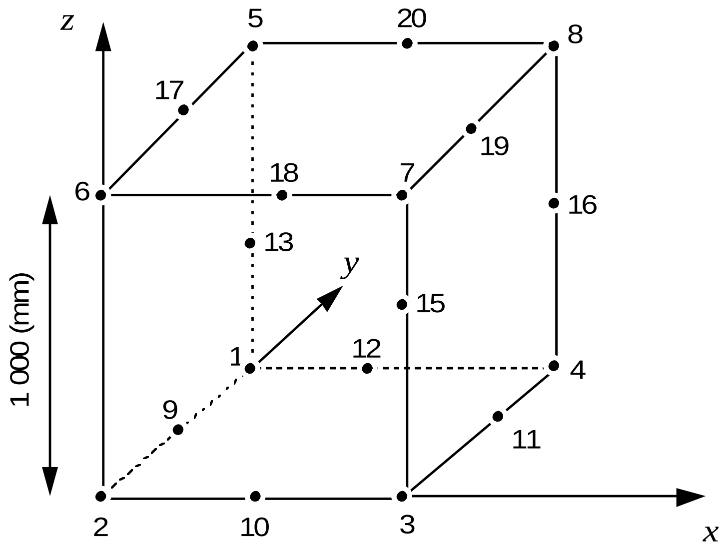
One will be able to refer to:

1) E. LORENTZ: A nonlinear behavior model hyper elastic - internal Note EDF DER HI-74/95/011/0

3 Modelization A

3.1 Characteristic of the voluminal

modelization Modelization: 1 mesh `HEXA20`
1 nets `QUAD8`



Boundary conditions:

$N2: \; U_x = U_y = U_z = 0 \qquad N9 \quad N13 \quad N14 \quad N5 \quad N17: U_x = 0$

$N1: \; U_x = U_z = 0$

$N6: \; U_x = U_y = 0$

Charge: Tension on the face $[3\,4\,8\,7\,11\,16\,19\,15]$

3.2 Characteristics of the mesh

Many nodes: 20

Number of meshes: 2

1 `HEXA20`
1 `QUAD8`

3.3 Quantities tested and results

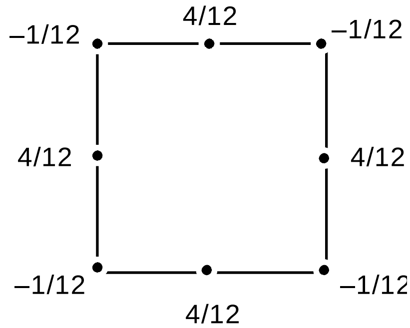
Identification	Reference	*Aster*	% difference
$t=2$ Displacement DX ($N8$)	100.100	.	0.
$t=2$ Displacement DY ($N8$)	– 37	– 37.005	0.013
$t=2$ Displacement DZ ($N8$)	– 37	– 37.005	0.013
$t=2$ Stresses $SIGXX$ ($PG1$)	1399.66	1399.67	0.001
$t=2$ Stresses $SIGYY$ ($PG1$)	11013.986	10-10	/
$t=2$ Forced $SIGZZ$ ($PG1$)	0	10-10	/
$t=2$ Forced $SIGXY$ ($PG1$)	0	10-12	/
$t=2$ Forced $SIGXZ$ ($PG1$)	0	10-12	/
$t=2$ Forced $SIGYZ$ ($PG1$)	0	10-11	/
$t=2$ Variable p $VARI$ ($PG1$)	8.9110-2	8.91 10-2	0.
$t=3$ Displacement DX ($N8$)	0	10-13	/
$t=3$ Displacement DY ($N8$)	0	10-13	/
$t=3$ Displacement DZ ($N8$)	0	10-14	/
$t=3$ Forced $SIGXX$ ($PG1$)	0	10-10	/
$t=3$ Forced $SIGYY$ ($PG1$)	0	10-11	/
$t=3$ Forced $SIGZZ$ ($PG1$)	0	10-11	/
$t=3$ Forced $SIGXY$ ($PG1$)	0	10-11	/
$t=3$ Forced $SIGXZ$ ($PG1$)	0	10-11	/
$t=3$ Forced $SIGYZ$ ($PG1$)	0	10-11	/
$t=3$ Variable p $VARI$ ($PG1$)	0	0	/
$t=2$ nodal Force DX ($N8$)	– 1.0817108	– 1.0817 108	– 0.003
$t=2$ nodal Force DY ($N8$)	0	10-5	/
$t=2$ nodal Force DZ ($N8$)	0	10-6	/

3.4 Remarks

Computation of the nodal force:

The applied force on the face $[3,4]$ F_x , is distributed between the various nodes according to following weighting:

- nodes tops: $-1/12 F_x$
- nodes mediums: $4/12 F_x$

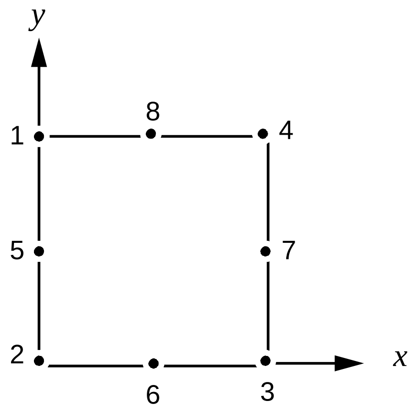


4 Modelization B

4.1 Characteristic of the modelization

Modelization $2D$ plane stresses:

1 mesh `QUAD8`
1 nets `SEG3`



Boundary conditions:

$N2$:	$U_x=0$	$U_y=0$
$N1$:	$U_x=0$	
$N5$:	$U_x=0$	

Loading:

Tension on the face $[3\,4\,7]$ (mesh `SEG3`)

4.2 Characteristic of the mesh

Many nodes: 8

Number of meshes: 2

1 `QUAD8`
1 `SEG3`

4.3 Quantities tested and results

Identification	Reference	*Aster*	% difference
$t=2$ Displacement DX ($N4$)	100.100		0
$t=2$ Displacement DY ($N4$)	– 37	– 37.004	0.013
$t=2$ Stresses $SIGXX$ ($PG1$)	1399.66	1399.67	0.001
$t=2$ Stresses $SIGYY$ ($PG1$)	0	□10 -12	/
$t=2$ Forced $SIGXY$ ($PG1$)	0	Variable □	10-12
$t=2$ / p $VARI$ ($PG1$)	8.9110-2	8.91 10-2	0
$t=3$ Displacement DX ($N4$)	0	□10 -14	/
$t=3$ Displacement DY ($N4$)	0	□10 -13	/
$t=3$ Forced $SIGXX$ ($PG1$)	0	□10 -10	/
$t=3$ Forced $SIGYY$ ($PG1$)	0	□10 -10	/
$t=3$ Forced $SIGXY$ ($PG1$)	0	Variable □	10-10
$t=3$ / p $VARI$ ($PG1$)	0	0	/

5 Summary of the results

the numerical and analytical results coincide remarkably. One can however be astonished by the execution time manifestly longer for the modelization in plane stresses ($123,8s$) that for 3D ($47,2s$). The difference is explained by a discretization in time much finer for the plane stresses, related to problems of convergence (the algorithm of resolution of the nonlinear scalar equation in p is still rudimentary).