

## HSNV102 - Thermo-metal-worker-plasticity coupled in simple tension

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### Summarized:

One treats the determination of the mechanical evolution of a cylindrical bar subjected to known and  $T(t)$  uniform evolutions  $Z(t)$  thermal and metallurgical (the metallurgical transformation is of martensitic type).

The elements used are axisymmetric elements and the behavior model is plasticity of von Mises with isotropic hardening (for the modelization B, one also takes account of the plasticity of transformation).

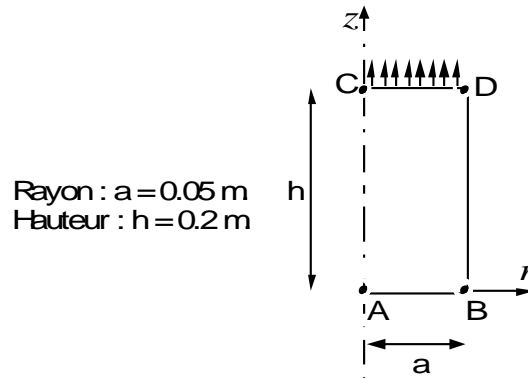
The yield stress and the slope of curve of tension depend on the temperature and the metallurgical composition.

The coefficient of thermal expansion  $\alpha$  depends on the metallurgical composition.

The metallurgical transformations take place with  $\dot{\epsilon}^p \neq 0$  (it is in the sense that the test **couple**s the plasticity of transformation of classical plasticity).

## 1 Problem of reference

### 1.1 Geometry



### 1.2 Properties of the materials

$$\begin{array}{lll}
 E = 200\,000 \cdot 10^6 \text{ Pa} & \sigma_y^{aust} = \sigma_o^{aust} + s^{aust} (T - T^o) & \text{notons } H(t) = \frac{\alpha(t) \cdot E(t)}{E(t) - \alpha(t)} \\
 \nu = 0.3 & \sigma_o^{aust} = 50 \cdot 10^6 \text{ Pa} & H^{aust} = H_o^{aust} + \lambda^{aust} (T - T^o) \\
 \alpha_{fbm} = 15 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1} & s^{aust} = 1.3 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1} & H_o^{aust} = 0 \text{ Pa} \\
 \alpha_{aust} = 23.5 \cdot 10^{-6} \text{ } ^\circ\text{C}^{-1} & \sigma_y^{fbm} = \sigma_o^{fbm} + s^{fbm} (T - T^o) & \lambda^{aust} = -1 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1} \\
 \varepsilon_{ref_{fbm}} = 2.52 \cdot 10^{-3} & \sigma_o^{fbm} = 50 \cdot 10^6 \text{ Pa} & H^{fbm} = H_o^{fbm} + \lambda^{fbm} (T - T^o) \\
 & s^{fbm} = 1 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1} & H_o^{fbm} = 0 \\
 cp = 2\,000\,000 \text{ J} \cdot \text{m}^{-3} \cdot ^\circ\text{C}^{-1} & \lambda = 9999.9 \text{ W} \cdot \text{m}^{-1} \cdot ^\circ\text{C}^{-1} & \lambda^{fbm} = -6 \cdot 10^6 \text{ Pa} \cdot ^\circ\text{C}^{-1} \\
 & & k^f = 0 \quad k^b = k^m = 1 \cdot 10^{-10} \text{ Pa}^{-1}
 \end{array}$$

#### Notes:

The indices or exponents *fbm* are relative to materials parameters phases ferrite - perlite, bainitic and martensitic and the indices or exponents *aust* are relating to austenite.

TRC to model a metallurgical evolution of martensitic type of the form:

$$\begin{cases} 0. & \text{si } t \leq \tau_1 & \tau_1 = 25 \text{ s} \\ 1 - e^{\varphi \lambda (t - \tau_1)} & \text{si } \tau_1 \leq t < \tau_2 & \tau_2 = 40 \text{ s} \quad \varphi = 0.03 \quad \lambda = -10^\circ\text{C} \cdot \text{s}^{-1} \\ 1. & \text{si } t \geq \tau_2 \end{cases}$$

### 1.3 Boundary conditions and loadings

$u_z = 0$  on the side *AB* (condition of symmetry).

tension imposed on the side *CD*  $p(t) = p_o t$   $p_o = 15 \cdot 10^6 \text{ Pa}$ .

$T = T^o + \mu t$ ,  $\mu = -10^\circ\text{C} \cdot \text{s}^{-1}$  on all structure.

### 1.4 Initial conditions

$$T^0 = 900^\circ\text{C}.$$

## 2 Reference solution

### 2.1 Method of calculating used for the reference solution

**Before transformation**, thermoelastic solution for  $t < \tau_1$  (not of metallurgical transformation  $\dot{Z} = 0$ ).

$$\sigma(t) = p_o t$$

$$\varepsilon_{zz}(t) = \varepsilon_{zz}^e(t) + \varepsilon_{zz}^{th}(t) = \frac{\sigma(t)}{E} + \alpha_{aust}(T - T^o)$$

$$\text{The yield stress is reached for } \tau_1 = \frac{\sigma_o^{aust}}{p_o - s^{aust} \times k} = 25 \text{ s.}$$

**During the transformation**, solution thermo-metal-worker-élasto-plastic, for  $\tau_1 < t < \tau_2$  with  $\tau_2 = 40 \text{ s}$ .

$$\sigma(t) = p_o t$$

$$\varepsilon_{zz}(t) = \varepsilon_{zz}^e(t) + \varepsilon_{zz}^{th}(t) + \varepsilon_{zz}^p(t) + \varepsilon_{zz}^{pt}(t)$$

$$\varepsilon_{zz}^e(t) = \frac{\sigma(t)}{E}$$

$$\varepsilon_{zz}^{th}(t) = Z_{aust} \times \alpha_{aust}(T - T^o) + Z_{fbm} \times \left[ \alpha_{fbm}(T - T^o) + \varepsilon_{ref_{fbm}} \right]$$

$$\varepsilon_{zz}^p(t) = \frac{\sigma(t) - \left( Z_{aust} \times \sigma_y^{aust}(T) + Z_{fbm} \times \sigma_y^{fbm}(T) \right)}{Z_{aust} \times H^{aust}(T) + Z_{fbm} \times H^{fbm}(T)}$$

$$\varepsilon_{zz}^{pt}(t) = k_m \times \left( p_o \times \tau_1 - \frac{k}{2 \times \lambda \times \varphi} \right) - k_m \times \left( p_o \times t - \frac{k}{2 \times \lambda \times \varphi} \right) (1 - Z)^2$$

**After transformation**, thermoelastoplastic solution, for  $t > \tau_2$

$$\sigma(t) = p_o t$$

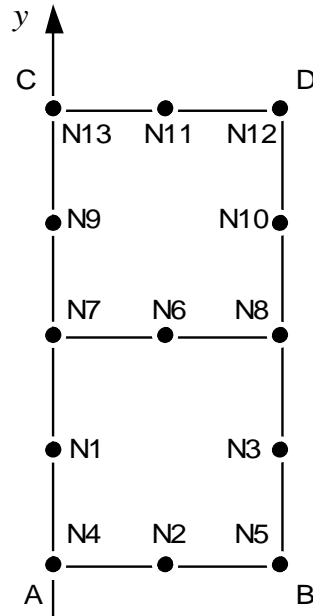
$$\varepsilon_{zz}(t) = \varepsilon_{zz}^e(t) + \varepsilon_{zz}^{th}(t) + \varepsilon_{zz}^p(t)$$

### 2.2 Results of reference

$\sigma$ ,  $\varepsilon_{zz}$ ,  $\varepsilon^p$ ,  $\chi$  with 24 s, 26 s, 40 s and 90 s.

## 3 Modelization A

### 3.1 Characteristic of the modelization



$$A=N4 \quad B=N5 \quad C=N13 \quad D=N12 .$$

### 3.2 Characteristics of the mesh

Many nodes: 13

Number of meshes and types: 2 meshes QUAD8, 6 meshes SEG3

## 4 Results of the modelization A

### 4.1 Values tested

One tests the parameters of the data structure results:

Identification	Reference	Test	Tolerance
INST for NUME ORDRE= 27	90	ANALYTIQUE	0,10%
ITER GLOB for NUME ORDRE=27	2	NON REGRESSION	0.00%

Identification	Reference	Aster	% difference
$\varepsilon^p$ $t=24 s$	0	0	-
$\chi$ $t=24 s$	0	0	-
$\sigma$ $t=24 s$	360. 106.360	. 106	0
$\varepsilon_{zz}$ $t=24 s$	- 3.84 10-3	- 3.84 10-3	0
$\varepsilon^p$ $t=26 s$	0.0372	- 3.7217 10-2	0.047
$\chi$ $t=26 s$	1	1	0.390
$\sigma$ $t=26 s$	. 106.390	. 106	0
$\varepsilon_{zz}$ $t=26 s$	3.428 10-2	3.4283 10-2	
$\varepsilon^p$ $t=40 s$	0.0625	6.2523 10-2	0.038
$\chi$ $t=40 s$	1	1	0.600
$\sigma$ $t=40 s$	. 106.600	. 106	0
$\varepsilon_{zz}$ $t=40 s$	0.06198	6.1977 10-2	- 0.068
$\varepsilon^p$ $t=90 s$	0.0741	7.4146 10-2	0.062
$\chi$ $t=90 s$	1	1	0
$\sigma$ $t=90 s$	1350. 106	1350. 106	0
$\varepsilon_{zz}$ $t=90 s$	0.069844	6.9854 10-2	0.014

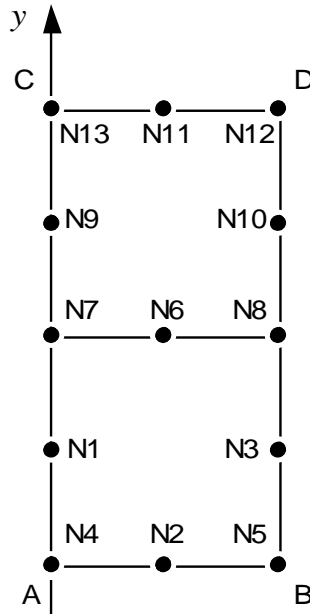
### 4.2 Remarks

In this modelization:

$$\varepsilon^{pt}(T, Z) = 0$$

## 5 Modelization B

### 5.1 Characteristic of the modelization



$$A=N4 \quad B=N5 \quad C=N13 \quad D=N12 .$$

### 5.2 Characteristics of the mesh

Many nodes: 13

Number of meshes and types: 2 meshes QUAD8, 6 meshes SEG3

## 6 Results of the modelization B

### 6.1 Values tested

Identification		Reference	Aster	% difference
$\varepsilon^p$	$t=24 s$	0	0	-
$\chi$	$t=24 s$	0	0	-
$\sigma$	$t=24 s$	360. 106.360	. 106	0
$\varepsilon_{zz}$	$t=24 s$	- 3.84 10-3	- 3.84 10-3	0
$\varepsilon^p$	$t=26 s$	0.0372	3.7217 10-2	0.047
$\chi$	$t=26 s$	1	1	0.390
$\sigma$	$t=26 s$	. 106	390.00 106	0.000
$\varepsilon_{zz}$	$t=26 s$	0.051507	5.098 10-2	- 1.009
$\varepsilon^p$	$t=40 s$	0.06252	6.2523 10-2	0.037
$\chi$	$t=40 s$	1	1	0.600
$\sigma$	$t=40 s$	. 106.600	. 106	0
$\varepsilon_{zz}$	$t=40 s$	0.10197	1.0093 10-1	- 1.018
$\varepsilon^p$	$t=90 s$	0.07407	7.4145 10-2	0.062
$\chi$	$t=90 s$	1	1	0
$\sigma$	$t=90 s$	1350. 106	1350. 106	0
$\varepsilon_{zz}$	$t=90 s$	0.10984	1.08806 10-1	- 0.942

### 6.2 Remarks

In this modelization, one takes into account the term due to the plasticity of transformation:

$$\dot{\varepsilon}^{pt}(T, Z) \neq 0 \text{ when } \dot{Z} \neq 0$$

## 7 Summary of the results

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the two modelizations give very good approximations of the reference solution.