HPLP310 – Biblio_35 Fissures radial intern in a thick cylinder under pressure and thermal loading

Abstract:

This test is resulting from the validation independent of version 3 in fracture mechanics.

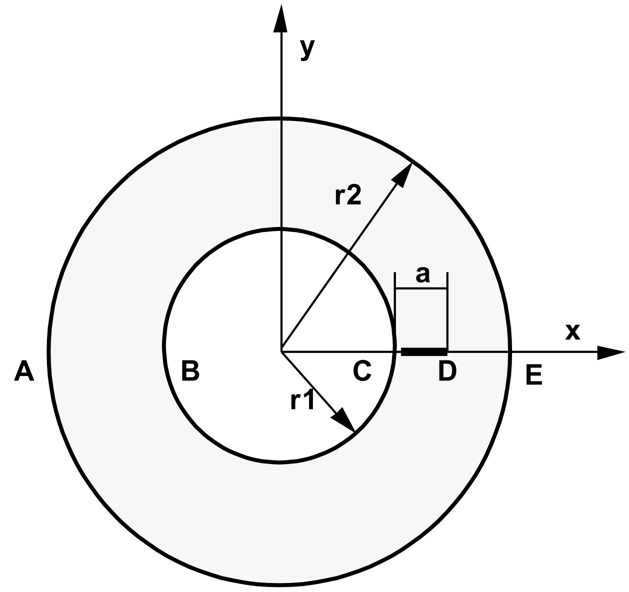
It is about a two-dimensional test in static in which one models to it not linearity of contact due to the reclosing partial of crack.

The behavior of structure is thermo-elastic linear isotropic.

The case test understands two modelizations $2D$ plane for which one studies the influence of the mechanical load factor α. In the first modelization a contact with a material infinimement rigid is used to represent the reclosing (symmetric) crack while in the second a boundary condition of type unilateral connection is put in work.

1 Problem of reference

1.1 Geometry



Cross-section of a thick tube presenting
a radial crack radius

interns Ratio $b=r_2/r_1=2\,(r_1=1\,mm,\,r_2=2\,mm)$

Depth of the crack $a/(r_2-r_1)=0{,}05$

1.2 Properties of the material

the material is thermo-elastic linear isotropic standard.

Young modulus $E=1000\,MPa$
linear $\nu=0{,}3$
Poisson's ratio Coefficient of thermal expansion $\alpha_T=1\text{E-6}$
Yield stress $\sigma_0=1\,MPa$ (being used to define the initial stress field created by the process of autofrettage on the assumption of a former behavior of elastoplastic type of Von Mises)

1.3 Boundary conditions and loadings

Boundary conditions (for a half-part in the area $y\geq 0$)

Blocking $UY=0$ on the segment AB and the ligament DE (symmetry).

Relation linear $UX(A)+UX(E)=0$ (to block the horizontal adjustment)

Loadings

Loading n° 1: radial tension $\sigma_{rr}(r_2)=\sigma_0$ on the external face; this mechanical loading produces the same one K_I as an internal pressure acting simultaneously on the internal radius r_1 and the lips of crack, without taking into account of nonthe linearity of contact.

Loading n° 2: thermal loading are equivalent to an autofrettage defined as follows:

$$T_1 = T_\rho + \frac{4\sigma_0}{\sqrt{3}} \bullet \frac{2(1-\nu)}{E\alpha_T} \bullet \ln\left(\frac{\rho}{r_1}\right)$$

$$T = T_1 - \frac{(T_1 - T_\rho)}{\ln\left(\frac{\rho}{r_1}\right)} \bullet \ln\left(\frac{r}{r_1}\right) \qquad r_1 \le r \le \rho$$

$$T = T_\rho \qquad \rho \le r \le r_2$$

In these formulas, ρ the maximum radius of the zone indicates having undergone autofrettage, T_1 the temperature with the radius r_1 and T_ρ the temperature with the radius $r=\rho$ in the thick tube not fissured. In the application concerned here, one takes $\rho=r_2$, which corresponds to the autofrettage of the totality of the section of the thick tube, and one does not take into account it not linearity of contact. One expects K negative for positive temperatures (put in compression of the tube not fissured).

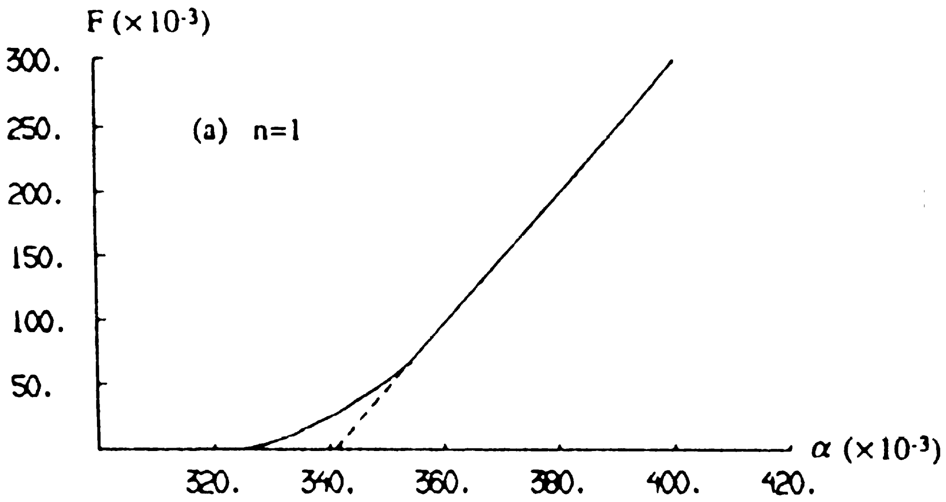
Loading n° 3: linear combination loading n° 2 + α * loading n° 1, α ($\neq \alpha T$!) indicating the mechanical load factor; one takes here into account it not linearity of contact, which supposes an incremental application of the mechanical load.

2 Reference solution

2.1 Method of calculating used for the reference solution

Computation by finite elements with code ABAQUS. Nonthe linearity of contact is modelled using one-way GAP elements. The factor of intensity of the stresses is calculated from the integral J.

2.2 Results of reference



adimensional Factor of intensity of the stresses according to the factor of loading mechanical, in the case of loading n° 3

Notation: $F_L = K_{IL} / \sigma_0 \sqrt{a}$ linear factor of intensity adimensional (obtained by linear combination of the effects of autofrettage and mechanical loading, in stopped feature)

$F_N = K_{IN} / \sigma_0 \sqrt{a}$ nonlinear factor of intensity adimensional (obtained by taking account of nonthe linearity of contact, in full feature).

$$K_I^2 = \frac{EJ}{\left(1-\nu^2\right)}$$

Empirical formula of the factor of intensity of the stresses under external radial tension

$$W = r_2 - r_1$$

$$K_{I0} = P\sqrt{\pi a} \bullet \frac{\dfrac{C_1}{(\ln b)^{0,2}} + \dfrac{C_2}{\ln b}}{\sqrt{1-1/b}} \qquad 0{,}01 \le \frac{a}{W} < 0{,}8 \quad et \quad 1{,}5 \le b \le 3{,}0$$

$$C_1 = 2{,}397 - 2{,}705\left(\frac{a}{W}\right)^{0,5} + 0{,}884\left(\frac{a}{W}\right)^2$$

$$C_2 = -0{,}244 + 1{,}447\left(\frac{a}{W}\right)^{0,5} + 0{,}809\left(\frac{a}{W}\right)^2$$

Formulates empirical factor of intensity of the stresses in autofrettage in full section

$$K_{Ia} = \sigma_0\sqrt{\pi a} \bullet \frac{C_1 + C_2(\ln b)^{0,75}}{\left(1{,}8 + \dfrac{1}{b^4}\right)^2} \qquad 0{,}01 \le \frac{a}{W} < 0{,}8 \quad et \quad 1{,}5 \le b \le 3{,}0$$

$$C_1 = \frac{\left(30{,}221 - 57{,}714\left(\frac{a}{W}\right)^{0,05} + 29{,}954\left(\frac{a}{W}\right)^{0,15} - 2{,}444\left(\frac{a}{W}\right)^{1,5}\right)}{\sqrt{1+\left(\frac{a}{W}\right)^{0,25}}}$$

$$C_2 = \frac{\left(-51{,}522 + 111{,}027\left(\frac{a}{W}\right)^{0,05} - 63{,}244\left(\frac{a}{W}\right)^{0,15} + 3{,}631\left(\frac{a}{W}\right)^{1,5}\right)}{\sqrt{1-\left(\frac{a}{W}\right)^{0,25}}}$$

2.3 bibliographical References

1) H.M. SHU, J. PETIT and G. BEZINE: Radial stress intensity factors for aces in thick walled cylinders. I. Symmetrical aces II. Combination of autofrettage and internal presses. Engng.Fract.Mechs., 49, n°4, 611-629, 1994. Modelization A

3 Characteristic

3.1 of the modelization The model consists

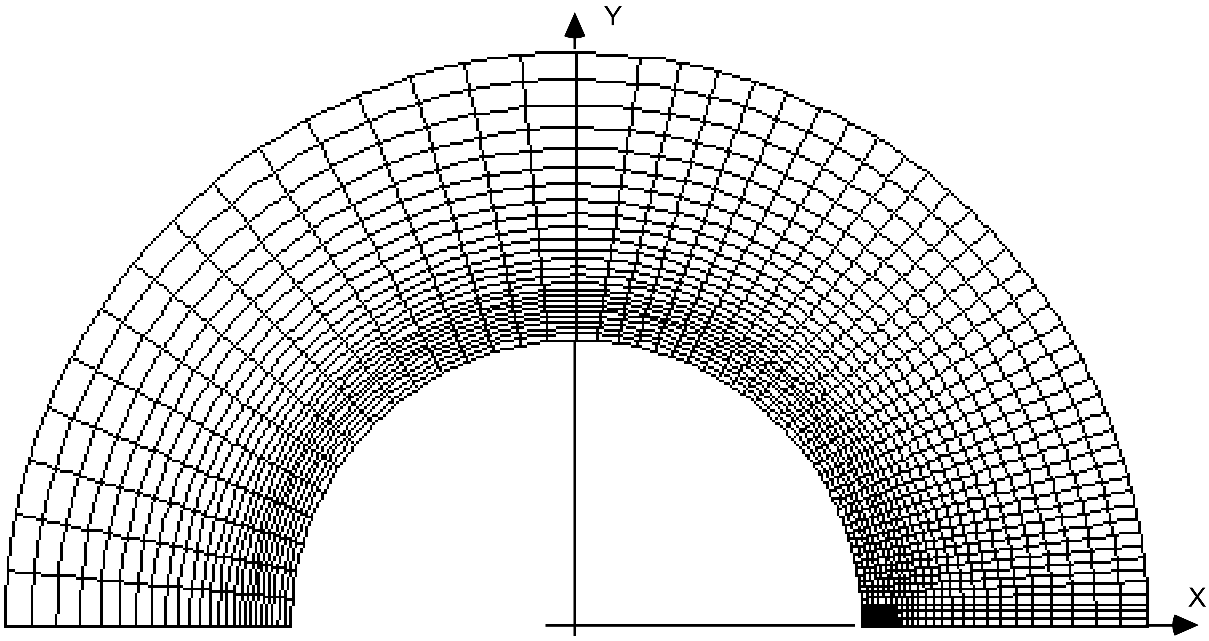
of quadrangles with 8 nodes and triangles with 6 nodes. It comprises 4877 nodes and 1598 elements. Characteristics

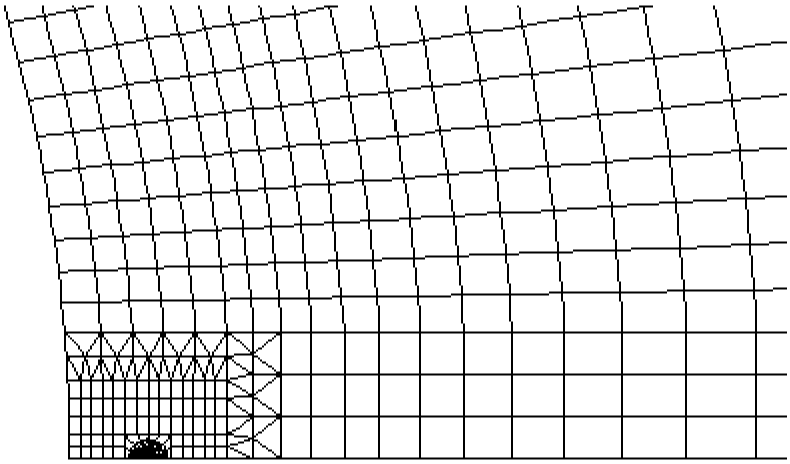
3.2 of the mesh Use of

procedure FISS2D_V1. The topological

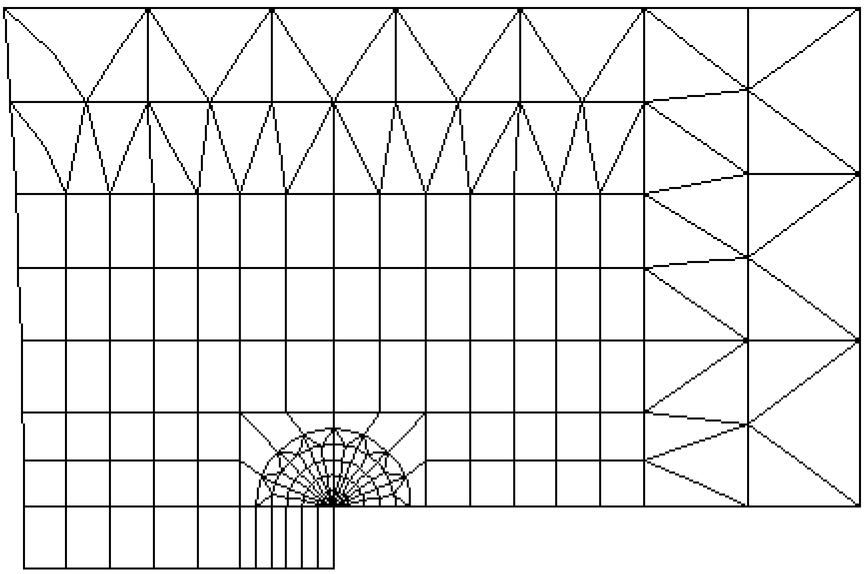
parameters concerning refinement around the crack tip are: (many contours

- $nc=4$) (many sectors
- $ns=8$) (many contours
- $nbcour=1$ of coarsening) Zoom of the fissured



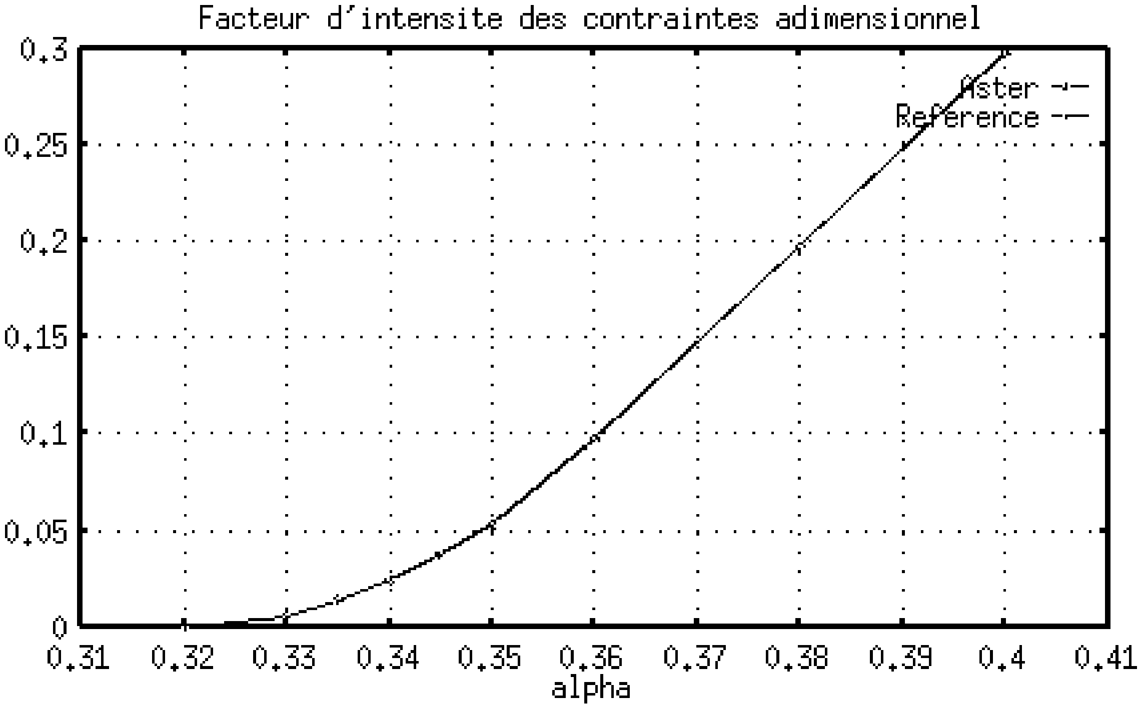


zone Zoom of the zone



fissured with “block of contact” Quantities tested

3.3 and results Standard Identification



Reference		of reference	Tolerance formulates	, loading
K_I, contour 0, contact neglected "SOURCE_EXTERNE	1,1482	"	2,0% formulates, loading	
K_I, contour 1, contact neglected "SOURCE_EXTERNE	1,1482	"	2,0% formulates, loading	
K_I, contour 2, contact neglected "SOURCE_EXTERNE	1,1482	"	2,0% formulates, loading	
K_I, contour 3, contact neglected "SOURCE_EXTERNE	1,1482	"	2,0% Identification	Standard

Reference		of reference	Tolerance formulates	, loading
K_I, contour 0, contact neglected ˜ "SOURCE_EXTERNE	0,41237	"	7,0% formulates, loading	
K_I, contour 1, contact neglected ˜ "SOURCE_EXTERNE	0,41237	"	7,0% formulates, loading	
K_I, contour 2, contact neglected ˜ "SOURCE_EXTERNE	0,41237	"	7,0% formulates, loading	
K_I, contour 3, contact neglected ˜ "SOURCE_EXTERNE	0,41237	"	7,0% Identification	Standard

Reference	of reference	Tolerance formulates	, loading
K_I, contact, formula, contour $\alpha=0{,}33$ "SOURCE_EXTERNE	"	6,0% formulates, loading	
K_I, contact, formula, contour $\alpha=0{,}335$ "SOURCE_EXTERNE	"	2,5% formulates, loading	
K_I, contact, formula, contour $\alpha=0{,}34$ "SOURCE_EXTERNE	"	1,0% formulates, loading	
K_I, contact, formula, contour $\alpha=0{,}345$ "SOURCE_EXTERNE	"	˜ 4,0% formula,	loading
K_I, contact, formula, contour $\alpha=0{,}35$ "SOURCE_EXTERNE	"	˜ 4,0% formula,	loading
K_I, contact, formula, contour $\alpha=0{,}36$ "SOURCE_EXTERNE	"	1,0% formulates, loading	
K_I, contact, formula, contour $\alpha=0{,}40$ "SOURCE_EXTERNE	"	1,0% Identification	Standard

Reference	of reference	Tolerance KI, loading	N
°3, contact, α = 0,33, contour 1 1,2075E-3 "SOURCE_EXTERNE	"	2,0% KI, loading	N
°3, contact, α = 0,335, contour 1 3,0187E-3 "SOURCE_EXTERNE	"	2,0% KI, loading	N
°3, contact, α = 0,34, crown 1 KI 5,4336E-3 "	SOURCE_ EXTERNE	" 1,0%, loading	N
°3, contact, α = 0,345, contour 1 8,5865E-3 "SOURCE_EXTERNE	"	˜ 4,0% KI, loading	N
°3, contact, α = 0,35, contour 1 1,2075E-2 "SOURCE_EXTERNE	"	˜ 4,0% KI, loading	N
°3, contact, α = 0,36, contour 1 2,1757E-2 "SOURCE_EXTERNE	"	1,0% KI, loading	N
°3, contact, α = 0,40, contour 1 6,6478E-2 "SOURCE_EXTERNE	"	1,0% Identification	Standard

Reference	of reference	Tolerance KI, loading	N
°3, contact, α = 0,33, contour 2 1,2075E-3 "SOURCE_EXTERNE	"	4,5% KI, loading	N
°3, contact, α = 0,335, contour 2 3,0187E-3 "SOURCE_EXTERNE	"	2,0% KI, loading	N
°3, contact, α = 0,34, contour 2 5,4336E-3 "SOURCE_EXTERNE	"	1,0% KI, loading	N
°3, contact, α = 0,345, contour 2 8,5865E-3 "SOURCE_EXTERNE	"	˜ 4,0% KI, loading	N
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°3, contact, α = 0,40, contour 2 6,6478E-2 "SOURCE_EXTERNE	"	1,0% Identification	Reference

Type		**of reference**	**Tolerance formulates**	**, loading**
K_I, contact, formula, contour "SOURCE_EXTERNE	$\alpha=0{,}335$	"	3,0% formulates, loading	
K_I, contact, formula, contour "SOURCE_EXTERNE	$\alpha=0{,}34$	"	1,0% formulates, loading	
K_I, contact, formula, contour "SOURCE_EXTERNE	$\alpha=0{,}345$	"	˜ 4,0% formula,	loading
K_I, contact, formula, contour "SOURCE_EXTERNE	$\alpha=0{,}35$	"	˜ 4,0% formula,	loading
K_I, contact, formula, contour "SOURCE_EXTERNE	$\alpha=0{,}36$	"	1,0% formulates, loading	
K_I, contact, formula, contour "SOURCE_EXTERNE	$\alpha=0{,}40$	"	1,0% Remarks	the tables

3.4 below

give rate of energy restitution for two values G of the coefficient which correspond α to nonthe separation of the lip of crack. (There is separation of the lip for). Identification $\alpha > 0,32$ Reference

G ASTER	, loading	n°
G 3, contact, crowns 0 0̃ $\alpha=0,30$ 8,7941 10̃	16	, loading n°
G 3, contact, crowns 1 0 4,4308 $\alpha=0,30$ 10̃ 15	,	loading n°
G 3, contact, crowns 2 0 3,3312 $\alpha=0,30$ 10̃ 15	,	loading n°
G 3, contact, crowns 3 0 4,4794 $\alpha=0,30$ 10 G̃	13	Identification

Reference ASTER	, loading	n°
G 3, contact, crowns 0 0̃ $\alpha=0,32$ 1,17E-14,	loading	n°
G 3, contact, crowns 1 0̃ $\alpha=0,32$ 3,26E-16,	loading	n°
G 3, contact, crowns 2 0̃ $\alpha=0,32$ 1,02E-15,	loading	n°
G 3, contact, crowns 3 0 4,23 $\alpha=0,32$ E-13 Modelization		B

4 Characteristic

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4.2 of the mesh Use of

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- $nc=4$) formula (many
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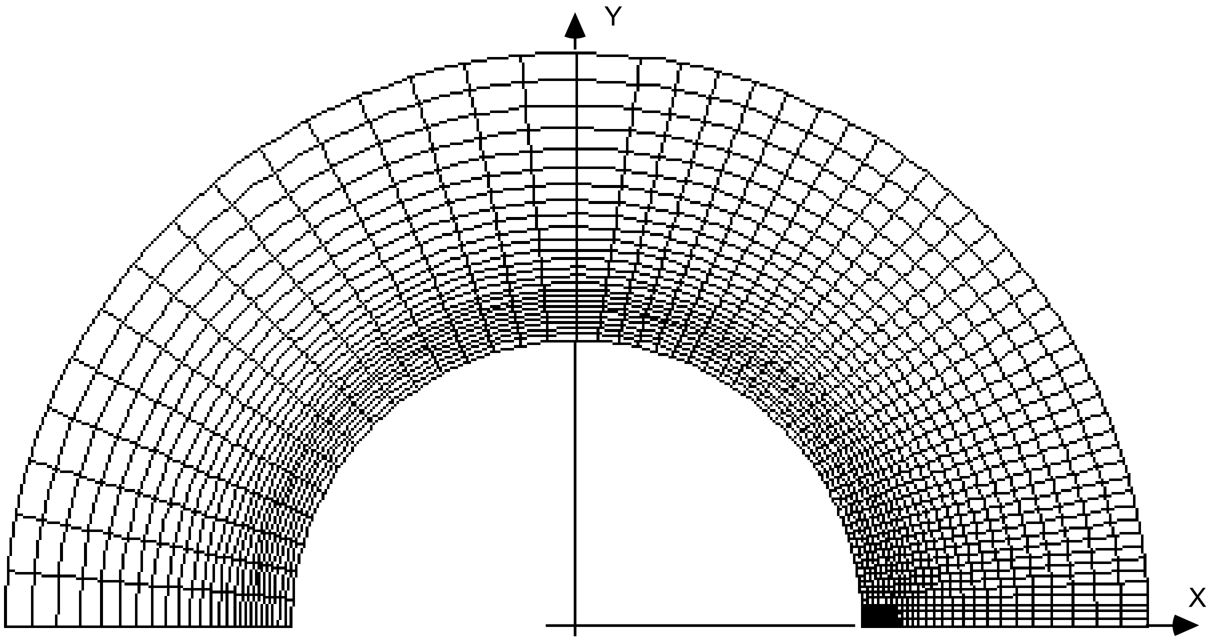
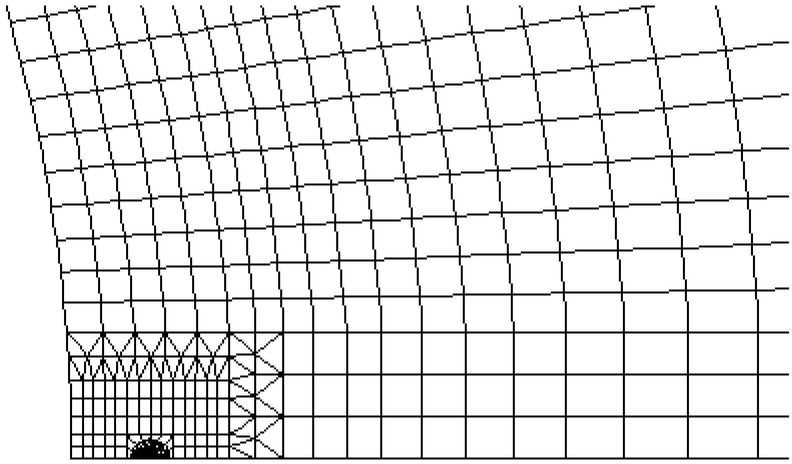
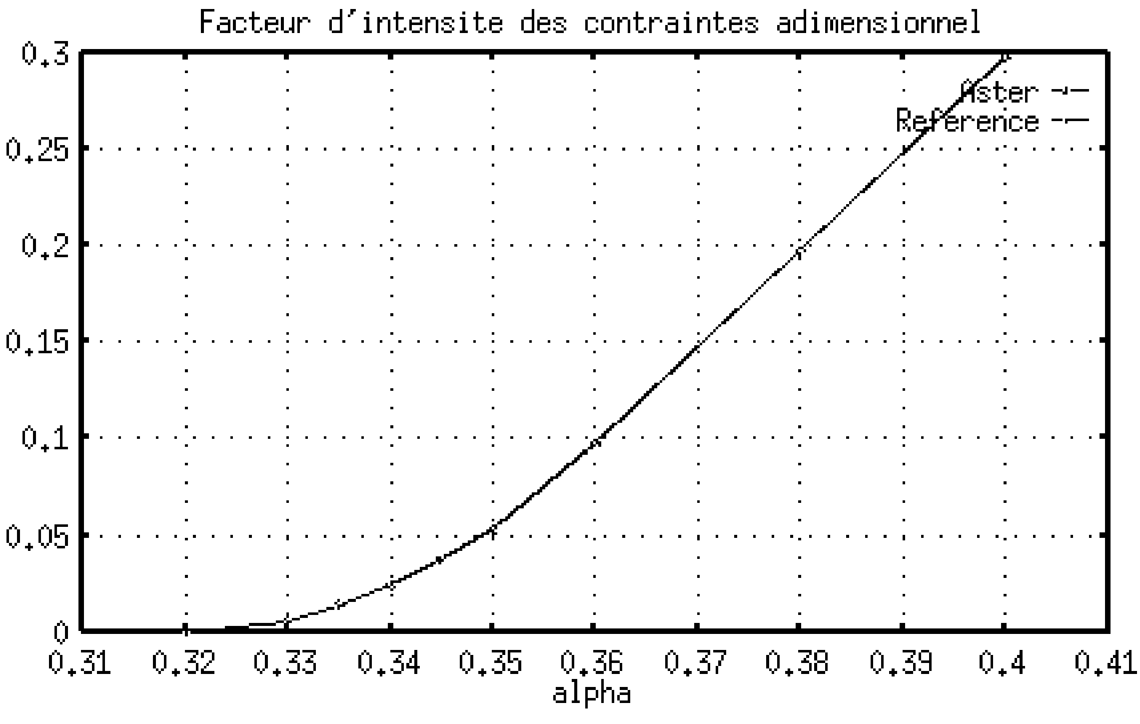




zone Quantities tested

4.3 and results Standard Identification



Reference		of reference	Tolerance formulates	, loading
K_I, contour 0, contact neglected “SOURCE_EXTERNE	1,1482	”	2,0% formulates, loading	
K_I, contour 1, contact neglected “SOURCE_EXTERNE	1,1482	”	2,0% formulates, loading	
K_I, contour 2, contact neglected “SOURCE_EXTERNE	1,1482	”	2,0% formulates, loading	
K_I, contour 3, contact neglected “SOURCE_EXTERNE	1,1482	”	2,0% Identification	Standard

Reference	of reference	Tolerance, loading	n°
K_I 2, contour 0, contact neglected ˜ 0,41237 “SOURCE_EXTERNE	”	7,0%, loading	n°
K_I 2, contour 1, contact neglected ˜ 0,41237 “SOURCE_EXTERNE	”	7,0%, loading	n°
K_I 2, contour 2, contact neglected ˜ 0,41237 “SOURCE_EXTERNE	”	7,0%, loading	n°
K_I 2, contour 3, contact neglected ˜ 0,41237 “SOURCE_EXTERNE	”	7,0% Identification	Standard

Reference	of reference	Tolerance, loading	n°
K_I 3, contact, crown 0 1,2075 $\alpha=0{,}33$ E-3 “SOURCE_EXTERNE	”	4,5%, loading	n°
K_I 3, contact, crown 0 3,0187 $\alpha=0{,}335$ E-3 “SOURCE_EXTERNE	”	3,0%, loading	n°
K_I 3, contact, crown 0 5,4336 $\alpha=0{,}34$ E-3 “SOURCE_EXTERNE	”	1,0%, loading	n°
K_I 3, contact, crown 0 8,5865 $\alpha=0{,}345$ E-3 “SOURCE_EXTERNE	”	˜ 4,0%, loading	n°
K_I 3, contact, crown 0 1,2075 $\alpha=0{,}35$ E-2 “SOURCE_EXTERNE	”	˜ 4,0%, loading	n°
K_I 3, contact, crown 0 2,1757 $\alpha=0{,}36$ E-2 “SOURCE_EXTERNE	”	1,0%, loading	n°
K_I 3, contact, crown 0 6,6478 $\alpha=0{,}40$ KI E-2 “	SOURCE_ EXTERNE	” 1,0%	Identification

Standard	Reference of	reference Tolerance, loading	N
°3, contact, α = 0,33, contour 1 1,2075E-3 “SOURCE_EXTERNE	”	4,5% KI, loading	N
°3, contact, α = 0,335, contour 1 3,0187E-3 “SOURCE_EXTERNE	”	3,0% KI, loading	N
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°3, contact, α = 0,345, contour 1 8,5865E-3 “SOURCE_EXTERNE	”	˜ 4,0% KI, loading	N
°3, contact, α = 0,35, contour 1 1,2075E-2 “SOURCE_EXTERNE	”	˜ 4,0% KI, loading	N
°3, contact, α = 0,36, contour 1 2,1757E-2 “SOURCE_EXTERNE	”	1,0% KI, loading	N
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°3, contact, α = 0,40, contour 2 6,6478E-2 “SOURCE_EXTERNE	”	1,0% Identification	Standard

Reference	of reference	Tolerance, loading	n°
K_I 3, contact, crowns 3 3,0187 $\alpha=0{,}335$ E-3 “SOURCE_EXTERNE	”	3,0%, loading	n°
K_I 3, contact, crowns 3 5,4336 $\alpha=0{,}34$ E-3 “SOURCE_EXTERNE	”	1,0%, loading	n°
K_I 3, contact, crowns 3 8,5865 $\alpha=0{,}345$ E-3 “SOURCE_EXTERNE	”	˜ 4,0%, loading	n°
K_I 3, contact, crowns 3 1,2075 $\alpha=0{,}35$ E-3 “SOURCE_EXTERNE	”	˜ 4,0%, loading	n°
K_I 3, contact, crowns 3 2,1757 $\alpha=0{,}36$ E-2 “SOURCE_EXTERNE	”	1,0%, loading	n°
K_I 3, contact, crowns 3 6,6478 $\alpha=0{,}40$ below E-2 “	SOURCE_ EXTERNE	” 1,0% Remarks	

4.4 the tables

give rate of energy restitution formulates for two G of the coefficient formulates which correspond α to nonthe separation of the lip of crack. (There is separation of the lip for formula). Identification $\alpha > 0{,}32$ Reference

G ASTER	formulates	, loading
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G , contact, formula, contour $\alpha=0{,}30$ 10˜ 15	formula	, loading
G , contact, formula, contour $\alpha=0{,}30$ 10˜ 15	formula	, loading
G , contact, formula, contour $\alpha=0{,}30$ 10˜ 13	Identification	Reference

G ASTER	formulates	, loading
G , contact, formula, contour $\alpha=0{,}32$ 1,17E-14 formula		, loading
G , contact, formula, contour $\alpha=0{,}32$ 3,26E-16 formula		, loading
G , contact, formula, contour $\alpha=0{,}32$ 1,02E-15 formula		, loading
G , contact, formula, contour $\alpha=0{,}32$ Summary		of the results

5 The computation of is

correct in G all the cases, including for a completely closed crack. The modelizations with block of contact and unilateral connection give similar results.