HPLA311 - Murakami 11.39. Fissure circular in the center of a sphere subjected to a uniform temperature on the lips

Summarized:

This test is resulting from the validation independent of version 3 in fracture mechanics.

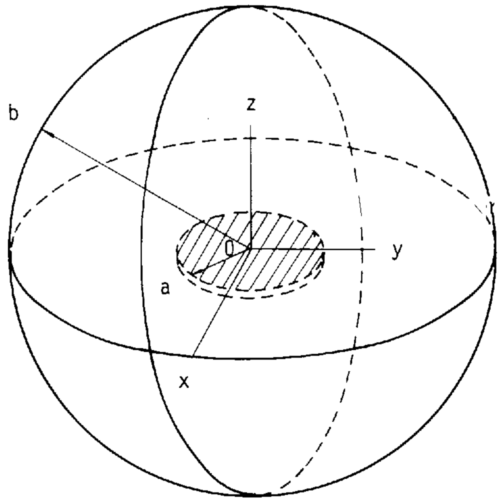
It is about a basic static test in axisymetry under steady thermal loading calculated by finite elements on the same mesh of a restricted field.

The behavior is thermo-elastic linear isotropic.

It understands two axisymmetric modelizations for which one varies a/b ratio, has being the radius of crack interns circular in the horizontal plane xoy and B radius of the sphere. The factors of intensity of the stresses K and rate of energy restitution are calculated by the method theta (operator `CALC_G`).

1 Problem of reference

1.1 Geometry



a: radius of crack interns circular in the horizontal plane xoy
b: radius of the sphere, with B = 2,5.10-3 Mr.

the radius has varies according to the modelization.

1.2 Properties of the material

modulus Young linear	E = 2 1011 Pa ν Poisson's ratio =
0,3 coefficient of thermal expansion	α = 1,2 10-5 °C-1

1.3 Boundary conditions and loadings

UX = ur = 0 on the axis of revolution X = R = 0
UY = uz = 0 in the horizontal plane Y = Z = 0, apart from the lips has ≤ R ≤ B

the lips are supposed to be free stresses (not closing partial of crack).

Temperature null on the surface of the sphere.

Reference temperature null (temperature to which the thermal strains are considered null).

Uniform and negative temperature T = - Tf on the lips of crack, crack tip understood. The steady thermal problem (of Dirichlet type) must be solved beforehand by finite elements on the same mesh as that intended for mechanical computation. One takes Tf = 100 °C.

2 Reference solution

2.1 Method of calculating used for the analytical reference solution

Computation by transform of Hankel.

2.2 Results of reference

For the reference solution, radius must check the condition a/b < 0,5.

$$\eta=\frac{a}{b}<0{,}5$$

$$K_I=\frac{E\,\alpha\,T_f}{1-\nu}\cdot\sqrt{\left(\frac{a}{\pi}\right)}\cdot F_I$$

$$F_I=1-0.6366\,\eta-0.4053\,\eta^2+2.0163\,\eta^3-0.6773\,\eta^4-3.8523\,\eta^5+4.1687\,\eta^6+3.2741\,\eta^7$$

2.3 Uncertainty on the Badly

definite solution. For the low values of a/b ratio, the solution must approach asymptotically the reference solution calculated for = 0, that is to say FI = 1, which is then exact (see MURAKAMI 11.23, page 1069).

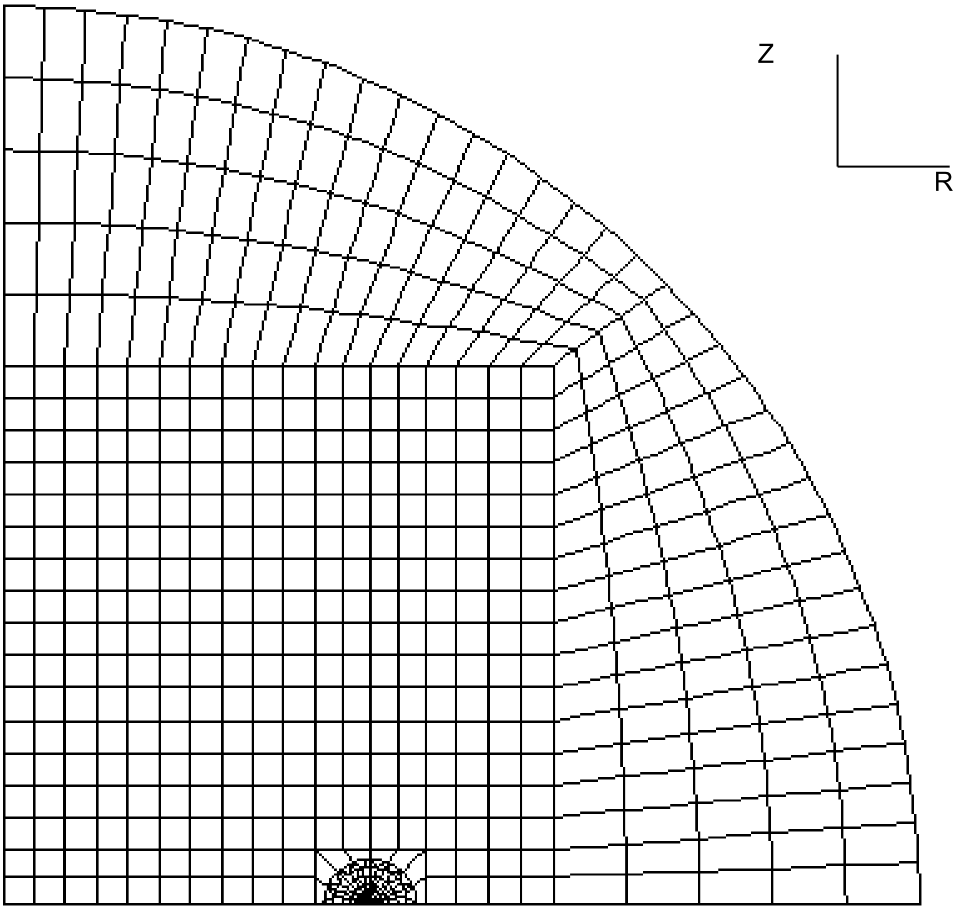
2.4 Bibliographical references

[1] Y. MURAKAMI: Stress Intensity Factors Handbook, box 11.39, pages 1089-1090. The Society of Materials Science, Japan, Pergamon Near, 1987.

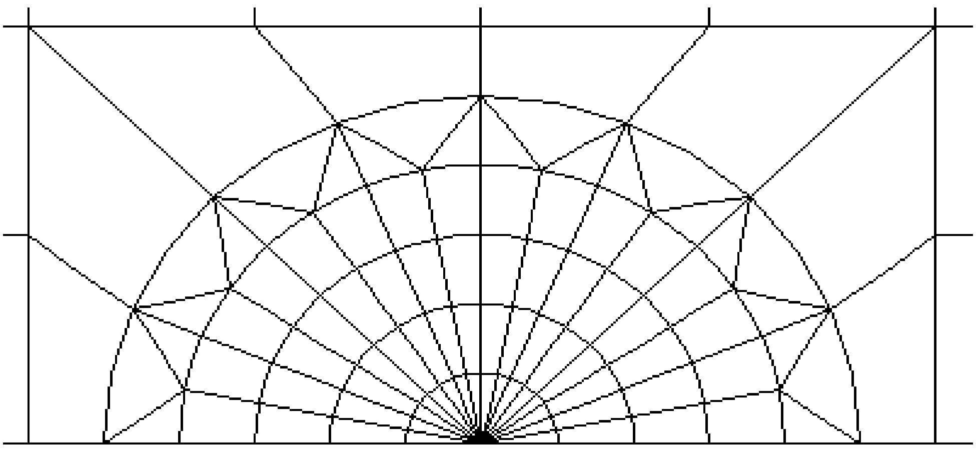
3 Modelization With

the modelization A corresponds to the case **a/b = 0,4**.

3.1 Characteristics of modelization



complete Mesh



Zoom of the point of crack

3.2 Characteristics of the mesh

1756 nodes and 569 elements whose 529 QUA8 and 40 TRI6

3.3 Functionalities tested

Commands

AFFE_MODELE	THERMAL	AXIS
AFFE_CHAR_THER	TEMP_IMPO	
AFFE_MODELE	MECANIQUE	AXIS
AFFE_MATERIAU	AFFE_VARC	NOM_VARC=' TEMP'
CALC_THETA	THETA_2D	
CALC_G	OPTION	CALC_G
CALC_G	OPTION	CALC_K_G

3.4 Definition of radius of contours

Several couples successive of radius for integration contours lower and higher are retained. These radius are to be specified in command `CALC_THETA` or factor key word `the THETA` of `CALC_G` :

	Crown n°1	Contour n°2	Contour n°3	Contour n°4
rinf	1.E-6	2.5E-5	5.E-5	7.5E-5
rsup	2.5E-5	5.E-5	7.5E-5	1.E-4

3.5 Reference solutions

For a ratio a/b = 0,4 (and = 10-3 m has), the reference solution for KI is:

$$K_I = 4.7419\,\mathrm{MPa}\cdot\sqrt{(\mathrm{m})}$$

To compute: the rate of refund of energy, one uses the formulas of IRWIN in plane strains:

$$G_{réf} = \frac{1-\nu^2}{E}(K_I^2 + K_{II}^2), \quad \text{with } K_{II}^2 = 0$$

is:

$$G_{réf} = 1.0231.10^2\,\mathrm{J.m^{-2}}$$

Note:

1) In the axisymmetric calculation case, the rate of energy restitution calculated with option `CALC_G` of `CALC_G` corresponds to a total rate of energy restitution *Gglob* for a radian. *Gglob* is equal to the local rate of energy restitution multiplied by the radius of the crack's point. One thus has:

$$G_{réf}^{glob} = a\cdot G_{réf} = 1.0231\,\mathrm{J.m^{-1}}$$

2) The rate of energy restitution calculated with option `CALC_K_G` of `CALC_G` corresponds as for him directly to local rate of energy restitution.

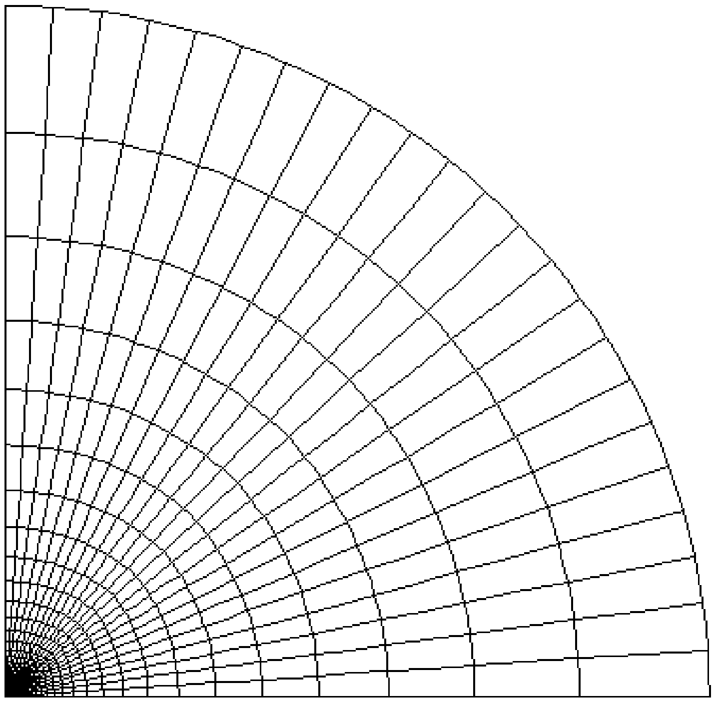
3.6 Quantities tested and results

Parameter	Unit	Option	Crowns	G	*Reference*	Aster %
difference	J.m -1 CALC_G	crowns	n°1 1,0231	0,9701	-5,18	-5.18
J.m -1	CALC_G	crowns	n°2 1,0231	1,0051	-1,74	-1.74
CALC_G	crowns		n°3 1,0231 1,0055	-	1,72	G J.m -1 CALC_G
	crowns	n°	4 1,0231 1,01	-1,71	1.01	-1.71
	crowns	n°2	1,0231.10 2 1,0052.10	2 -	1,75 K1 MPa ·	m -2 CALC_K_G
	crowns	n°1	4,7419 4,4145	- 6,89	K1 MPa.	m -2 CALC_K_G
	crowns	n°2	4,7419 4,7571	0,30 K1	MPa. m	-2 CALC_K_G
	crowns	n°3	4,7419 4,7913	1,04 K1	MPa. m	-2 CALC_K_G
	B	crowns	n°4 4,7419	4,8244 1,74		

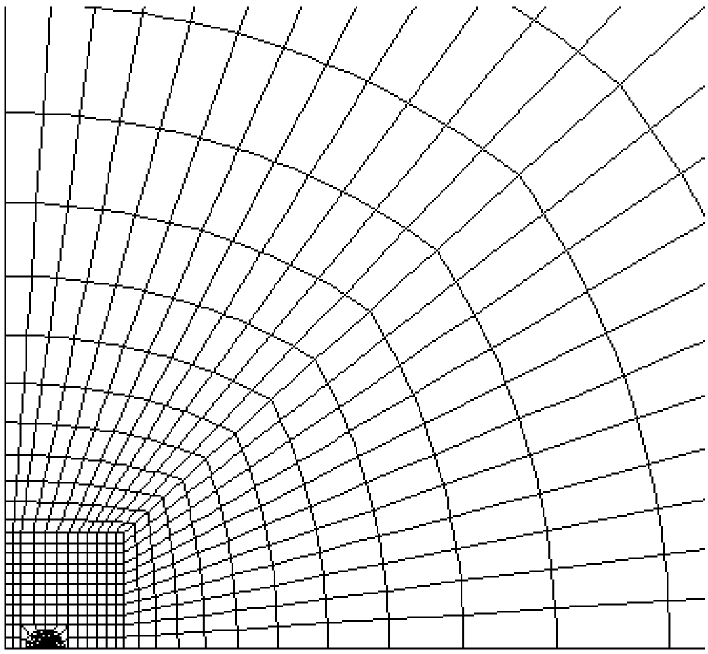
4 Modelization The modelization

B corresponds to the case a/b = 0,01. Characteristics

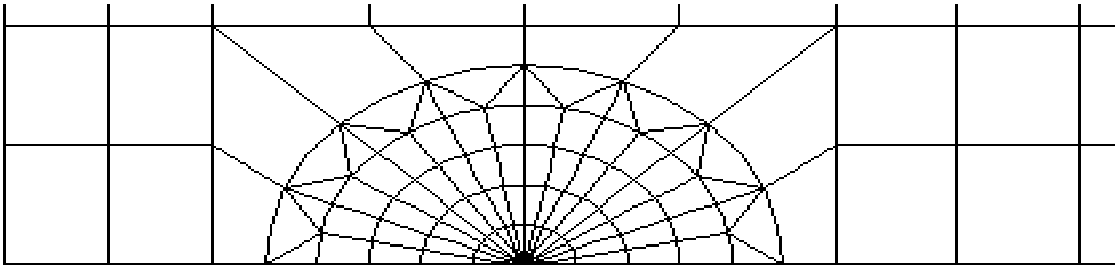
4.1 of the modelization complete Mesh



Zoom Zoom of



the point



of crack Characteristics

4.2 of the mesh 2095 nodes and

680 elements whose 640 QUA8 and 40 TRI6 Functionalities

4.3 tested Commands AFFE_MODELE

THERMAL

AXIS	AFFE_CHAR_THER	
TEMP_IMPO	MECANIQUE	
AXIS	AFFE_MATERIAU	
AFFE_VARC NOM_	VARC=' TEMP	"CALC_THETA
CALC_G	OPTION	
CALC_G	CALC_G	OPTION
CALC_K_G	Definition	of

4.4 radius of contours Several couples

successive of radius for contours D" integration lower and higher are retained. These radius are to be specified in command CALC_THETA or `factor key word` the THETA of CALC_G : `Crown` n° 2.5

	0 Contour	n°1 Contour	n°2 Contour	n°3 Contour	n°4 rinf 1.E-6
E-5 2.75	E-5	3.E-5	3.25E-5	rsup 2.5	E-5 2.75
E-5	3.E-5	3.25E-5	3.5E-5	Note:	: For

the computation of the factors of intensity of the stresses with option CALC_K_G of CALC_G , `it is not necessary` that the radius of contours is larger than the radius of the crack's point. The radius of the crack's point being here equal to 2.5E-5, only contour n°0 can be used. Reference solutions

4.5 For a ratio

a/b = 0,01 (and = 2,5.10-5 m have), the reference solution for KI is: To compute:

$$K_I = 0.9609\,\text{MPa}\cdot\sqrt{(\text{m})}$$

the rate of refund of energy, one uses the formulas of IRWIN in plane strains: , with is

$$G_{réf} = \frac{1-\nu^2}{E}(K_I^2 + K_{II}^2) : \quad \text{Note: } K_{II}^2 = 0$$

$$G_{réf} = 4.2019\,\text{J.m}^{-2}$$

In the axisymmetric calculation case

3) , the rate of energy restitution calculated with option CALC_G of CALC_G corresponds to a total rate of energy restitution Gglob for *a radian* . Gglob is equal *to the local* rate of energy restitution multiplied by the radius of the crack's point. One thus has: The rate of energy restitution

$$G_{réf}^{glob} = a\cdot G_{réf} = 1.0505\cdot 10^{-4}\,\text{J.m}^{-1}$$

4) calculated with option CALC_K_G of CALC_G corresponds as for him directly to local rate of energy restitution. Quantities tested

4.6 and results Parameter Unit

Option	Crowns	G	Reference	Aster %		difference J.m -1 CALC_G crowns
	N	°1 1,0505.10	- 4 1,0387.10	- 4 -5,18	G J.m -1 CALC_G	-5.18
	N °2	1,0505.10	- 4 1,0388.10	- 4 -1,74 G	J.m -1 CALC_G crowns	-1.74
	n°3 1,0505.10		- 4 1,0388.10-4	-1,72 G J.m -1	CALC_G crowns	-1.72
°	4 1,0505.10	- 4	1,0388.10-4 -1,71	G J.m -2 CALC_K_G	crowns	-1.71
0	4,2019	4,1653 -0,87	K1 MPa. m	-2 CALC_K_G	The computation	-0.87
crowns	N °0	0,9609 0,9653	-0,46	Summary of		-0.46

5 of K and G in axisymetry

- in the presence of a steady thermal loading, gives good results since the maximum change for G is of 1,75% (out the first contour) for = 0,4. The results of K and ν G for
- =0,01 (modelization B) are ν better than for = 0,4 (modelization A). The computation ν of G is slightly less
- less sensitive to the choice of integration contours than the computation of K.