SSND105 - Constitutive law visco-élasto-plastic with effect of memory

Summarized:

The problem is quasi-static nonlinear in structural mechanics. The models tested, VMIS_CIN2_MEMO and VISC_CIN2_MEMO, are models with nonlinear kinematic hardening, isotropic hardening, and memory of maximum hardening. One analyzes the response in a material point, with a pre-hardening, then a cyclic loading.

The modelization A allows to validate the effect of memory with VMIS_CIN2_MEMO in a case where hardening is purely isotropic, for a simple tension. The reference solution for this modelization is analytical.

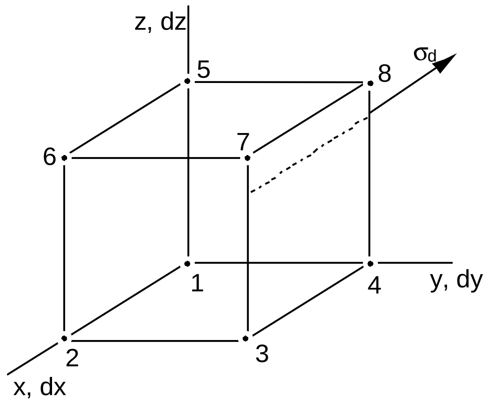
The modelization B compares the results got with effect of memory, and without effect of memory between models VISC_CIN2_MEMO and VISCOCHAB, for a cyclic loading with pre-hardening.

The modelization C is similar to the modelization B, and allows to validate the two models into axisymmetric.

The modelization D is similar to the modelization C, and allows to check that the models being able to take into account the effect of nonproportionality give in this case of the results identical to the preceding models.

1 Problem of reference

1.1 Geometry



Face YZ : (1, 4, 5, 8)
Face XZ : (1, 2, 5, 6)
Face 1YZ : (2, 3, 6, 7)
Face 1XZ : (4, 3, 8, 7)

σ_d : pression

1.2 Properties of the materials

isotropic Elasticity $E=145\,000\,MPa$ $\nu=0.3$

Elastoplasticity with effect of memory (modelization A): model R_0

isotropic VISC_CIN2_MEMO			
Hardening	$35\,MPa$	B	12
Memory			
MU	19	Q_0	140MPa
ETA	0.5	Q_M	460MPa
Kinematic hardening (modelization A)			
C1	0	G1_0	0
C2	0	G2_0	0

Viscoplasticity with effect of memory (modelizations B and C): model VISC_CIN2_MEMO
Parameters identical to the preceding values, except:

LEMAITRE			
UN_SUR_K	$1/70(MPa\,S^{1/N})^{-1}$ =0.0142857	N	24
Kinematic hardening (modelization B)			
C1	1950 MPa	G1_0	50
C2	65000 MPa	G2_0	1300

model Viscoplasticity VISCOCHAB (modelizations B and C)

k	$35\,MPa$	B	12	ETA	0.5	C2	$65000\,MPa$
A_K	0	M_R	1	C1	$1950\,MPa$	M_2	1
A_R	1	G_R	0	M_1	1	D2	1
K_0	$70\,MPa\,S^{1/N}$	MU	19	D1	1	G_X2	0
N	24	Q_M	460	G_X1	0	G2_0	$1300\,MPa$
ALP	0 MPa	Q_0	40 MPa	G1_0	$50\,MPa$	A_I	1

	QR_0	200 MPa	

1.3 Boundary conditions and loadings

$N6 \quad dy=dz=0$
$N2 \quad dy=0$
$FACE1YZ \quad dx=0$

Tension (modelization A): $FACEYZ \quad Fx=-0.25\times coef \qquad Coef=120$ for $t=8s$.

Pre-hardening (modelization B) $FACEYZ \quad Sxx=250\,MPa\times coef2 \qquad Sxx=250\,Mpa\times coef2$

$coef\ 2=1$ for $t=10s$, then discharge ($coef2=0$) for $t=11s$.

From $11s$, 20 cycles in imposed strain (+ 0.5%)

2 Reference solution

2.1 Method of calculating used for the reference solution

One can calculate the analytical solution corresponding to pre-hardening (tension monotonous, modelization A):

The system of equations of the problem with effect of memory is written (20 equations) [R5.03.04]:

Elasticity : $\tilde{\sigma}=2\mu(\tilde{\varepsilon}-\varepsilon^p)$

Yielding $\left(\tilde{\boldsymbol{\sigma}}-\frac{2}{3}C_1\boldsymbol{\alpha_1}-\frac{2}{3}C_2\boldsymbol{\alpha_2}\right)_{eq}=R_0+R(p)$

plasticity criterion: $\dot{\boldsymbol{\varepsilon}}^{\mathbf{p}}=\dot{p}\,\mathbf{n}$ with $\mathbf{n}=\frac{3}{2}\frac{\tilde{\boldsymbol{\sigma}}-\frac{2}{3}C_1\boldsymbol{\alpha_1}-\frac{2}{3}C_2\boldsymbol{\alpha_2}}{\left(\tilde{\boldsymbol{\sigma}}-\frac{2}{3}C_1\boldsymbol{\alpha_1}-\frac{2}{3}C_2\boldsymbol{\alpha_2}\right)_{eq}}$

isotropic Hardening : $\dot{R}=b(Q-R)\dot{p}$

Memory of maximum hardening: $Q=Q_0+(Q_m-Q_0)\left(1-e^{-2\mu q}\right)$

where Q is determined by:

•a field $F\left(\varepsilon^p,\xi,q\right)=\frac{2}{3}J_2\left(\varepsilon^p-\xi\right)-q\leq 0$ characterizing the plastic strains maximum ones, whose Q measures the radius and ξ the center

• ξ is calculated according to a model of normality i.e.: $\dot{\xi}=\dot{q}\mathbf{n}^*$, with $\mathbf{n}^*=\frac{3}{2}\frac{\boldsymbol{\varepsilon}^{\mathbf{p}}-\xi}{J_2\left(\boldsymbol{\varepsilon}^{\mathbf{p}}-\xi\right)}$

On the surface of the field of maximum hardening, one A. $F=0$ By applying the condition $dF=0$, one obtains the statement of velocity: $\dot{q}=\eta\left\langle\mathbf{n}:\mathbf{n}^*\right\rangle\dot{p}$

For a material point in uniaxial load, the fields (uniform) have as components:

$$\boldsymbol{\sigma}=\sigma\begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{pmatrix}\qquad \boldsymbol{\varepsilon}^{\mathbf{p}}=p\begin{pmatrix}1 & 0 & 0\\ 0 & -\frac{1}{2} & 0\\ 0 & 0 & -\frac{1}{2}\end{pmatrix}$$

In this case, at the time of the first uniaxial load in direction X:

$$\xi^{-}=0$$
$$q^{-}=0$$
$$\Delta q=\eta\varepsilon_{x}{}^{p}$$

In this case $q=\frac{1}{2}\Delta\boldsymbol{\varepsilon}^{\mathbf{p}}{}_{\max}$, implies that $\eta=\frac{1}{2}$. In this case, $\Delta\boldsymbol{\xi}=\frac{1}{2}\left(\boldsymbol{\varepsilon}^{\mathbf{p}}\right)$

Moreover, in the case of a cycle of symmetric tension compression (in plastic strain), one obtains, during the first symmetric discharge (with $\eta=\frac{1}{2}$):

$$\xi^{-}=\frac{1}{2}\varepsilon^{p}{}_{\max}$$
$$q^{-}=\frac{1}{2}\varepsilon^{p}_{xx\max}$$
$$\Delta q=\eta\left(\frac{2}{3}J_{2}\left(\varepsilon^{p}\right)-q^{-}\right)=\eta\left(\left|\varepsilon^{p}_{xx\min}-\xi^{-}\right|-\frac{1}{2}\varepsilon^{p}_{xx\max}\right)=\frac{1}{2}\left|\varepsilon^{p}_{xx\min}\right|$$

$$q=q^{-}+\Delta q=\varepsilon^{p}_{xx\max}=\frac{1}{2}\Delta\varepsilon^{p}_{xx}$$
$$\Delta\xi=\frac{(1-\eta)\Delta q\left(\boldsymbol{\varepsilon}^{\mathbf{p}}-\boldsymbol{\xi}^{-}\right)}{\eta q^{-}+\Delta q}=-\frac{1}{2}\Delta\varepsilon^{p}_{xx\max}$$

$\xi=\xi^{0}+\Delta\xi=0$ what corresponds well to result waited (cf [bib2]): field $F=0$ centered on the origin, and of radius the half-amplitude of plastic strain.

In the case of an increasing tension, and if kinematic hardening is neglected, the equations to be solved become:

$$\sigma\leq R_{0}+R(p)$$

The function thus should be calculated $R(p)$, such as:

$$dR=b(Q-R)dp \qquad \text{with } Q=Q_{0}+(Q_{m}-Q_{0})\left(1-e^{-2\mu q}\right).$$

Moreover, it is considered that one is in load, therefore $F\left(\varepsilon^{p},\xi,q\right)=0$

$$dq=\eta dp$$

It is thus necessary to integrate the differential equation:

$$dR = b\left(Q_0 + (Q_m - Q_0)\left(1 - e^{-2\mu q}\right) - R\right)dp$$

what is integrated in the following way:

$$dR + bR = 0 \;\Rightarrow\; R = \lambda e^{-bp}$$

Méthode of variation of the constant: $R = \lambda(p)e^{-bp}$

$$d\lambda e^{-bp} = b\left(Q_m - (Q_m - Q_0)e^{-2\mu \eta p}\right)dp$$
$$d\lambda = bQ_m e^{bp}dp + b(Q_0 - Q_m)e^{(b-2\mu \eta)p}dp$$

while integrating:

$$\lambda = Q_m e^{bp} + \frac{b(Q_0 - Q_m)}{(b - 2\mu \eta)}e^{(b-2\mu \eta)p} + K$$

from where

$$R(p) = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu \eta)}e^{-2\mu \eta p} + Ke^{-bp}$$

the constant K is defined by the initial conditions: for $p=0$, $R=0$

$$0 = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu \eta)} + K \text{ that is to say } K = \frac{b(Q_m - Q_0)}{(b - 2\mu \eta)} - Q_m = \frac{-bQ_0 + 2\mu \eta Q_m}{(b - 2\mu \eta)}$$

Finally:

$$R(p) = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu \eta)}e^{-2\mu \eta p} + \frac{2\mu \eta Q_m - bQ_0}{(b - 2\mu \eta)}e^{-bp}$$

One thus has in load: $\sigma = R_0 + R(p)$

2.2 Results of reference

Modelization a:

Value of $SIXX$ to final moment: $\sigma = R_0 + R(p)$

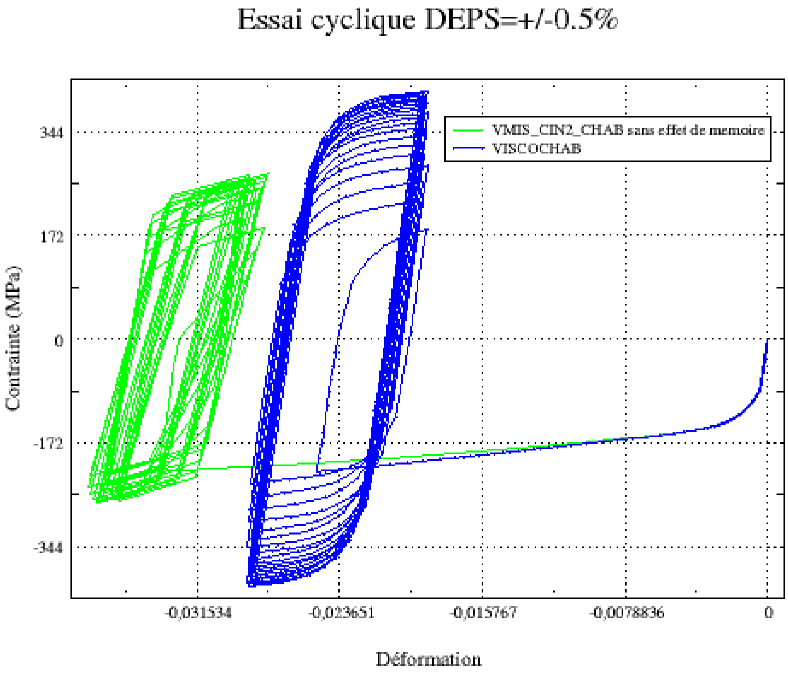
with $R(p) = Q_m + \frac{b(Q_0 - Q_m)}{(b - 2\mu \eta)}e^{-2\mu \eta p} + \frac{2\mu \eta Q_m - bQ_0}{(b - 2\mu \eta)}e^{-bp}$

$t=8\text{s}$, one must find $SIXX = 120\,Mpa$.

For that one computation $R(p)$ from the value of p at time $t=8\text{s}$.

Modelization b:

One will compare the results obtained with `VISC_CIN2_MEMO` with those obtained with `VISCOCHAB`, at the end of pre-hardening and at the end of 10 cycles. The curves below highlight of the effect of memory (by comparison with `VISC_CIN2_CHAB` which does not model it): after a pre-hardening, the cycles with imposed strain are stabilized with an amplitude of stresses higher than that obtained without effect of memory:



2.3 Uncertainty on the solution

- analytical Modelization a:
- Modelization b: intercomparison between VISCOCHAB and VISC_CIN2_MEMO : accuracy of the numerical integration, estimated at less 1% .
- Modelization C: validation ddes behaviors in 2D AXIS ; the results must be identical to those of the modelization B.

2.4 bibliographical References

[1] R5.03.04 “Behaviors élasto-visco-plastics of J.L.Chaboche”. J.M.PROIX “Behavior

[2] viscoplastic taking into account it not proportionality of loading” EDF R & D - CR-AMA12-284, 12/12/12 Modelization A Characteristic

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3.1 the modelization Modelization 3D, 1

hexa8. Simple tension. Quantities tested and

3.2 results Identification Reference

tolerance	formulates 0,20%	0,10%
σ_{xx}	120	0.20%
p	$3.70925\,E-2$	0.10%

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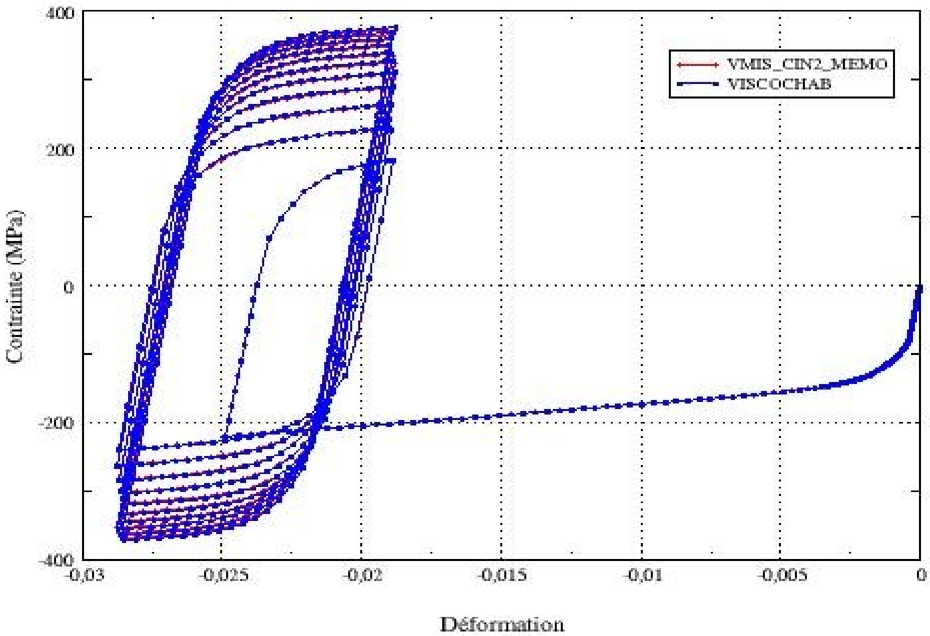
4.1 the modelization Pre-hardening in

tension then cycles with imposed strain, comparison VISCOCHAB and VISC_CIN2_MEMO . 250 `time step for` 10 cycles. Quantities tested and

4.2 results Identification Time

VISCOCHAB VISC_CIN2_MEMO		difference	formulates	% 0 formula 0
σ_{xx}	10	220	220	
σ_{xx}	11		0	Remarks
σ_{xx}	113.5	3.75459E+02	3.72353E+02	−0.8
ε_{xx}	113.5	−1.87638E−02	−1.87638E−02	

Essai cyclique DEPS=+/-0.5%



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the stresses 0.8% to final moment grows blurred if one refines time step: with one time step 2 times finer, the variation become. Modelization C Characteristic 0.4%

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5.1 the modelization Pre-hardening in

tension then cycles with imposed strain, comparison VISCOCHAB and VISC_CIN2_MEMO . 250 time step for 10 cycles. Modelization 2D AXIS . Quantities tested and

5.2 results Identification Time

VISCOCHAB VISC_CIN2_MEMO		difference	formulates	% formula Modelization
σ_{xx}	113.5	3.75459E+02	3.72353E+02	−0.8
ε_{xx}	113.5	−1.87638E−02	−1.87638E−02	

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6.1 the modelization This modelization

is identical to the modelization C, with models of the type NRAD (not radiality). The results of models VISC_MEMO_NRAD and VISC _CIN2 _NRAD can be compared with those of the modelization C, since the effect of nonradiality must be inoperative here. The tests of VMIS_MEMO_NRAD, VMIS _CIN2_NRAD (without viscosity) are of non regression. Quantities tested and

6.2 Urgent results Behavior

VISC_MEMO _NRAD Identification

Urgent	Reference	VISC_CIN2_MEMO Behavior
σ_{xx}	113.5	369.679
ε_{xx}	113.5	−1.8773E−02

VISC_CIN 2_NRAD Identification

Urgent	Reference	VISC_CIN2_CHAB Behavior
σ_{xx}	113.5	269.6
σ_{xx}	10	220

VMIS_MEMO _NRAD Identification

Reference	(non regression	) (
σ_{xx}	113.5	372.2 analytical) Behavior
σ_{xx}	10	220 Urgent

VMIS_CIN 2_NRAD Identification

Reference	VISC_CIN2_MEMO	(non regression) (
σ_{xx}	113.5	225.254 analytical) Summary
σ_{xx}	10	220 of the results

7 the four modelizations

make it possible to validate, on a material point, the behaviors of the kinematical type nonlinear for purpose of memory, in plasticity and viscoplasticity.