COMP001 – Test of elastoplastic behaviors. Simulation in a Summarized

material point:

This test implements a simulation of a way of loading in stresses or strains in a material point, i.e. on a model such as the stress states and of strains are homogeneous at any moment. It thus makes it possible to test a certain number of elastoplastic models of behavior, with an aim of checking the robustness of their numerical integration, their insensitivity compared to a change of units, the good taking into account of the command variables whose the coefficients depend on the model, invariance compared to a total rotation applied to the problem, the accuracy of the tangent matrix.

Modelization a: this modelization makes it possible to the model validate VMIS_ISOT_LINE in 3D and C_PLAN.
Modelization b: this modelization makes it possible to the model validate VMIS_ISOT_TRAC in 3D and C_PLAN.
Modelization C: this modelization makes it possible to validate VMIS_CINE_LINE in 3D the model.
Modelization D: this modelization makes it possible to the model validate VMIS_ECMI_LINE in 3D and C_PLAN.
Modelization E: this modelization makes it possible to the model validate VMIS_ECMI_TRAC in 3D and C_PLAN.
Modelization F: this modelization makes it possible to validate VMIS_CIN1_CHAB in 3D the model.
Modelization G: this modelization makes it possible to validate VMIS_CIN2_CHAB in 3D the model.
Modelization H: this modelization makes it possible to validate VMIS_ISOT_PUIS in 3D the model.

1 Problem of reference

1.1 Geometry

the geometry (generated automatically in macro-command SIMU_POINT_MAT [U4.51.12] is single and simple: it acts in 3D of a tetrahedron on side 1, and in 2D of a triangle on side 1, with the nodes of which one applies linear relations to obtain a stress state and of homogeneous strain.

1.2 Properties of the material

the characteristics of the materials are defined for each behavior via command DEFI_MATERIAU. The elastic characteristics and of isotropic hardening selected are those of standard steel 16MND5 :

- $E=200\,000\,MPa$
- $\nu=0.3$
- $\sigma_y=437\,MPa$.

The other parameters describing the models were selected starting from the cases test of ASTER. The two following tables and the summarize all the models of code ASTER considered associated parameters:

Modeling	elastoplastic models of code_Aster	parameters selected	criteria retained for the choice of the parameters
A	VMIS_ISOT_LINE	$SY=437\,MPa$, $DSY=2024\text{MPa}$	Material characteristics 16MND5
B	VMIS_ISOT_TRAC	curve of tension with $100°C$ $16\,MND5$	Material characteristics 16MND5
C	VMIS_CINE_LINE	$SY=437\,MPa$, $DSY=2024\text{MPa}$	Material characteristics 16MND5
D	VMIS_ECMI_LINE	$SY=437\,MPa$ $DSY=2024\text{MPa}$ $C_{PRAG}=1486.9$.	Material characteristics 16MND5
E	VMIS_ECMI_TRAC	curve of tension with $100°C$ $16\,MND5$ $C_{PRAG}=1486.9$.	Material characteristics 16MND5
F	VMIS_CIN1_CHAB	$SY=437.0$; $Rinf=758.0$; $b=2.3$; $Cinf=63767.0$ $Gamma0=341.0$	hardening: *données16MND5* other parameters: ssnv101c
G	VMIS_CIN2_CHAB	$SY=437.0$; $Rinf=758.0$; $b=2.3$; $C1inf=63767.0/2.0$ $C2inf=63767.0/2.0$ $Gam1=341.0$ $Gam2=341.0$	Hardening given 16MND5 other parameters ssnv101c kinematical Choice $X1+X2=X$ of VMIS_CIN1_CHAB
H	VMIS_ISOT_PUIS	$SY=437.0$; $APUI=1.3$ $NPUI=3.5$	

1.3 Boundary conditions and loadings

1.3.1 Characteristic of the ways of loading

Two ways of loading were defined to treat 3D case and the 2D plane one. They are common to all the constitutive laws. Each one of them respects, the following criteria:

- a plastic strain cumulated p , from 4 to 5% on the group of the way,
- an increase of 1% of p during a portion of the way,

This calibration was carried out on model `VMIS_ISOT_LINE`, then deferred on the other models.

The loading suggested varies in a way decoupled each component of the tensor of the strains by successive stage. One proposes a cyclic way charges discharge with it by covering the states with tension and compression as well as an inversion with the signs with the shears in order to test a broad range of values.

Schematically, it follows a path on 8 segments $[O-A-B-C-O-C'-B'-A'-O]$ where the second part of way [O-C'-B'-A'-O] is symmetric compared to the origin of the first $[O-A-B-C-O]$.

1.3.2 Application of the requests

One under investigation brings back material point (by means of macro-command `SIMU_POINT_MAT)` by requesting a homogeneous element of way while imposing:

- in 3D, 6 components of the strain tensor:

$$\bar{\varepsilon}=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz}\\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz}\end{bmatrix}$$

- into 2D the three components of the tensor

$$\bar{\varepsilon}=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy}\\ \varepsilon_{xy} & \varepsilon_{yy}\end{bmatrix}$$

For a more general writing, the tensor of the strains imposed will be broken up into a hydrostatic and deviatoric part on bases of shears:

$$\bar{\varepsilon}=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy}\\ \varepsilon_{xy} & \varepsilon_{yy}\end{bmatrix}=p\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}+d\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}+\varepsilon_{xy}\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix} \text{ in 2D,}$$

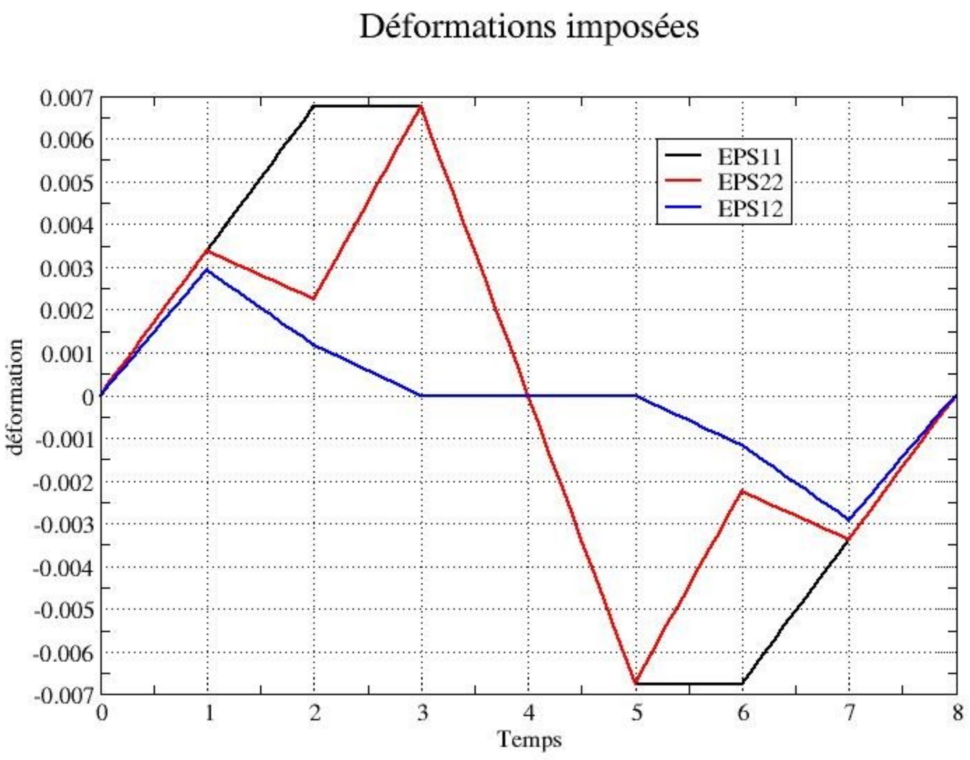
$$\bar{\varepsilon}=\begin{bmatrix}\varepsilon_{xx} & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{xy} & \varepsilon_{yy} & \varepsilon_{yz}\\ \varepsilon_{xz} & \varepsilon_{yz} & \varepsilon_{zz}\end{bmatrix}=p\begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}+d_1\begin{bmatrix}1 & 0 & 0\\ 0 & -1 & 0\\ 0 & 0 & 0\end{bmatrix}+d_2\begin{bmatrix}0 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & -1\end{bmatrix}+\begin{bmatrix}0 & \varepsilon_{xy} & \varepsilon_{xz}\\ \varepsilon_{xy} & 0 & \varepsilon_{yz}\\ \varepsilon_{xz} & \varepsilon_{yz} & 0\end{bmatrix} \text{ in 3D}$$

1.3.3 Description of the way of strain imposed in 2D

the way applied is described in the table below, the values of strains are gauged with respect to the elastic modulus:

times	1	2	3	4	5	6	7	8
Point of loading	A	B	C	O	C'	B'	A'	O
$\varepsilon_{xx} \times E$	675	1350	1350	0	-1350	-1350	-675	0.675.450
$\varepsilon_{yy} \times E$			1350	0	-1350	-450	-675	0.450.180
$\varepsilon_{xy} \times E/(1+\nu)$			0	0	0	-180	-450	0.675.900
P			1350		-1350	-900	-675	0
D	0	0.450		0	0	-450	0	0

This way is illustrated by the following graph:



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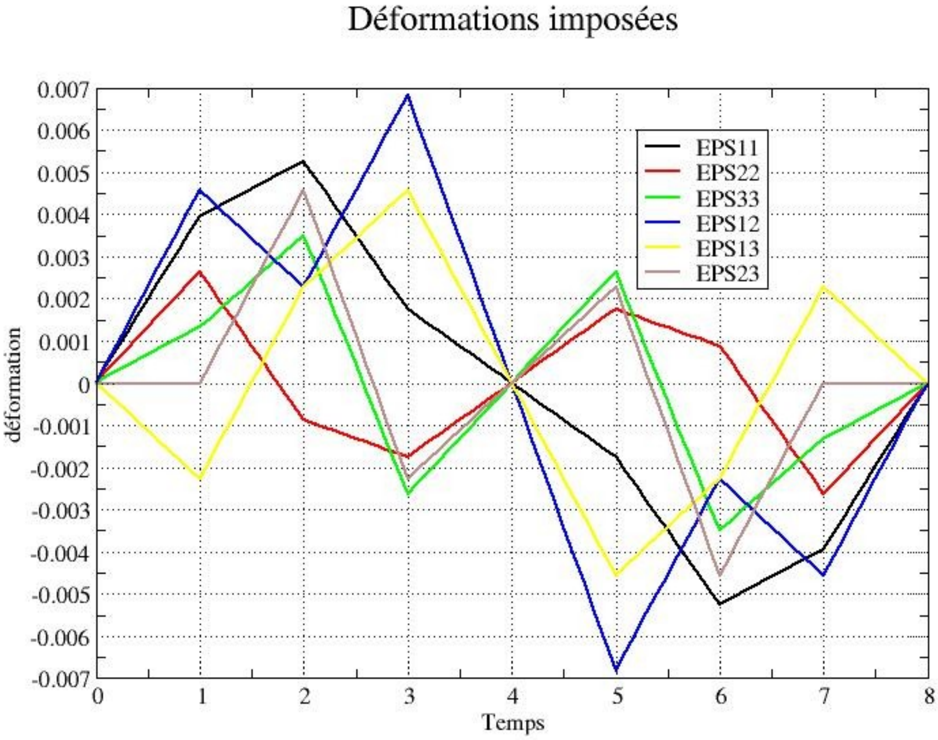
1.3.4 Description of the way of strain imposed in 3D

the way applied is described in the table below, the values of strains applied are gauged with respect to the elastic modulus:

N° segment	1	2	3	4	5	6	7	8
Segment	$0-A$	$A-B$	$B-C$	O	C'	B'	A'	O
$\varepsilon_{xx}\times E$	787.5	1050	350	0	-350	-1050	-787.5	0
$\varepsilon_{yy}\times E$	525.0	-175	-350	0.350.175			525	0
$\varepsilon_{zz}\times E$	262.5	700	-525	0.525		-700	-262.5	0.700.350
$\varepsilon_{xy}\times E/(1+\nu)$			1050	0	-1050	-350	-700	0
$\varepsilon_{xz}\times E/(1+\nu)$	-350	350.700		0	-700	-350	700	0
$\varepsilon_{yz}\times E/(1+\nu)$	0.700		-350	0.350		-700	0	0.525.525
P			-175	0.175		-525	-525	0
$d1$	262.5	525.525		0	-525	-525	-262.5	0
$d2$	262.5	-175	350	0	-350	175	-262.5	0

This way is illustrated by the following graph:



1.4 Forced

initial conditions and null strains.

2 Reference solution

This test proceeds, for each modelization, with an intercomparison between the reference solution (obtained with one time step very fine), the solution with a fairly coarse discretization, the solution with effect of the temperature (or another command variable), the solution by changing the system of units (Pa into MPa), and that obtained after rotation or symmetry.

2.1 Definition of the cases tests of robustness

One proposes 3 angles of analysis to test the robustness of the integration of the constitutive laws:
- studies of equivalent problems
- checking of the tangent matrix
- study of the discretization of time step

For each one of them, one studies the evolution the relative differences between several computations using the same model but presenting parameters or different computation options. The operating relates to the invariants of the tensor of the stresses: trace tensor, stress of Von-Put and the local variables of scalar nature: generally it is cumulated plasticity.

The total convergence criteria are the values envisaged by default by ASTER. (`RESI_GLOB_RELA=10-6, ITER_GLOB_MAXI=10`). One adopted a usual diagram of Newton for the reactualization of the tangent matrix: with each converged increment (`REAC_INC=1`) and all the 1 iterations (`REAC_ITER=1`).

2.2 Studies of equivalent problems

For a coarse discretization of the ways: 1 time step for each segment of the way, the solution obtained for each model is compared with 3 strictly equivalent problems for the state of the material point:
- Tpa , even way with a change of unit, one substitutes to them Pa for MPa in the data materials and the possible parameters of the model
- $Trot$, way by imposing the same tensor $\bar{\varepsilon}$ after a rotation: ${}^t R \cdot \bar{\varepsilon} \cdot R$ where R is a matrix of rotation. For the case 2D, the swing angle will be $\alpha = 0.9\, radian$, for the configuration 3D, one chose the Eulerian angles with the arbitrary values $\{ \Psi = 0.9\, radian$ $\theta = 0.7\, radian$, and $\varphi = 0.4\, radian \}$
- $Tsym$, way by imposing the tensor $\bar{\varepsilon}$ after a symmetry: permutation of the axes x and y in 2D, permutation of x in y , y z and z x of 3D.

For each one of these problems, the solution (invariants of the stresses, cumulated equivalent plastic strain) must be identical to the basic solution, obtained with the same discretization in time. The value of reference of the variation is thus 0. That means in practice that the found variation must be about the machine accuracy is approximately $1.\mathrm{E}-15$.

2.3 Test of the tangent matrix

One also tests for each behavior the tangent matrix, by difference with the matrix obtained by disturbance. There still, the value of reference is 0.

2.4 Study of the discretization of time step (A2)

One studies the behavior of the integration of the models according to the discretization. For the same modelization, therefore a given behavior, one studies several different discretizations in time here, while multiplying by 5 the number of steps of the way of loading. In the reference [1], the discretization is pushed up to 3125 increments per segment on the same principle. Here, to limit the period of the tests, one limits oneself to 3 successive refinements. This led to the following discretization:

Number of interval per segment of loading	1	5	25
Number of total step on the group of the way	8	40.200	
Computation	$T0$	$T1$	$Tréf$ reference solution

the reference solution $Tréf$, that is obtained for $N=25$, that is to say 200 steps for the totality of the way. These various solutions make it possible time step to judge sensitivity to large and robustness of integration. To reveal the velocity of convergence according to time step, one defers here the solutions put forward in [1], up to 3125 time step for each of the 8 segments of the way of loading.

2.4.1 VMIS_LINE

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.70e-02	1.38e-02	3.37e-03	6.82e-04	1.14e-04	0.00e+00
$VMIS$	4.34e-03	1.86e-03	4.72e-04	9.72e-05	1.64e-05	0.00e+00
$TRAC$	1.19e-01	6.89e-02	1.70e-02	3.45e-03	5.80e-04	0.00e+00

2.4.2 VMIS_ISOT_TRAC

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.58e-02	1.34e-02	3.26e-03	6.60e-04	1.11e-04	0.00e+00
$VMIS$	6.38e-03	2.38e-03	5.81e-04	1.18e-04	2.00e-05	0.00e+00
$TRAC$	1.20e-01	7.69e-02	1.67e-02	3.30e-03	5.53e-04	0.00e+00

2.4.3 VMIS_CINE_LINE

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$VMIS$	3.91e-03	1.05e-03	2.16e-04	4.15e-05	6.91e-06	0.00e+00
$TRAC$	7.48e-14	7.44e-14	7.44e-14	7.69e-14	5.87e-14	0.00e+00

2.4.4 VMIS_ECMI_LINE

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.71e-02	1.39e-02	3.40e-03	6.88e-04	1.15e-04	0.00e+00
$VMIS$	3.63e-03	2.00e-03	4.79e-04	9.53e-05	1.61e-05	0.00e+00
$TRAC$	1.64e-01	9.16e-02	2.18e-02	4.33e-03	7.30e-04	0.00e+00

2.4.5 VMIS_ECMI_TRAC

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.70e-02	1.38e-02	3.38e-03	6.84e-04	1.15e-04	0.00e+00
$VMIS$	2.36e-03	1.18e-03	2.98e-04	6.00e-05	1.01e-05	0.00e+00
$TRAC$	1.46e-01	8.51e-02	2.13e-02	4.31e-03	7.22e-04	0.00e+00

2.4.6 VMIS_CIN1_CHAB

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.32e-02	1.12e-02	2.57e-03	5.10e-04	8.52e-05	0.00e+00
$VMIS$	9.04e-02	3.24e-02	7.45e-03	1.48e-03	2.49e-04	0.00e+00
$TRAC$	3.34e-14	3.31e-14	3.27e-14	3.48e-14	3.86e-14	0.00e+00

2.4.7 VMIS_CIN2_CHAB

Variations	$N1$	$N5$	$N25$	$N125$	$N625$	$N3125$
$V1_N$	3.32e-02	1.12e-02	2.57e-03	5.10e-04	8.52e-05	0.00e+00
$VMIS$	9.04e-02	3.24e-02	7.45e-03	1.48e-03	2.49e-04	0.00e+00
$TRAC$	3.72e-14	3.69e-14	3.69e-14	3.83e-14	4.75e-14	0.00e+00

2.5 REFERENCES

1) P.LEVASSEUR: “Applicative Third party maintenance of the code _Aster” Checking of the robustness and the reliability of the integration of constitutive laws in ASTER. Ratio PRINCIPIA RET.693.127.01 December 2006.

3 Modelization A

3.1 Characteristic of the modelization

the behavior tested is VMIS_ISOT_LINE , in 3D and C_PLAN .

3.2 Quantities tested and results

Modelization C_PLAN :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0.3.4		1	0
VMIS	0	0	0.0.4. 0.1			0
TRACE	0	0	0	0	0	0

Modelization 3D :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0.2.5. 0.9			0
$VMIS$	0	0	0.0.4. 0.2			0
$TRACE$	0	0	0	0	0	0

tangent Matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	2.E-9

4 Modelization B

4.1 Characteristic of the modelization

the behavior tested is VMIS_ISOT_TRAC , in 3D and C_PLAN .

4.2 Quantities tested and results

Modelization C_PLAN :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0.3.3		1	0
$VMIS$	0	0	0.0.6. 0.2			0
$TRACE$	0	0	0	0	0	0

Modelization 3D:

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0.2.5. 0.9			0
$VMIS$	0	0	0.0.3. 0.1			0
$TRACE$	0	0	0	0	0	0

tangent Matrix:

Variations	$N25$
$Max(Ktgte - Kpert)$	1.6 E-9

5 Modelization C

5.1 Characteristic of the modelization

the behavior tested is `VMIS_CINE_LINE` , in 3D.

5.2 Quantities tested and Modelization

results `3D`:

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$VMIS$	0	0	0.0.4		0.08	0
$TRACE$	0	0	0	0	0	0

tangent Matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	7.7 E-10

6 Modelization D

6.1 Characteristic of the modelization

the behavior tested is `VMIS_ECMI_LINE` , in 3D and `C_PLAN` .

6.2 Quantities tested and results

Modelization `C_PLAN` :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0.3.4. 1.1			0
$VMIS$	0	0	0.0.3. 0.2			0
$TRACE$	0	0	0	0	0	0

Modelization `3D`:

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0.2.4. 0.9			0
$VMIS$	0	0	0.0.4. 0.1			0
$TRACE$	0	0	0	0	0	0

tangent Matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	1. E-9

7 Modelization E

7.1 Characteristic of the modelization

the behavior tested is VMIS_ECMI_TRAC , in 3D and C_PLAN .

7.2 Quantities tested and results

Modelization C_PLAN :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0.3.4. 1.1			0
$VMIS$	0	0	0.0.2		0.08	0
$TRACE$	0	0	0	0	0	0

Modelization 3D :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0.2.5. 0.9			0
$VMIS$	0	0	0.0.4		0.09	0
$TRACE$	0	0	0	0	0	0

tangent Matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	2.6 E-9

8 Modelization F

8.1 Characteristic of the modelization

the behavior tested is `VMIS_CIN1_CHAB` , in 3D.

8.2 Quantities tested and Modelization

results `3D` :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0.3.1. 0.9			0
$VMIS$	0	0	0	8	2	0
$TRACE$	0	0	0	0	0	0

tangent Matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	0.031

9 Modelization G

9.1 Characteristic of the modelization

the behavior tested is VMIS_CIN2_CHAB , in 3D.

9.2 Quantities tested and Modelization

results 3D :

Variations (%)	T_{Pa}	T_{sym}	T_{rot}	$N1$	$N5$	$N25$
$V1_P$	0	0	0.3.1. 0.9			0
$VMIS$	0	0	0	8.2.5		0
$TRACE$	0	0	0	0	0	0

tangent Matrix:

Variations	$N25$
$Max(Ktgte-Kpert)$	0.031

10 Synthesis

For all the elastoplastic behaviors tested in the modelizations A with G, the results are satisfactory:

- the results are valid during a physical change of unit of the problem (Pa in Mpa), or following a rotation or a symmetry of the loading
- the results converge correctly with time step, and the diagrams of integration (implicit for the studied elastoplastic behaviors) are robust, since they make it possible to use large time step
- tangent matrixes are correct because similar to the matrixes tangent calculated by disturbance.