SSNS107 – Roll with reinforcements under pressure

Summarized:

One considers a cylinder with reinforcements under pressure. More exactly, a hollow roll, 3D, is supplemented on its external face by a circumferential reinforcement (skin 2D). This structure is charged by an internal pressure. The computation is elastic linear.
Simulation is compared with an analytical solution in order to validate the modelizations of reinforcements of the type GRILLE_MEMBRANE on a structure.

All the numerical values are given in unit IF.

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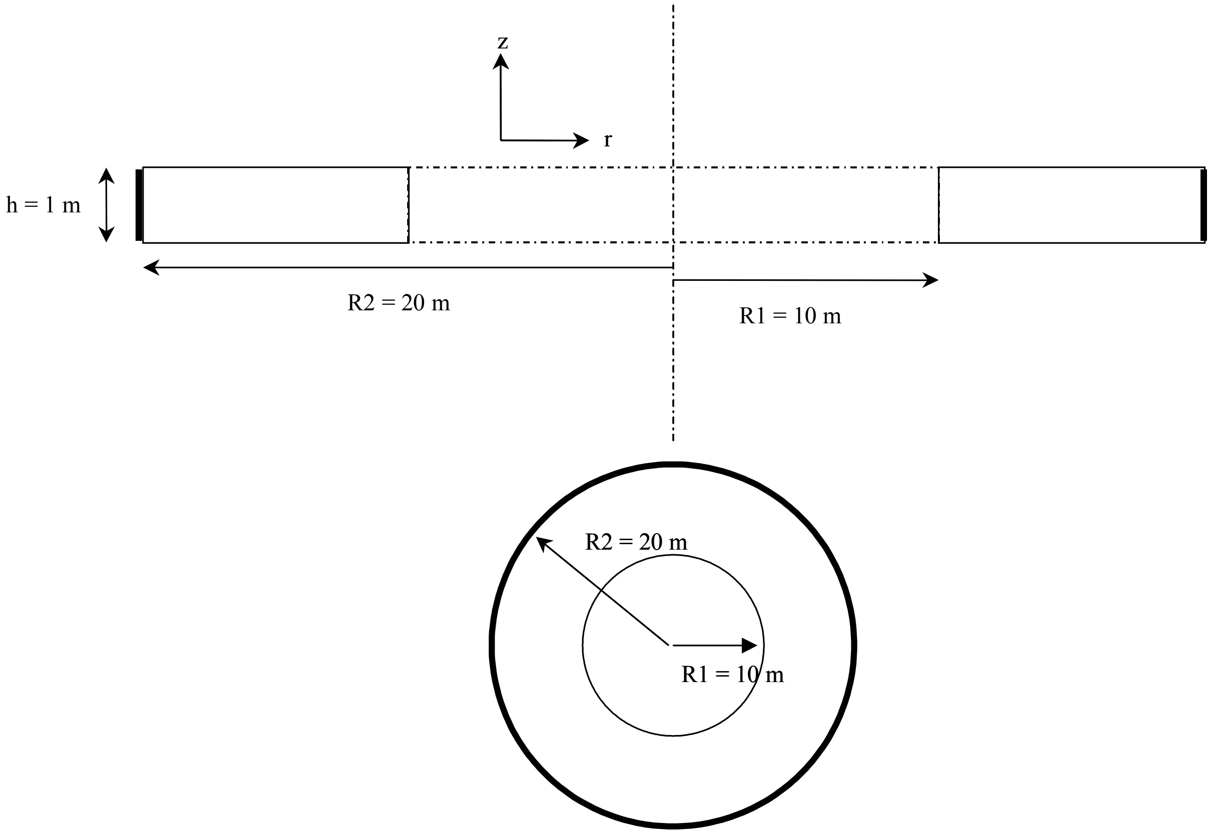
1 Problem of reference

1.1 Geometry



the cylinder has an interior radius $R_1 = 10\,m$, an external radius $R_2 = 20\,m$, a height $h = 1\,m$. Circumferential reinforcement has a section per unit of length of $e = 1\,m^2 / ml$.

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1.2 Properties of the materials

the characteristic materials of the concrete (hollow roll) are $E=2\text{E}+10\,Pa$ and $\nu=0.2$; those of steel (circumferential reinforcement) $E=2\text{E}+11\,Pa$ $\nu=0$, $E_t=2.\text{E}+09$ (tangent modulus), $\sigma_y=2.\text{E}+11\,Pa$ (forced very large plasticity to remain in the elastic domain).

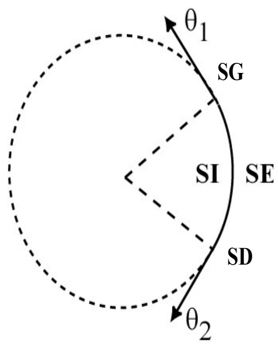
The three-dimensions function of reinforcement has its vertical principal direction and a section per linear meter of $1\,m$.

1.3 Boundary conditions and loading

One is interested only in one angular section of the cylinder.

The following conditions are imposed:

1) a normal displacement no one on the side sides of the angular section of cylinder (SG and SD on the schematic figure) (ortho-radial displacement no one)
2) a vertical displacement no one on the low face of the section of the cylinder (simple bearing)
3) a pressure imposed of $10^6\,Pa$ on the inner face of the section of the cylinder (SI) on the schematic figure) (pressure interns)



schematic Designation of surfaces of the cylinder

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2 Reference solutions

For the solution without reinforcement, and seen symmetries of the problem, displacement can be written in the form:

$$U(r,z)=g(r)u_r+h(z)u_z$$

With:

$$g(r)=(a+3b)\frac{r}{2}+\frac{c}{r}$$
$$h(z)=-az$$

$$\varepsilon_{rr}(r)=\frac{1}{2}(a+3b)-\frac{c}{r^2}$$
$$\varepsilon_{\theta\theta}(r)=\frac{1}{2}(a+3b)+\frac{c}{r^2}$$
$$\varepsilon_{zz}(r)=-a$$

$$\sigma_{rr}(r)=A-2\mu\frac{c}{r^2}$$
$$\sigma_{\theta\theta}(r)=A+2\mu\frac{c}{r^2}$$
$$\sigma_{zz}(r)=B$$

The boundary conditions then make it possible to clarify the constants:

$$\sigma_{rr}(R_1)=-P_1$$
$$\sigma_{rr}(R_2)=-P_2$$
$$\sigma_{zz}=0$$

with R_2 the radius on the level of the interface enters the concrete and the circumferential three-dimensions function of reinforcements.

$$A=\frac{P_1\,R_1^2-P_2\,R_2^2}{R_2^2-R_1^2}$$
$$B=0$$
$$2\mu c=R_1^2R_2^2\,\frac{P_1-P_2}{R_2^2-R_1^2}$$
$$b=\frac{2(P_1R_1^2-P_2R_2^2)}{3(3\lambda+2\mu)(R_2^2-R_1^2)}$$
$$a=\frac{\lambda(P_1R_1^2-P_2R_2^2)}{\mu(3\lambda+2\mu)(R_2^2-R_1^2)}$$

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One gives the solution with non-zero P_2 here because that will be useful in the continuation.

If there is circumferential reinforcement on the external face of the cylinder, this reinforcement puts pressure. It is thus necessary to determine this pressure (function of displacement), then to apply the preceding results.

One applies the “formula of coppersmiths” (presence of reinforcements):

$$P=\sigma_{\theta\theta}^{armature}\frac{e}{2R_2}$$

To connect the stress to solution displacement, one passes by the strain (strain of reinforcements being strain of the cylinder):

$$\mathrm{P}=\frac{\mathrm{E}^{\mathrm{armature}}\mathrm{e}}{2\mathrm{R}_2}\left[\frac{\mathrm{P}_1\mathrm{R}_1^2-\mathrm{P}_2\mathrm{R}_2^2}{(3\lambda+2\mu)(\mathrm{R}_2^2-\mathrm{R}_1^2)}\left(\frac{\lambda}{2\mu}+1\right)+\frac{\mathrm{R}_1^2}{\mathrm{R}_2^2-\mathrm{R}_1^2}\frac{\mathrm{P}_1-\mathrm{P}_2}{2\mu}\right]$$

By applying the preceding results with $P_2=P$, one obtains the sought solution.

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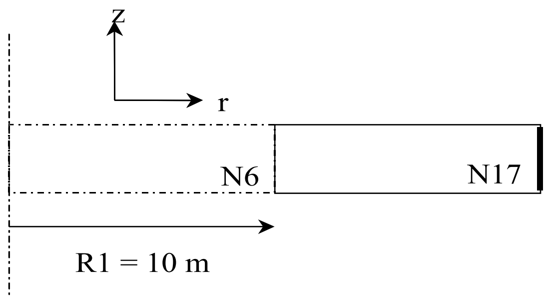
3 Modelization With

the modelization relates to only one angular sector of the cylinder, an element in the section, an element in the height, 100 elements enters R_1 and R_2

3.1 Values tested

One tests the values of radial displacements of the points located on the interior face of the concrete cylinder ($N6$ on figure Ci below) and on the outside of the cylinder of concrete ($N17$ on the figure below) and one compared to the analytical solution.

The cylinder is with a grid with elements TRIA3.



Node	Reference
$N6$	6.76923E-04
$N17$	3.8462E-04

4 Modelization B

The modelization B is identical to modelization A. Seuls the elements used for the mesh B change (QUAD4 instead of TRIA3

) the results of the modelization are the following:

Node	Reference
$N6$	6.76923E-04
$N17$	3.8462E-04

5 Modelization C

The modelization C is identical to modelization A. Seuls the elements used for the mesh C change (quadratic elements instead of linear

) the results of the modelization are the following:

Node	Reference
$N6$	6.76923E-04
$N17$	3.8462E-04

6 Modelization D

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The modelization D is identical to the modelization B. Only the elements used for the mesh D change (quadratic elements instead of linear

) the results of the modelization are the following:

Node	Reference
$N6$	6.76923E-04
$N17$	3.8462E-04

7 Synthesis

the various modelizations of this case test validate behavior `GRILLE_MEMBRANE` for a structure supplements (cylinder with reinforcement)

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