SSNS100 - Behavior nonlinear of a three-dimensions function of reinforcements under thermal loading

Abstract:

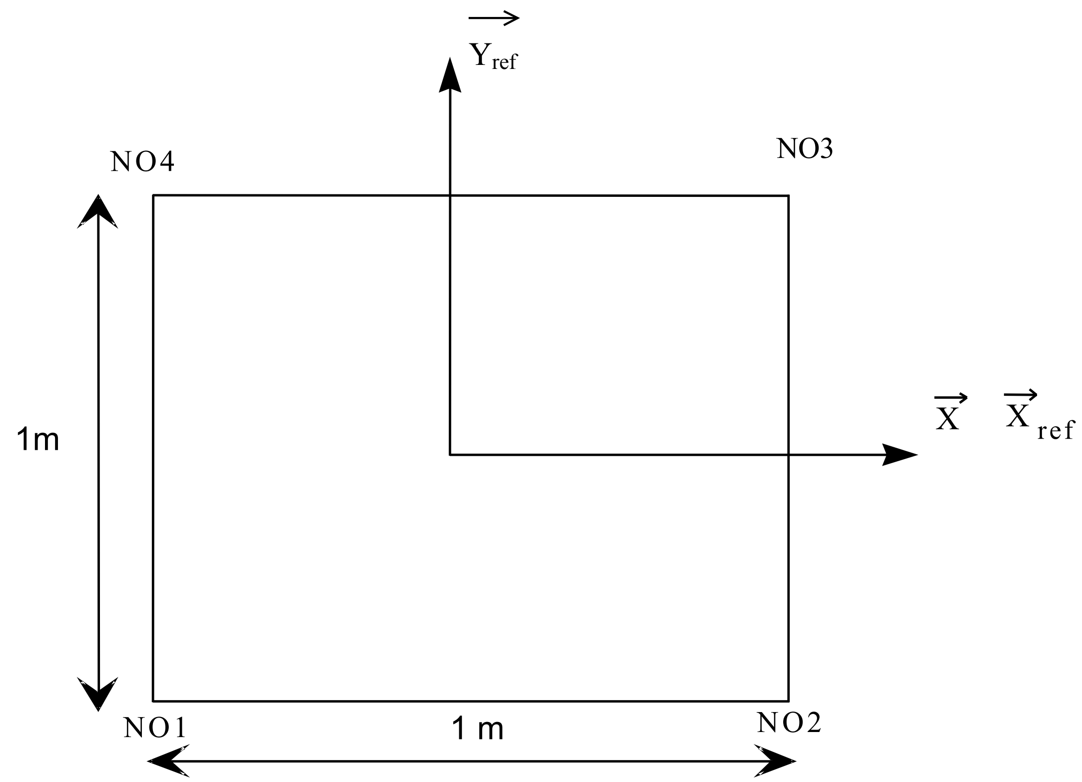
A three-dimensions function of reinforcements not offset compared to the average average and embedded on its four sides is subjected to a thermal loading. The directional senses of reinforcements are confused with the axes (X, Y) of the total reference.

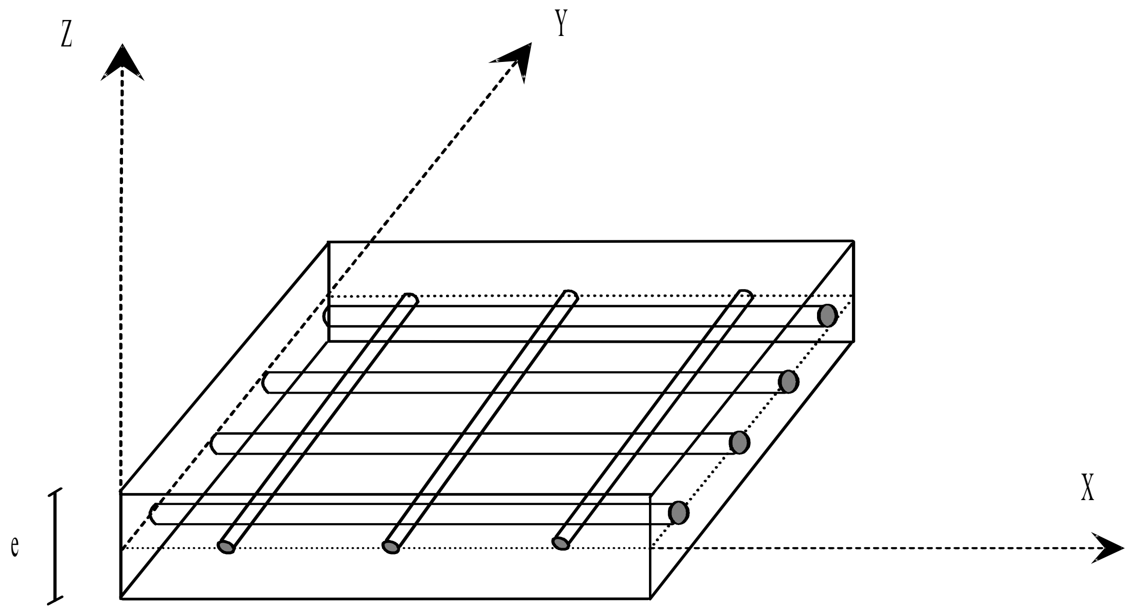
The principal interest of this test is to validate the numerical integration of the models of behavior élasto - plastic `GRILLE_ISOT_LINE`, `GRILLE_CINE_LINE` and `GRILLE_PINTO_MEN` of a three-dimensions function of reinforcements associated with the finite element `GRILLE_EXCENTRE` (plate with eccentring compared to the datum-line), in general algorithm `STAT_NON_LINE`.

In order to obtain reference solutions, analytical solutions were established for two elastoplastic behaviors with linear and kinematical hardening isotropic linear. The behavior of Pinto-Menegotto is validated by non regression numerical results got with *Aster* in version 5-3 (cf [§1.3.3]).

1 Problem of reference

1.1 Geometry of the plate





1.2 Characteristics of the modelizations

This case test is composed of 8 modelizations. The table below summarizes their characteristics:

1) Directional sense of reinforcements: longitudinal direction (L) : OX; transverse direction (T) : OY
2) Eccentring = $0\,m$
3) Section per meter linear = $0{,}01\,m^2/ml$ (even thickness in the directions transversal and longitudinal in the event of presence of transverse reinforcements)

Modelization	Constitutive law	Presence of transverse reinforcement	Application mode of the linear
temperature	A isotropic	Yes	to the nodes
B	kinematical linear	Yes	with the nodes
C	Pinto Menegotto	Yes	with the nodes
D	Pinto Menegotto	Yes	with the elements
E	Pinto Menegotto	Not	with the elements
F	isotropic linear	Not	with the nodes
G	kinematical linear	Not	with the nodes
H	Pinto Menegotto	Not	***

*** For the case test H, the loading in temperature is replaced by a mechanical loading (displacement imposed on the nodes).

1.3 Properties of the materials

1.3.1 Properties common to all the modelizations

Modulus Young: $E=2.10^{11}\,MPa$

Poisson's ratio: $\nu=0$

Elastic limit: $\sigma_y=2.10^8\,MPa$

Thermal coefficient of thermal expansion: $\alpha=10^{-5}(°C^{-1})$

1.3.2 Behavior plastic isotropic and kinematics

For the behaviors isotropic (`GRILLE_ISOT_LINE`) and kinematics (`GRILLE_CINE_LINE`)
Slope of hardening: $E_T=2.10^{10}\,MPa$

1.3.3 Behavior of Pinto Menegotto

For behavior PINTO MENEGOTTO (`GRILLE_PINTO_MEN`)

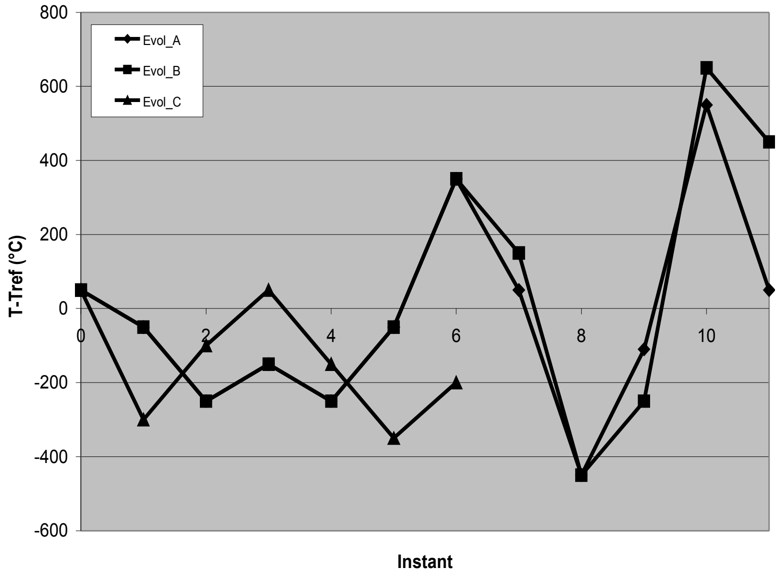
`EPSI_ULTM`	:	3,0. 10-2
`SIGM_ULTM`	:	2,58. 108
`EPSP_HARD`	:	0,0023
`R_PM`	:	20,0
`EP_SUR_E`	:	0,01
`A1_PM`	:	18,5
`A2_PM`	:	0,15
`ELAN`	:	4,9
`A6_PM`	:	620,0
`C_PM`	:	0,5
`A_PM`	:	0,008

1.4 Boundary conditions and loading

the plate is entirely embedded for the modelizations A with G (thermomechanical). For the modelization H (mechanical), one blocks all displacements and all rotations with the nodes safe UX for the nodes $NO2$ and $NO3$.

The loading is of origin thermal for the modelizations A with G. the change of the temperature according to time is given for each modelization in the following table. The temperature is applied to the nodes or the elements, according to the modelization.

Time	Evolution A $T°$	Evolution B $T°$	Evolution C $T°$
0	50	50	50
1	– 50	– 50	– 300
2	– 250	– 250	– 100
3	– 150	– 150	50
4	– 250	– 250	– 150
5	– 50	– 50	– 350
6.350.350			– 200
7	50.150		
8	– 450	– 450	
9	– 110	– 250	
10.550.650			
11	50.450		



One took for all the tests a reference temperature of $50°$.

For the modelization H, one applies a nodal force FX to the nodes $NO2$ and $NO3$ (directed according to the vector UX) by controlling computation by displacement UX of $NO3$ so that it follows the following evolution:

Time	0.001	0.0023	0.03	0.2.0.4.6.4			7.92
Ux (m)	0.001	0.0023	0.03	0.0296667	0.02	-0.04	-0.033

Time	17	19	20	21	22	25	50
Ux (m)	0.01	0.03	0.0298	0.026	0.022	0.025	0.05

2 isotropic Reference solutions

2.1 Behavior plastic

the reference solution is calculated analytically.

One notes T^+, ε_p^+, p^+, ε^+ the temperature, the plastic strain, the cumulated plastic strain and the total deflection at the time of computation, and the T^-, ε_p^-, p^- same quantities at previous time. $T_{réf}$ indicate the reference temperature.

The solution is calculated in the following way:

$$\sigma_e = E\left[\varepsilon^+ - \alpha\left(T^+ - T_{ref}\right) - \varepsilon_p^-\right]$$

$$R(p) = \frac{E.E_T}{E - E_T}\,p + \sigma_y$$

si $|\sigma_e| \le R(p^-)$

$$\varepsilon_p^+ = \varepsilon_p^-\,;\, p^+ = p^-\,;\, \sigma^+ = \sigma_e$$

sinon

si $\sigma_e > R(p^-)$

$$\varepsilon_p^+ = \frac{E - E_T}{E}\left[\varepsilon^+ - \alpha\left(T^+ - T_{ref}\right) - \frac{\sigma_y}{E}\right] + \frac{E_T}{E}\left[\varepsilon_p^- - p^-\right]$$

$$p^+ = p^- + \varepsilon_p^+ - \varepsilon_p^-$$

$$\sigma^+ = R(p^+)$$

sinon

$$\varepsilon_p^+ = \frac{E - E_T}{E}\left[\varepsilon^+ - \alpha\left(T^+ - T_{ref}\right) + \frac{\sigma_y}{E}\right] + \frac{E_T}{E}\left[\varepsilon_p^- + p^-\right]$$

$$p^+ = p^- - \varepsilon_p^+ + \varepsilon_p^-$$

$$\sigma^+ = -R(p^+)$$

This calculation is done in each direction. For the treated case, $\varepsilon^+ = 0$ at any moment.

The stress memorized in *Aster* is the real stress existing in each grid of this direction. The local variables (ε_p et p) are calculated starting from the equations above.

2.2 Behavior plastic kinematics

the reference solution is calculated analytically.

One notes $T^+ ,\varepsilon_p^+, \varepsilon^+, X^+$ the temperature, the plastic strain and the variable of kinematic hardening at the time of computation, and the $T^- ,\varepsilon_p^-\ X^-$ same quantities at previous time.

The solution is calculated in the following way:

$$\sigma_e = E\left[\varepsilon^+ - \alpha\left(T^+ - T_{ref}\right) - \varepsilon_p^-\right]$$

si $\left|\sigma_e - X^-\right| \le \sigma_y$

$$\varepsilon_p^+ = \varepsilon_p^- ; X^+ = X^- ; \sigma^+ = \sigma_e$$

sinon

si $\sigma_e - X^- > \sigma_y$

$$\varepsilon_p^+ = \frac{E - E_T}{E}\left[\varepsilon^+ - \alpha\left(T^+ - T_{ref}\right) - \frac{\sigma_y}{E}\right]$$

$$X^+ = E_T\left[\varepsilon^+ - \alpha\left(T^+ - T_{ref}\right) - \frac{\sigma_y}{E}\right]$$

$$\sigma^+ = E_T\left[\varepsilon^+ - \alpha\left(T^+ - T_{ref}\right)\right] + \frac{E - E_T}{E}\sigma_y$$

sinon

$$\varepsilon_p^+ = \frac{E - E_T}{E}\left[\varepsilon^+ - \alpha\left(T^+ - T_{ref}\right) + \frac{\sigma_y}{E}\right]$$

$$X^+ = E_T\left[\varepsilon^+ - \alpha\left(T^+ - T_{ref}\right) + \frac{\sigma_y}{E}\right]$$

$$\sigma^+ = E_T\left[\varepsilon^+ - \alpha\left(T^+ - T_{ref}\right)\right] - \frac{E - E_T}{E}\sigma_y$$

This calculation is done in each direction. For the treated case, $\varepsilon^+ = 0$ at any moment.
The stress memorized in *Aster* is the real stress existing in each grid of this direction.

2.3 Behavior Pinto Ménégotto

the reference solution is that obtained by an Aster computation with the same mesh to which one applies cycles of load/discharge, in imposed displacement, allowing to recreate the strains resulting from thermomechanical computations presented hereafter. The test corresponding is thus only one test of non regression, by comparing the stresses obtained by these two types of modelization: on the one hand mechanical, and on the other hand thermomechanical.

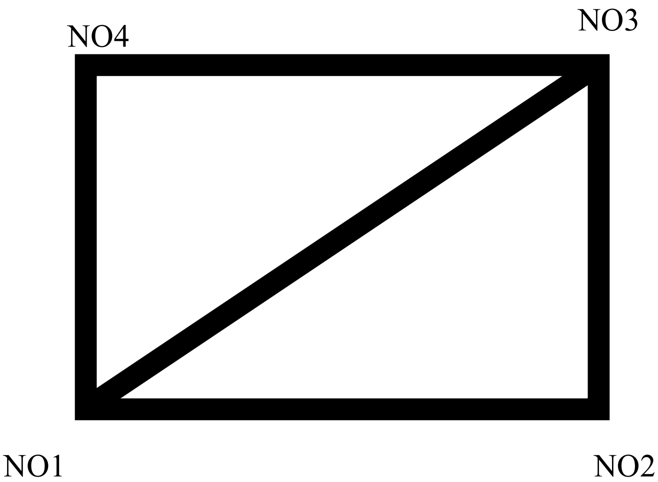
The stress memorized in *Aster* is the real stress existing in each grid of this direction.

3 Modelization With

the test-tube is with a grid with two elements TRIA3 with three nodes for all the modelizations.

- 2 elements grids to model fibers in the direction OX
- 2 elements grids to model fibers in the direction OY

the nodes of the elements are common for each three-dimensions function of reinforcement.



3.1 Modelization results A (linear isotropic hardening)

3.1.1 thermal Loading for the modelization A

Reference temperature: 50
History of the loading: $Evolution_A$ (cf [§1.4])
the temperatures returned like a field at nodes.

3.1.2 Results

One belongs to the values to $SIGXX$ the node $NO1$ (in the direction of longitudinal reinforcements) and $SIGXX$ to the node $NO1$ (in the direction of transverse reinforcements) and those to the Maximum and minimum of the local variables $V1$ and $V3$.

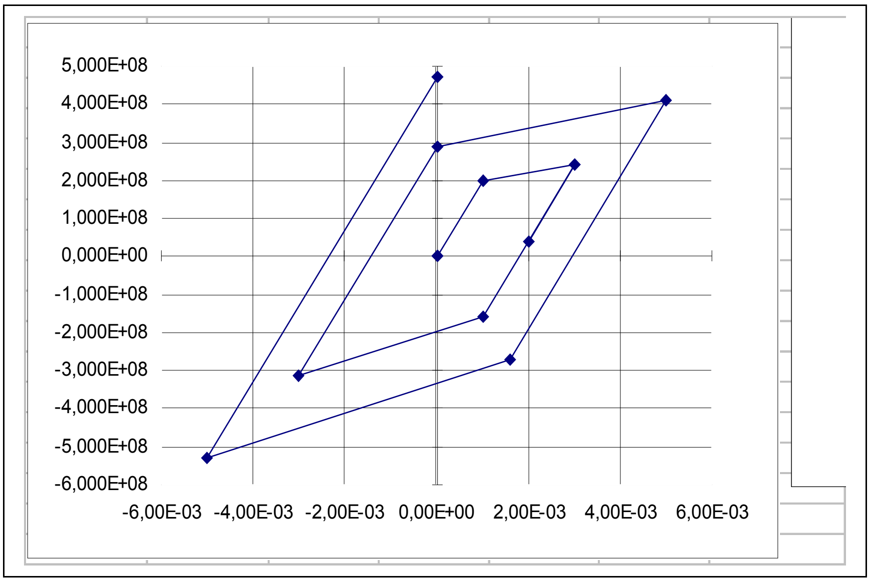
	$SIGXX$ in meshes modelling directed fibers following OX			$SIGXX$ in meshes modelling fibers directed according to OY		
Time	Reference	*Code_Aster*	Variation	Reference	*Code_Aster*	Variation
1	2,0000E+08	2,0000E+08	0	2,0000E+08	2,0000E+08	0
2	2,4000E+08	2,4000E+08	0	2,4000E+08	2,4000E+08	0
3	4,0000E+07	4,0000E+07	0	4,0000E+07	4,0000E+07	0
4	2,4000E+08	2,4000E+08	0	2,4000E+08	2,4000E+08	0
5	– 1,6000E+08	– 1,6000E+08	0	– 1,6000E+08	– 1,6000E+08	0
6	– 3,1200E+08	– 3,1200E+08	0	– 3,1200E+08	– 3,1200E+08	0
7	2,8800E+08	2,8800E+08	0	2,8800E+08	2,8800E+08	0
8	4,0960E+08	4,0960E+08	0	4,0960E+08	4,0960E+08	0
9	– 2,7040E+08	– 2,7040E+08	0	– 2,7040E+08	– 2,7040E+08	0
10	– 5,2768E+08	– 5,2768E+08	0	– 5,2768E+08	– 5,2768E+08	0
11	4,7232E+08	4,7232E+08	0	4,7232E+08	4,7232E+08	0

Time	Local variables	Minimum		Maximum	
		Code_Aster	Reference	*Code_Aster*	Reference
1	V1	0	0	0	0
	V3	0	0	0	0
2	V1	1.8E-03	1.8E-03	1.8E-03	1.8E-03
	V3	1.8E-03	1.8E-03	1.8E-03	1.8E-03
3	V1	1.8E-03	1.8E-03	1.8E-03	1.8E-03
	V3	1.8E-03	1.8E-03	1.8E-03	1.8E-03
4	V1	1.8E-03	1.8E-03	1.8E-03	1.8E-03
	V3	1.8E-03	1.8E-03	1.8E-03	1.8E-03
5	V1	1.8E-03	1.8E-03	1.8E-03	1.8E-03
	V3	1.8E-03	1.8E-03	1.8E-03	1.8E-03
6	V1	5.04E-03	5.04E-03	5.04E-03	5.04E-03
	V3	5.04E-03	5.04E-03	5.04E-03	5.04E-03
7	V1	5.04E-03	5.04E-03	5.04E-03	5.04E-03
	V3	5.04E-03	5.04E-03	5.04E-03	5.04E-03
8	V1	9.432E-03	9.432E-03	9.432E-03	9.432E-03
	V3	9.432E-03	9.432E-03	9.432E-03	9.432E-03
9	V1	9.432E-03	9.432E-03	9.432E-03	9.432E-03
	V3	9.432E-03	9.432E-03	9.432E-03	9.432E-03
10	V1	1.47456E-02	1.47456E-02	1.47456E-02	1.47456E-02
	V3	1.47456E-02	1.47456E-02	1.47456E-02	1.47456E-02
11	V1	1.47456E-02	1.47456E-02	1.47456E-02	1.47456E-02
	V3	1.47456E-02	1.47456E-02	1.47456E-02	1.47456E-02

Note:

The results presented are given in the reference of reference (X_{ref}, Y_{ref}) *which forms an angle from* $0°$ *ratio to* (X, Y) *.*

The case studied test corresponds to the following diagram in a plane forced strain:



4 Modelization B

Case with linear kinematic hardening.

4.1 Thermal loading for the modelization B

Reference temperature: 50°C
History of the loading: $Evolution_B$ (cf [§1.4)
the temperatures returned like a field at nodes.

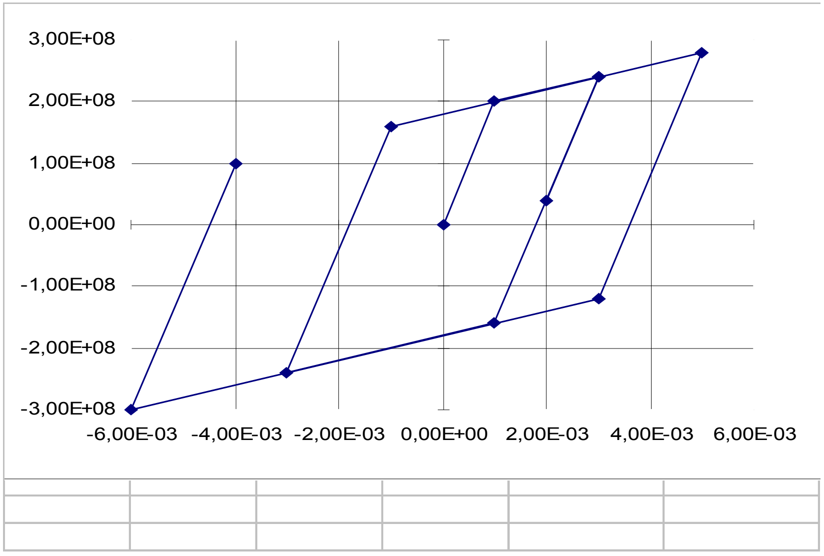
4.2 Results

One records the values of $SIGXX$ with the node $NO1$ (direction of longitudinal reinforcements) and $SIGXX$ with the node $NO1$ (direction of transverse reinforcements)

	SIGXX in meshes modelling directed fibers following OX			$SIGXX$ in meshes modelling fibers directed according to OY		
Time	Reference	*Code_Aster*	Variation	Reference	*Code_Aster*	Variation
1	2,00E+08	2,00E+08	0	2,00E+08	2,00E+08	0
2	2,40E+08	2,40E+08	0	2,40E+08	2,40E+08	0
3	4,00E+07	4,00E+07	0	4,00E+07	4,00E+07	0
4	2,40E+08	2,40E+08	0	2,40E+08	2,40E+08	0
5	– 1,60E+08	– 1,60E+08	0	– 1,60E+08	– 1,60E+08	0
6	– 2,40E+08	– 2,40E+08	0	– 2,40E+08	– 2,40E+08	0
7	1,60E+08	1,60E+08	0	1,60E+08	1,60E+08	0
8	2,80E+08	2,80E+08	0	2,80E+08	2,80E+08	0
9	– 1,20E+08	– 1,20E+08	0	– 1,20E+08	–1,20E+08	0
10	– 3,00E+08	– 3,00E+08	0	– 3,00E+08	– 3,00E+08	0
11	1,00E+08	1,00E+08	0	1,00E+08	1,00E+08	0

Note:

The results presented are given in the reference of reference (X_{ref}, Y_{ref}) which forms an angle from $0°$ ratio to (X, Y) .
The case studied test corresponds to the following diagram in a plane forced strain:



5 Modelization C

Case models of Pinto-Menegotto.

5.1 Thermal loading for the modelization C

Reference temperature: 50°C
History of the loading: $Evolution_C$ (cf [§1.4])
the temperatures returned like a field at nodes.

5.2 Results

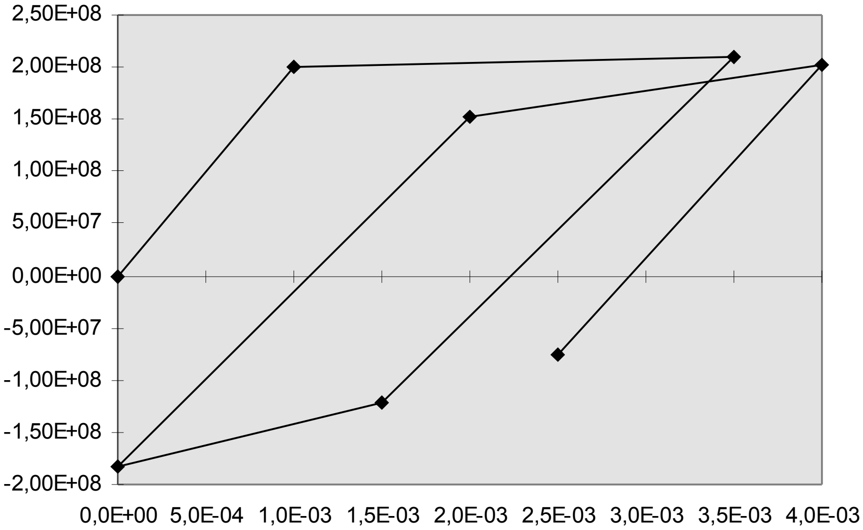
One records the values of $SIGXX$ with the node $NO1$ (direction of longitudinal reinforcements) and $SIGXX$ with the node $NO1$ (direction of transverse reinforcements)

	$SIGXX$ in meshes modelling directed fibers following OX			$SIGXX$ in meshes modelling fibers directed according to OY		
Time	Reference	*Code_Aster*	Variation	Reference	*Code_Aster*	Variation
1	2.09416E+08	2.09416E+08	0	2.09416E+08	2.09416E+08	0
2	– 1.21555E+08	– 1.21555E+08	0	– 1.21555E+08	– 1.21555E+08	0
3	– 1.82862E+08	– 1.82862E+08	0	– 1.82862E+08	– 1.82862E+08	0
4	1.52164E+08	1.52164E+08	0	1.52164E+08	1.52164E+08	0
5	2.02506E+08	2.02506E+08	0	2.02506E+08	2.02506E+08	0
6	– 7.59307E+07	– 7.59307E+07	0	– 7.59307E+07	– 7.59307E+07	0

Note:

The results presented are given in the reference of reference (X_{ref}, Y_{ref}) which forms an angle from $0°$ ratio to (X, Y).

The case studied test corresponds to the following diagram in a plane forced strain:


2,50E+08
2,00E+08
1,50E+08
1,00E+08
5,00E+07
0,00E+00
-5,00E+07
-1,00E+08
-1,50E+08
-2,00E+08
0,0E+00
5,0E-04
1,0E-03
1,5E-03
2,0E-03
2,5E-03
3,0E-03
3,5E-03
4,0E-03

6 Modelization D

The modelization D is the same one as the modelization C, with the difference which the temperatures are defined by a card.

The results are identical.

7 Modelization E

The modelization E approaches the modelization D, with the difference that there is no transverse reinforcement.

7.1 Results

One records the values of $SIGXX$ with the node $NO1$ (not of test on $SIGYY$ in the absence of transverse reinforcements because the stresses are only calculated in the principal direction of reinforcements)

$SIGXX$

Urgent	Reference	*Code_Aster*	Variation
1	2.09416E+08	2.09416E+08	0
2	– 1.21555E+08	– 1.21555E+08	0
3	– 1.82862E+08	– 1.82862E+08	0
4	1.52164E+08	1.52164E+08	0
5	2.02506E+08	2.02506E+08	0
6	– 7.59307E+07	– 7.59307E+07	0

Note:

The results presented are given in the reference of reference (X_{ref}, Y_{ref}) *which forms an angle from* $0°$ *ratio to* (X, Y) *.*

8 Modelization F

The modelization F approaches the modelization A, with the difference that there is no transverse reinforcement.

8.1 Results

One records the values of $SIGXX$ with the node $NO1$ (not of test on $SIGYY$ in the absence of transverse reinforcements)

SIGXX

Urgent	Reference	*Code_Aster*	Variation
1	2.00000E+08	2.00000E+08	0
2	2.40000E+08	2.40000E+08	0
3	4.00000E+07	4.00000E+07	0
4	2.40000E+08	2.40000E+08	0
5	– 1.60000E+08	– 1.60000E+08	0
6	– 3.12000E+08	– 3.12000E+08	0
7	2.88000E+08	2.88000E+08	0
8	4.09600E+08	4.09600E+08	0
9	– 2.70400E+08	– 2.70400E+08	0
10	– 5.27680E+08	– 5.27680E+08	0
11	4.72320E+08	4.72320E+08	0

Note:

The results presented are given in the reference of reference (X_{ref}, Y_{ref}) *which forms an angle from* $0°$ *ratio to* (X, Y) *.*

9 Modelization G

The modelization G approaches the modelization B, with the difference that there is no transverse reinforcement.

9.1 Results

One records the values of $SIGXX$ with the node $NO1$ (not of test on $SIGYY$ in the absence of transverse reinforcements)

SIGXX

Urgent	Reference	*Code_Aster*	Variation
1	2,00E+08	2,00E+08	0
2	2,40E+08	2,40E+08	0
3	4,00E+07	4,00E+07	0
4	2,40E+08	2,40E+08	0
5	– 1,60E+08	– 1,60E+08	0
6	– 2,40E+08	– 2,40E+08	0
7	1,60E+08	1,60E+08	0
8	2,80E+08	2,80E+08	0
9	– 1,20E+08	– 1,20E+08	0
10	– 3,00E+08	– 3,00E+08	0
11	1,00E+08	1,00E+08	0

Note:

The results presented are given in the reference of reference (X_{ref}, Y_{ef}) *which forms an angle from* $0°$ *ratio to* (X, Y) *.*

10 Modelization H

The modelization H is purely mechanical (application of a nodal force on $NO2$ and $NO3$). It is here about a case test of non regression.

10.1 Results

One records the values of: $SIXX$ with the node $NO2$
-- $EPXX$ with the Urgent $NO2$

	$SIGXX$			$EPXX$		
node	Reference	*Code_Aster*	Variation	Reference	*Code_Aster*	Variation
0.001	2E+08	2E+08	0	1E-03	1E-03	0
0.0023	2E+08	2E+08		0.2.3E-03	2.3E-03	0
0.03	2.58E+08	2.58E+08	0	3rd-02	3rd-02	0.0.2
	1.93156E+08	1.93156E+08	0	2.96667E-02	2.96667E-02	0.0.4
	-1.43311E+08	-1.43311E+08	0	2nd-02	2nd-02	0.6.4
	-2.77424E+08	-2.77424E+08	0	4th-02	4th-02	0
7.92	1.01029E+08	1.01029E+08	0	-3.3E-02	-3.3E-02	0
17	2.1647E+08	2.1647E+08	0	1E-02	1E-02	0
19	2.571E+08	2.571E+08	0	3rd-02	3rd-02	0
20	2.17745E+08	2.17745E+08	0	2.98E-02	2.98E-02	0
21	-8.0318E+07	-8.0318E+07		0.2.6E-02	2.6E-02	0
22	-1.28487E+08	-1.28487E+08		0.2.2E-02	2.2E-02	0
25	2.04332E+08	2.04332E+08		0.2.5E-02	2.5E-02	0
50	2.97691E+08	2.97691E+08	0	5th-02	5th-02	0

11 Summary of the results

For elastoplastic behaviors with linear hardening, the analytical solution is found perfectly.

The behavior of Pinto-Menegotto is validated by non regression.