SSNV509 – Catenary in contact rubbing with quadratic X-FEMs

Abstract:

The purpose of this test is of the contact validating the taking into account on the lips of a curved crack, while being limited if the crack crosses completely the structure (interface). One places oneself in the frame of the continuous method adapted to the method X-FEM with resolution by Augmented Lagrangian. This test brings into play a parallelepipedic block in compression. The interface is represented by a crossing circular level set of the elements with variable angles. It utilizes the elements X-FEM $P2$ in displacement and $P1$ pressure which have degrees of freedom of displacement in each node and degrees of freedom of contact/friction on the nodes tops.

1 Problem of reference in 2D

1.1 Geometry

the structure is an operational square plate on side $L=10\,\mathrm{m}$.

The interface is represented by a circular level set introduced into the command file using operator DEFI_FISS_XFEM [U4.82.08] given in his analytical form: $(x-5)^2+(y-11{,}77)^2=64{,}27$. It was selected of way has to cross structure completely but is not identified with an inclusion [V6.04.507].

The position of the points of reference east:

	x	y
A	0.5,5	
B	5	3,75
C	10.5,5	

1.2 Material properties

Poisson's ratio: 0
Young modulus: $1.10^6\,N/m^2$

1.3 Boundary conditions and loadings

the lower face is blocked by a fixed support and a displacement $u_y=-1.10^{-6}\,m$ is imposed on the upper face

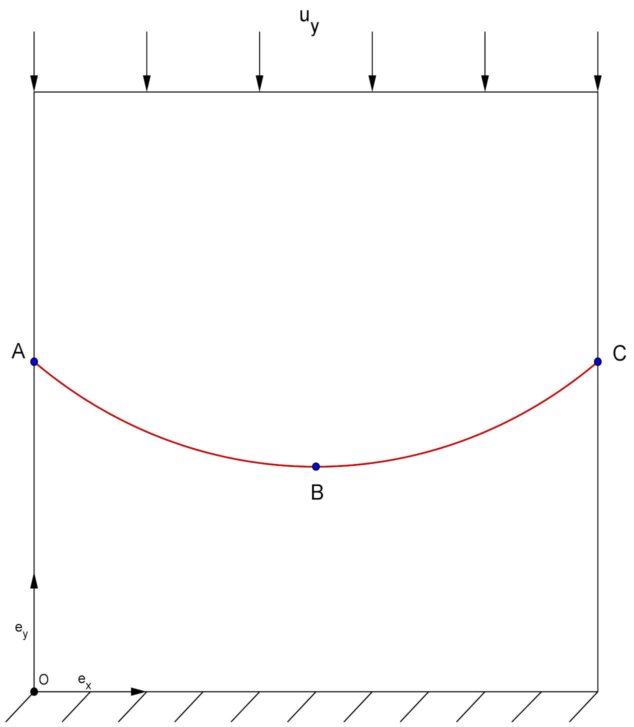

Figure 1.3 - a: Geometry of structure and positioning of the interface and loadings

2 Reference solution: analytical

the norm with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}, \qquad \tau=\begin{pmatrix}-\sin\theta\\ \cos\theta\end{pmatrix} \qquad \textbf{éq 2.1-1}$$

with $\theta=\arctan\dfrac{y-11{,}77}{x-5}$ **éq 2.1-2**

In this modelization, one considers the assumption of plane stresses (although here, the Poisson's ratio being null, it does not have there differences between stresses and plane strains).

The method of Augmented Lagrangian is used for the processing of the contact/friction.

The interface presents a slope which varies from one end to another of the plate. However at the places where the slope is strongest, near as of side edges, there is likely to be sliding. To avoid that, one increases the dependancy via the coefficient of kinetic friction of Coulomb: one takes $\mu=2$.

The value of the contact pressure on the interface is function of the norm n :

$$\lambda=\boldsymbol{n}.\boldsymbol{\sigma}.\boldsymbol{n}=n_y\sigma_{yy}n_y \qquad \textbf{éq .2.1-3}$$

- where n_y is the component following y of $\boldsymbol{n}$
- where σ_{yy} is the stress following y in the plane of norm $\boldsymbol{e_y}$ in structure without interface: $\sigma_{yy}=E\dfrac{u_y}{L_y}$

The semi-multiplier of friction $\boldsymbol{\Lambda}$ is defined by:

$$\boldsymbol{r_\tau}=\lambda\mu\boldsymbol{\Lambda} \qquad \textbf{éq 2.1.1-4}$$

With the density of tangential stress being written as follows:

$$\boldsymbol{r_\tau}=(\boldsymbol{\tau}.\boldsymbol{\sigma}.\boldsymbol{n})\boldsymbol{\tau} \qquad \textbf{éq 2.1-5}$$

From where:

$$\boldsymbol{\Lambda}=\left(\frac{1}{\mu}\frac{\boldsymbol{\tau}.\boldsymbol{\sigma}.\boldsymbol{n}}{\boldsymbol{n}.\boldsymbol{\sigma}.\boldsymbol{n}}\right)\boldsymbol{\tau}=\left(\frac{1}{\mu}\frac{\tau_y}{n_y}\right)\boldsymbol{\tau} \qquad \textbf{éq 2.1-6}$$

With the numerical values previously introduced:

$$\lambda(x,y)=\frac{-1}{10}\sin^2\left(\arctan\frac{11{,}77-y}{5-x}\right)Pa \qquad \textbf{éq 2.1-7}$$

$$\Lambda(x,y)=\boldsymbol{\Lambda}.\boldsymbol{\tau}=\frac{x-5}{2(y-11{,}77)} \qquad \textbf{éq 2.1-8}$$

3 Modelization a: Mesh quadrangle

3.1 Characteristics of the modelization

Modelization: `C_PLAN` .

The structure is a healthy square, into which an interface is introduced directly into the command file using operator `DEFI_FISS_XFEM` [U4.82.08]. The interface has the shape of an arc of a circle.

The contact/friction is treated with quadratic `elements` X-FEM $P2$ (displacement) and $P1$ (pressure), i.e. carrying the degrees of freedom of displacement on all the nodes and the Lagrange of contact/friction on the nodes tops.

3.2 Characteristics of the mesh

Many nodes: 1281
Number of meshes and types: 400 `QUAD8` for the plate and 80 `SEG3` for edges.

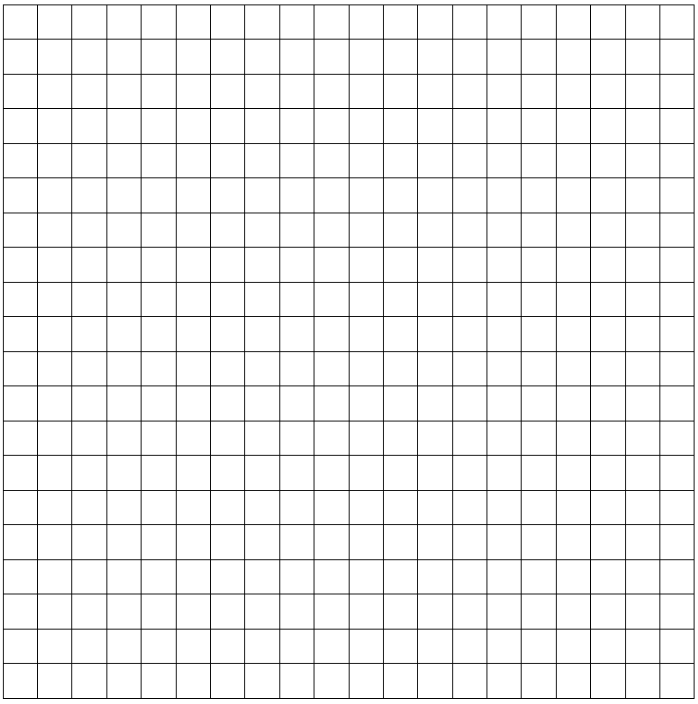

Figure 3.2 - a: Mesh 2D quadrangle

In operator `DEFI_CONTACT`, one stipulates FROTTEMENT='COULOMB' then, and the coefficient of kinetic friction is selected equal to 2.

3.3 Quantities tested and results

One post-draft values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of operator `STAT_NON_LINE`, the loop on the active stresses and the loop on the thresholds of friction. The values appear only with the nodes of the interface resulting from the new mesh.

Standard	identification	Reference	% tolerance
$\mathrm{LAGS\_C}$ to the points A and C (MIN)	-0,1	Analytical	0.10
$\mathrm{LAGS\_C}$ at point B (MAX)	-0.06110	Analytical	0.10
$\mathrm{LAGS\_F1}$ at point A (MAX)	0.39894	Analytical	0.50
$\mathrm{LAGS\_F1}$ at the point B (MIN)	0	Analytical	0.10
	Non regression	9.07e-9	1,00E-06
	1.01e-3	Non regression	1,00E-06
	10.06	Non regression	1,00E-06
	20.85	Non regression	1,00E-06

3.4 Comments

This valid test:

- the computation of the stiffness matrix (the good shift during the filling of the matrix because the nodes do not increase the same number of degrees of freedom),
- the computation of the contact matrixes (integration on `a SE3` with Gauss points),
- under cutting (configurations in curved interface and elements on right board),
- postprocessing `X-FEM` of the elements $\mathrm{P2P1}$.

The MUMPS solver detects a singularity in the matrix. This problem already arose on many other cases tests `X-FEM` in contact. The readjustment of the level set norm makes it possible to improve conditioning of the matrix but to the detriment of a too great error of discretization of the level set. One thus keeps the option of deactivation of the detection of singularity of the solver before suggesting a more satisfactory solution.

4 Modelization b: Mesh triangle

4.1 Characteristics of the modelization

Modelization: C_PLAN .

The structure is a healthy square, into which an interface is introduced directly into the command file using operator DEFI_FISS_XFEM [U4.82.08]. The interface has the shape of an arc of a circle.

The contact/friction is treated with quadratic elements X-FEM $P2$ (displacement) and $P1$ (pressure), i.e. carrying the degrees of freedom of displacement on all the nodes and the lagranges of contact/friction on the nodes tops.

4.2 Characteristics of the mesh

Many nodes: 1681
Number of meshes and types: 800 TRIA6 for the plate and 80 SEG3 for edges.

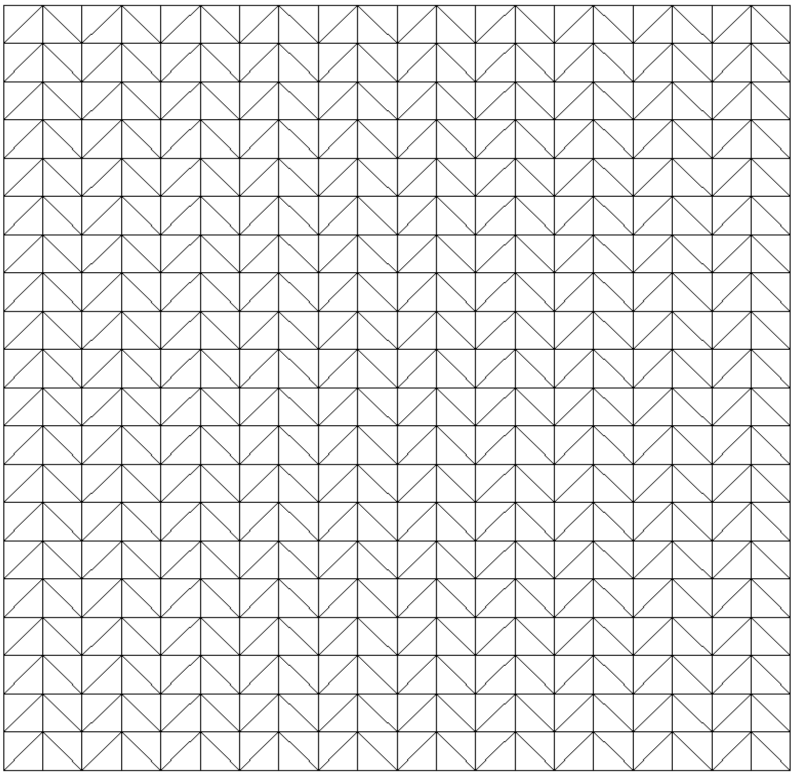

Figure 4.2 - a: Mesh 2D triangle

In operator DEFI_CONTACT, one stipulates FROTTEMENT='COULOMB' then, and the coefficient of kinetic friction is selected equal to 2.

4.3 Quantities tested and results

One post-draft values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of operator `STAT_NON_LINE`, the loop on the active stresses and the loop on the thresholds of friction. The values appear only with the nodes of the interface resulting from the new mesh.

Standard	identification	Reference	% tolerance
$\mathrm{LAGS\_C}$ to the points A and C (MIN)	-0,1	Analytical	0.10
$\mathrm{LAGS\_C}$ at point B (MAX)	-0.06110	Analytical	0.10
$\mathrm{LAGS\_F1}$ at point A (MAX)	0.39894	Analytical	0.50
$\mathrm{LAGS\_F1}$ at the point B (MIN)	0	Analytical	0.10
	Non regression	6.74e-8	1,00E-06
	1.24e-3	Non regression	1,00E-06
	11.58	Non regression	1,00E-06
	22.45	Non regression	1,00E-06

4.4 Comments

This valid test:

- the computation of the stiffness matrix (the good shift during the filling of the matrix because the nodes do not increase the same number of degrees of freedom),
- the computation of the contact matrixes (integration on `a SE3` with Gauss points),
- under cutting (configurations in curved interface and elements on right board),
- postprocessing `X-FEM` of the elements $\mathrm{P2P1}$.

5 Summaries of the results

the purposes of this test are reached.

- It was a question of showing the feasibility of the taking into account of the contact rubbing on the lips of the catenary with the continuous method adapted to the frame X-FEM. Only the case of an interface crossing structure completely was considered (interface).
- The method is validated in $2D$ $P2$ (displacement) $P1$ (pressure) on a mesh quadrangle and triangle.
- The method is validated with the method of Augmented Lagrangian for the processing of the contact/friction.