SSNV508 – Block in plane stresses with interface, tension and side compression, for quadratic elements X-FEM

Abstract:

The purpose of this test is validating the deformed shape of an interface introduced into a rectangular parallelepipedic very thin plate in a frame X-FEM. The structure is requested in tension and is subjected to a linear side pressure. The interface is represented by a level set plane and horizontal cutting elements or coinciding with their edges. It utilizes the elements X-FEM [R7.02.12] $P2$ (displacement) which have degrees of freedom of displacement in each node. With the problem is dealt in 2D and 3D.

1 Problem of reference

1.1 Geometry 2D

the structure is a rectangle made up of two of the same plates material, separated by an interface.

Dimensions of the plate, to which the pressures are applied, are:
$L_X=2\text{m}$, $L_Y=1{,}8\,m$
The second plate has following dimensions:
$L_X=2\text{m}$, $L_Y{}'=1{,}2\,m$

The position of the points of reference east:

	x	y
A	-1	0
B	1	0
C	1.1,8	
D	-1.1,8	
O	0	0

1.2 Geometry 3D

the structure is a rectangular parallelepiped made up by two of the same plates material, separated by an interface.

Dimensions of the plate of the top, to which the pressures are applied, are:
$L_X=2\text{m}$ $L_Y=1{,}8\,m$, $L_Z=0{,}01\,m$
the second plate in lower part has following dimensions:
$L_X=2\text{m}$ $L_Y{}'=1{,}2\,m$, $L_Z=0{,}01\,m$
the position of the points of reference east:

	x	y	z
A	-1	0	0
B	1	0	0
C	1.1,8		0
D	-1.1,8		0
O	0	0	0
A'	-1	0	-0,01
B'	1	0	-0,01
C'	1.1,8		-0,01
D'	-1.1,8		-0,01
O'	0	0	-0,01

1.3 Material properties

Poisson's ratio: 0.2
Young modulus: $1.10^{10} N/m^2$

1.4 Boundary conditions and loadings

the lower plate $(y<0)$ is blocked by a fixed support of its lower face.
The plane $ABCD$ is blocked in the direction e_z .
In 3D case, the plane $(x=0)$ is blocked in the direction e_x , and in the case 2D, it is the line $(x=0)$ which is blocked in the direction e_x .
The higher plate $(y>0)$ is subjected to a horizontal distributed pressure acting on the side sides $P=\pm(-5.10^4 y+1.10^5) N/m^2$ (according to the principle of compression). One applies a quadratic displacement of tension to it to the upper face $d=-2{,}5.10^{-6} x^2+1.10^{-5} m$.

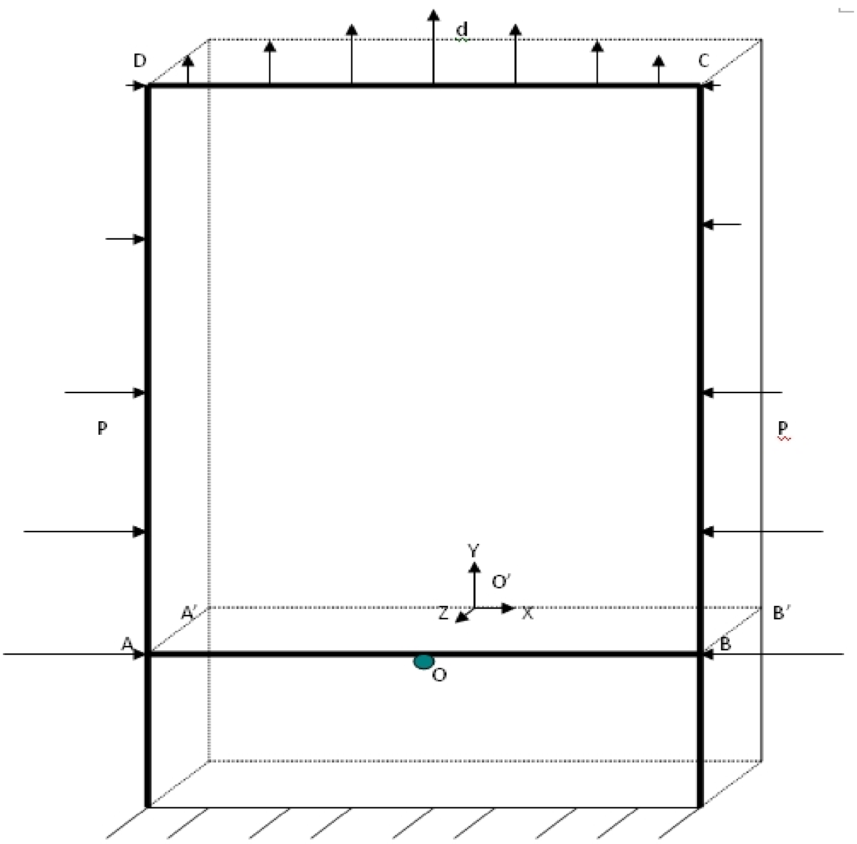

Figure 1: Geometry of structure and positioning of the interface and loadings 3D

2 Reference solution: analytical in plane stresses

2.1 Solution 2D

While being placed in the cartesian coordinate system (x, y), the displacement of any point $M(x, y)$ of the higher plate is written:

$$u(x, y)=u_x(x, y)\vec{e}_x+u_y(x, y)\vec{e}_y$$ **éq 2-1**

Note:

- *The higher plate and the lower plate are dissociated owing to the fact that the interface separates the plate into two completely. The plate lower being embedded than its base, it results from it that it is completely motionless and that one makes carry the analytical study only on the higher plate.*

One breaks up the components of displacement in the base $\{1, x, y, x^2, y^2, xy, x^2 y, xy^2\}$:

$$u_x(x, y)=I_1+I_2 x+I_3 y+I_4 x^2+I_5 y^2+I_6 xy+I_7 x^2 y+I_8 x y^2$$ **éq 2-2**

$$u_y(x, y)=J_1+J_2 x+J_3 y+J_4 x^2+J_5 y^2+J_6 xy+J_7 x^2 y+J_8 x y^2$$ **éq the 2-3**

problem has a geometrical symmetry and of loading compared to the y-axis. That implies:

$$I_1=I_3=I_4=I_5=I_7=J_2=J_6=J_8=0$$ **éq 2-4**

the balance equations room expressed in the cartesian coordinate system gives:

$$I_8=J_7=0$$ **éq 2-5**

$$I_6=-\frac{2(1-\nu)}{1+\nu}J_4-\frac{4}{1+\nu}J_5$$ **éq 2-6**

By applying the limiting conditions of Dirichlet of the upper face $d=d_2 x^2+d_0$, one from of deduced:

$$J_4=d_2=-2{,}5.10^{-6}$$ **éq 2-7**

$$J_1+J_3 L_y+J_5 L_y^2=d_0$$ **éq 2-8**

By applying the limiting conditions of Neumann of side edges $P=p_1 y+p_0$ to the stresses resulting from the generalized Hooke's law:

$$J_5=-\frac{d_2}{2+\nu}+\frac{p_1(1+\nu)^2}{2E(2+\nu)}=-0{,}5.10^{-6}$$ **éq 2-9**

$$I_6=-\frac{2d_2(1-\nu)}{1+\nu}p_0-\frac{4J_5}{1+\nu}=-\frac{5}{3}.10^{-6}$$ **éq 2-10**

$$I_2=-\frac{1-\nu^2}{E}p_0-\nu J_3$$ **éq the 2-11**

interface is a free edge, i.e. the vector forced in any point of this surface in the normal direction external with structure is null:

$$J_3=\frac{\nu p_0}{E}=2.10^{-6}$$ **éq 2-12**

$$I_2=\frac{-p_0}{E}=-1.10^{-6}$$ **éq 2-13**

Consequently, by combining the results and statements obtained, one draws J_1:

$$J_1=d_0-J_3 L_y-J_5 L_y^2=8{,}02.10^{-6}$$ **éq the 2-14**

solution obtained is the following one:

$$u_x(x, y) = -1.10^{-6}(10x + \frac{5}{3}xy)$$ **éq 2-15**

$$u_y(x, y) = 1.10^{-6}(8{,}02 + 2y - 2{,}5x^2 - 0{,}5y^2)$$ **éq 2-16**

Is on the interface result according to:

$$u_x(x, y=0) = -1.10^{-5}x$$ **éq 2-17**

$$u_y(x, y=0) = 1.10^{-6}(8{,}02 - 2{,}5x^2)$$ **éq 2-18**

2.2 Solution 3D

According to the assumption of the plane stresses the stress field 3D does not vary in the direction z , which implies that the strains of it are also independent. The problem can then be brought back to the problem in 2D (plane $ABCD$) for the resolution of the stresses and strains.

In 3D case, the solution on u_x and u_y thus has the following form:

$$u_x(x, y, z) = 1.10^{-6}(-10x - \frac{5}{3}xy + h_x(z))$$ **éq 2-19**

$$u_y(x, y, z) = 1.10^{-6}(8{,}02 + 2y - 2{,}5x^2 + h_y(z))$$ **éq 2-20**

Moreover, the strain on e_z is written:

$$\epsilon_{zz}(x, y, z) = -\frac{\nu}{1-\nu}(\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y}) = 1.10^{-6}(2 + \frac{2}{3}y)$$ **éq 2-21**

With:

$$\epsilon_{zz}(x, y, z) = \frac{\partial u_z(x, y, z)}{\partial z}$$ **éq 2-22**

Consequently, by combining the results and statements obtained, one obtains:

$$u_z(x, y, z) = 1.10^{-6}[(2 + \frac{2}{3}y)z + g(x, y)]$$ **éq 2-23**

According to $\epsilon_{xz} = 0$ and $\epsilon_{yz} = 0$, one obtains:

$$h_x(z) = C_1 z + C_0$$ **éq 2-24**

$$h_y(z) = -\frac{z^2}{3} + C_3 z + C_5$$ **éq 2-25**

$$g(x, y) = -C_{1x} - C_3 y + C_4$$ **éq the 2-26**

plane $ABCD$ is blocked on the direction e_z , one obtains:

$$u_z(x, y, z=0) = 0$$ **éq 2-27**

That implies: $C_1 = C_3 = C_4 = 0$.

The plane is blocked on the direction e_x , one obtains:

$$u_x(x=0, y, z) = 0$$ **éq 2-28**

That implies: $C_0 = 0$.

Moreover, the displacement imposed on the upper surface leads to: $C_5 = 0$.

Finally, one obtains:

$$u_x(x, y, z) = -(10x + \frac{5}{3}xy).10^{-6}$$ **éq 2-29**

$$u_y(x, y, z) = (8{,}02 + 2y - 2{,}5x^2 - 0{,}5y^2 - \frac{1}{3}z^2).10^{-6}$$ **éq 2-30**

$$u_z(x,y,z)=(2+\frac{2}{3}y)z.10^{-6}$$ **éq 2-31**

Is on the interface result according to:

$$u_x(x,y=0,z)=-1.10^{-5}x$$ **éq 2-32**

$$u_y(x,y=0,z)=(8{,}02-2{,}5x^2-\frac{1}{3}z^2).10^{-6}$$ **éq 2-33**

$$u_z(x,y=0,z)=2.10^{-6}z$$ **éq 2-34**

3 Modelization A

3.1 Characteristic of the modelization

Modelization: `C_PLAN` .

The structure is a healthy rectangle, into which an interface is introduced directly into the command file using operator `DEFI_FISS_XFEM` [U4.82.08]. The interface is present at a distance $L_Y = 1,8\,m$ from higher edge of the plate.

3.2 Characteristics of the mesh

Many nodes: 661
Number of meshes and types: 200 `QUAD8` for the plate and 50 `SEG3` for edges.

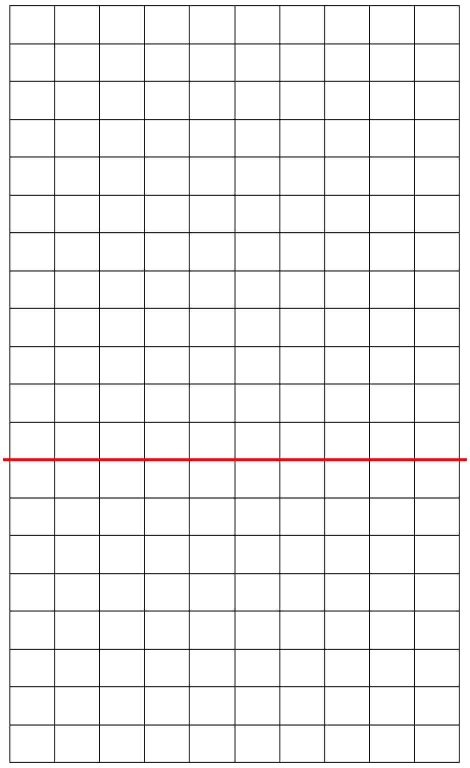

Figure 3.2 - a: Mesh 2D quadrangle and position of the interface

3.3 Quantities tested and results

displacements resulting from operator `STAT_NON_LINE` are post-treaties so as to recover the values with the nodes of crack resulting from the new mesh.

Identification	Reference	Aster	tolerance
DX to the item A	1.10-5	Analytical	1.10-12
DX as in point B	-1.10-5	Analytical	1.10-12
DY as in point A	5,52.10-6	Analytical	1.10-12
DY as in point B	5,52.10-6	Analytical	1.10-12
DY as in Analytical O	point	8,02.10-6	1.10-12

3.4 Comments

This valid test:

- the computation of the stiffness matrix and the second member vectors (taken into account of the distributed pressure on quadratic edge elements),
- postprocessing X-FEM of the elements $P2$.

4 Modelization B

4.1 Characteristic of the modelization

Modelization: C_PLAN .

The structure is a healthy rectangle, into which an interface is introduced directly into the command file using operator DEFI_FISS_XFEM [U4.82.08]. The interface is present at a distance $L_Y=1,8\,m$ from higher edge of the plate.

4.2 Characteristics of the mesh

Many nodes: 597
Number of meshes and types: 180 QUAD8 for the plate and 46 SEG3 for edges.

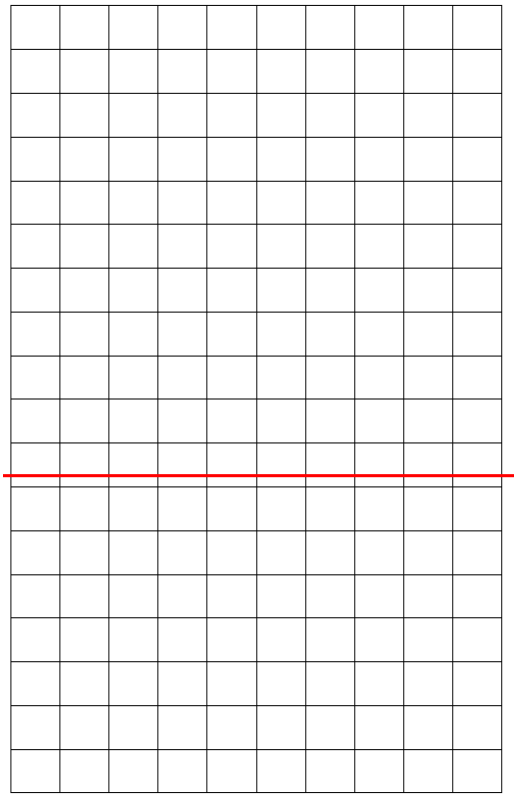

Figure 4.2 - a: Mesh 2D quadrangle and position of the interface

4.3 Quantities tested and results

displacements resulting from operator STAT_NON_LINE are post-taities in order to way to recover the values with the nodes of crack resulting from the new mesh.

Identification	Reference	Aster	tolerance
DX to the item A	1.10-5	Analytical	1.10-12
DX as in point B	-1.10-5	Analytical	1.10-12
DY as in point A	5,52.10-6	Analytical	1.10-12

DY as in point B	5,52.10-6	Analytical	1.10-12
DY as in Analytical O	point	8,02.10-6	1.10-12

4.4 Comments

This valid test:

- the computation of the stiffness matrix and the second member vectors (taken into account of the distributed pressure on quadratic edge elements),
- under cutting (configuration in right interface and elements on right board),
- postprocessing X-FEM of the elements $P2$.

5 Modelization C

5.1 Characteristic of the modelization

Modelization: `3D` .

The structure is parallelepipedic rectangular healthy, into which an interface is introduced directly into the command file using operator `DEFI_FISS_XFEM` [U4.82.08]. The interface is present at a distance $L_Y=1{,}8\,m$ from higher edge of the plate.

5.2 Characteristics of the mesh

Many nodes: 8644
Number of meshes and types: 6989
of which `TRIA6`: 2600
of which `TETRA10` : 4389

5.3 Quantities tested and results

displacements resulting from operator `STAT_NON_LINE` are post-treaties so as to recover the values with the nodes of crack resulting from the new mesh.

Identification	Reference	Aster	tolerance
DX over line AA'	the 1.10-5	Analytical	1.10-10
DX on line BB'	the -1.10-5	Analytical	1.10-10
DY to the item A'	5,52.10-6	Analytical	1.10-10
DY as in point B'	5,52.10-6	Analytical	1.10-10
DY as in Analytical O	point	8,02.10-6	Analytical
DZ 1.10-10 on $A'B'$	line	the -2,0.10-8	1.10-10

6 Modelization D

6.1 Characteristic of the modelization

Modelization: 3D .

The structure is parallelepipedic rectangular healthy, into which an interface is introduced directly into the command file using operator DEFI_FISS_XFEM [U4.82.08]. The interface is present at a distance $L_Y=1,8\,m$ from higher edge of the plate.

6.2 Characteristics of the mesh

Many nodes: 5653
Number of meshes: 3800
of which SEG3: 100
of which TRIA6: 2400
of which QUAD8: 100
of which PENTA15: 1200

6.3 Quantities tested and results

displacements resulting from operator STAT_NON_LINE are post-treaties so as to recover the values with the nodes of crack resulting from the new mesh.

Identification	Reference	Aster	tolerance
DX over line AA'	the 1.10-5	Analytical	1.10-10
DX on line BB'	the -1.10-5	Analytical	1.10-10
DY to the item A'	5,52.10-6	Analytical	1.10-10
DY as in point B'	5,52.10-6	Analytical	1.10-10
DY as in Analytical O	point	8,02.10-6	Analytical
DZ 1.10-10 on $A'B'$	line	the -2,0.10-8	1.10-10

7 Modelization E

7.1 Characteristic of the modelization

Modelization: `3D` .

The structure is parallelepipedic rectangular healthy, into which an interface is introduced directly into the command file using operator `DEFI_FISS_XFEM` [U4.82.08]. The interface is present at a distance $L_Y=1,8\,m$ from higher edge of the plate.

7.2 Characteristics of the mesh

Many nodes: 4453
Number of meshes: 2000

of which	`SEG3:`	100
of which	`QUADS:`	1300
of which	`HEXA20:`	600

7.3 Quantities tested and results

displacements resulting from operator `STAT_NON_LINE` are post-treaties so as to recover the values with the nodes of crack resulting from the new mesh.

Identification	Reference	Aster	tolerance
DX over line AA'	the 1.10-5	Analytical	1.10-10
DX on line BB'	the -1.10-5	Analytical	1.10-10
DY to the item A'	5,52.10-6	Analytical	1.10-10
DY as in point B'	5,52.10-6	Analytical	1.10-10
DY as in point O	8,02.10-6	Analytical	1.10-10
DZ on line $A'B'$	the -2,0.10-8	Analytical	1.10-10

8 Summary of the results of modelization

the purposes of this test are reached.

- It was a question of showing the feasibility of the taking into account of enrichment by the Heaviside function of the classical shape functions on quadratic elements. Only the case of a crack crossing structure completely was considered (interface).
- The method is validated in $2\mathrm{D}$, $P2$ on a mesh quadrangle.
- The method is validated in $3\mathrm{D}$, $P2$ on a rectangular parallelepipedic mesh.
- One obtains a better solution with modelization `C_PLAN` than the modelization 3D.