SSNV507 – Rotation of a rigid inclusion with X-FEM

Summarized:

This test is used to validate the approach X-FEM on the assumption of the taking into account of large rotations (DEPLACEMENT = "GROT_GDEP" in operator STAT_NON_LINE).

Whatever the modelization, one considers a square structure having a discontinuity defining a first disc inside a square. One defines as rigid inclusion, a second disc inside the first. One then imposes a rotation finished on this inclusion.

The modelizations A B C D, E and F consist in of the contact operating large rotation without taking into account. For that, one blocks displacements of rigid bodies by applying boundary conditions of blocking to edges of the square. All the points belonging to first disc are pulled by the rotation of inclusion and undergo same rotation. Outside the disc, the points do not undergo any displacement. The validation is done on the values of displacements of the points which are inside the disc and which must correspond to the rotation movement finished imposed.

The modelizations G, H and I consist in operating large rotation by activating the contact. One applies a displacement to edges of the square. This displacement is selected so as to obtain a uniform normal stress on the level of discontinuity. The validation is done on the values of Lagrange of contact.

1 Problem of reference

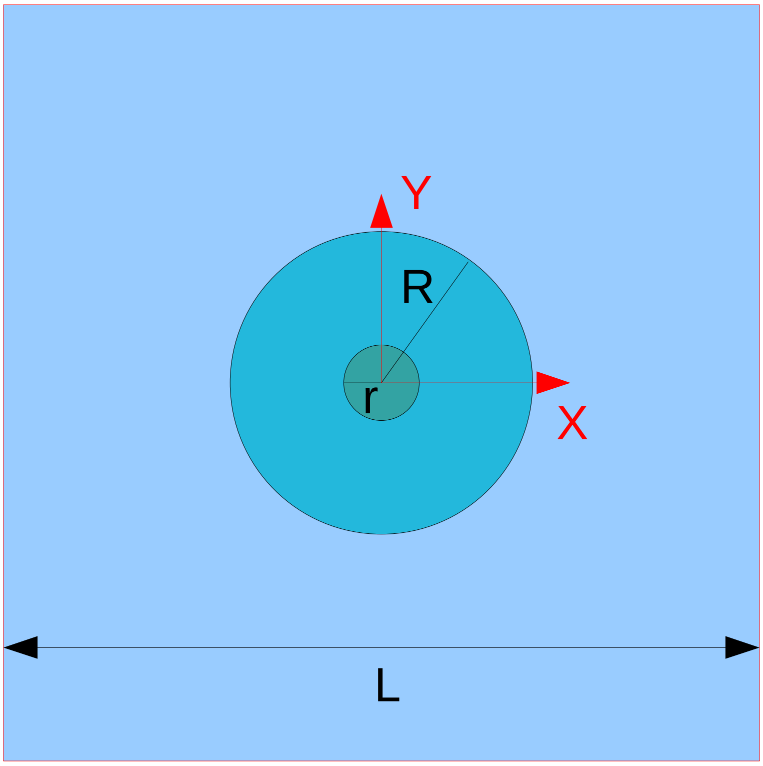
1.1 Geometry

the structure is a healthy square in which a circle of radius R defines an interface (discontinuity). A circle of radius R_1 delimits inclusion and the material. Dimensions of structure are:

$L=20\,m$

$R=8\,m$

$R_1=3\,m$



Appear 1.1-a: Geometry of structure.

1.2 Properties of the material

- Modulus Young: $E=100\,MPa$
- Without contact: Poisson's ratio: $\nu=0.3$
- With contact: Poisson's ratio: $\nu=0$

1.3 Boundary conditions and loadings

1.3.1. Without contact

the blocking of horizontal displacements is imposed on the sides left and right square (Figure 1.3-a). The blocking of vertical displacements is imposed on with dimensions the inferior and superior of the square.

A time finished rotation of a full rotation is imposed on central inclusion. Its application is done according to a function crawls classical, in 4 steps of load. The numerical values of imposed rotation are thus:

$$\omega=(90°,180°,270°,360°)$$

Concretely, on the step of time i , for a node pertaining to the inclusion, which has as real coordinates of reference (X,Y) , one imposes following displacements:

$$Depl_X(X,Y,i)=\sqrt{X^2+Y^2}(\cos(\arctan(\frac{Y}{X})-\omega(i)))-X$$
$$Depl_Y(X,Y,i)=\sqrt{X^2+Y^2}(\sin(\arctan(\frac{Y}{X})-\omega(i)))-Y$$ éq 1.1

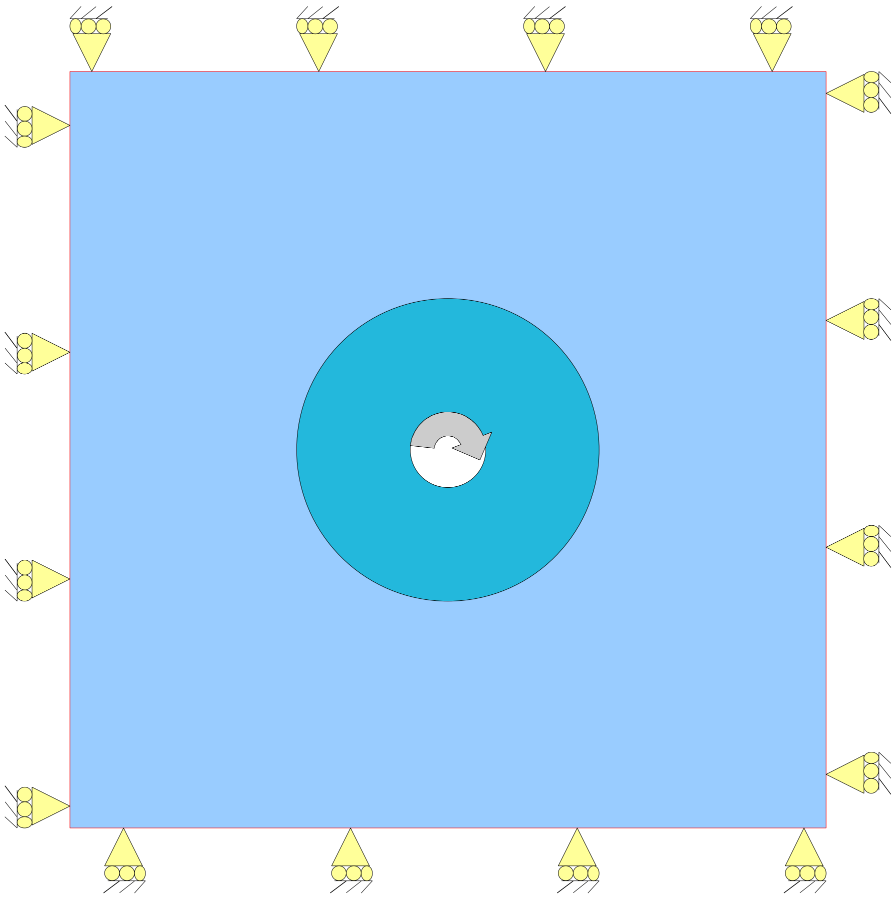


Figure 1.3-a: Illustration of the boundary conditions and the loadings.

1.3.2. With contact

a displacement u_r is imposed on all the edges of the square, it is equivalent to a radial displacement applied to a circle of radius $R_2=\frac{L}{2}$:

$$u_r=ar+\frac{b}{r}$$ éq 1. the 2

coefficients a and b are selected so as to satisfy $\sigma_{\theta\theta}(r=R_1)=0$ and $\sigma_{rr}(r=R_2)=-p$ where $p=1.10^6\,Pa$:

$$a=\frac{-p}{E}\frac{R_2^{\ 2}}{R_1^{\ 2}+R_2^{\ 2}}$$
$$b=\frac{p}{E}\frac{R_1^{\ 2}R_2^{\ 2}}{R_1^{\ 2}+R_2^{\ 2}}$$ éq 1.3

a time finished rotation of a quarter of turn is imposed on central inclusion. Its application is done according to a function crawls classical, in 4 steps of load. The numerical values of imposed rotation are thus:

$$\omega=(\frac{\pi}{8},\frac{2\pi}{8},\frac{3\pi}{8},\frac{\pi}{2})$$

Concretely, on the step of time i , for a node pertaining to the inclusion, which has as real coordinates of reference (X,Y) , one imposes following displacements:

$$Depl_X(X,Y,i)=\sqrt{X^2+Y^2}(\cos(\arctan(\frac{Y}{X})-\omega(i)))-X$$
$$Depl_Y(X,Y,i)=\sqrt{X^2+Y^2}(\sin(\arctan(\frac{Y}{X})-\omega(i)))-Y$$ éq 1.4

2 Reference solution

2.1 Without contact

One tests each node inside the disc delimited by the interface. For each one of these nodes, the reference solution is provided analytically by the computation of its displacement. This displacement must correspond to the rotation imposed on inclusion: it is thus calculated by means of equation 1.1.

2.2 With contact

One tests each node pertaining to the interface. For each one of these nodes, the reference solution is provided analytically by the computation of the contact pressure. It is calculated by means of equation 1.2:

$$\lambda=aE-\frac{bE}{R^2}$$ éq 2.1

3 Modelization A

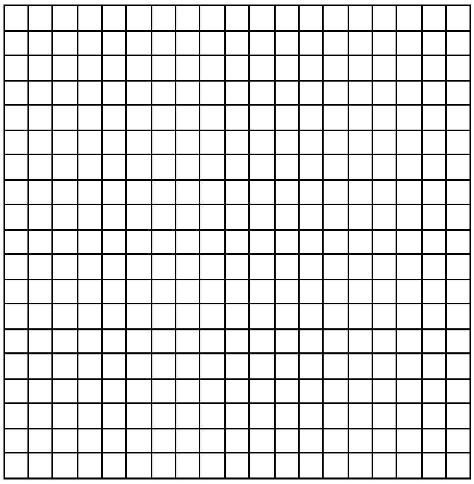
3.1 Characteristic of the modelization

It acts of a modelization X-FEM , in plane strains (D_PLAN). Discontinuity is defined by a function of level (level set noted norm Ln) directly introduced into the command file using operator DEFI_FISS_XFEM [U4.82.08]. This function represents the circle of radius R :

$$\ln = R^2 - X^2 - Y^2 \qquad \text{Éq 3.1}$$

3.2 Characteristics of the mesh

The mesh (Figure 3.2-a) comprises 361 meshes type QUAD4.



Appear 3.2-a: The mesh of modelization A.

3.3 Grandeurs tested and results

For each step of load, one calculates for all the nodes inside the disc, the absolute difference between the displacement obtained by computation aster and that calculated analytically. For each one of these nodes, the difference must be null. To go more quickly one tests the sum of all these differences which must also be worth zero.

Not	Standard	Identification	Reference	Tolerance (%)
1	SOMM	0	ANALYTIQUE	1.00E-008
2	SOMM	0	ANALYTIQUE	1.00E-008
3	SOMM	0	ANALYTIQUE	1.00E-008
4	SOMM	0	ANALYTIQUE	1.00E-008

3.4 Comments

One imposed a rigid rotation on the nodes of inclusion. The test shows that inclusion involves all the other nodes of the disc (nonconcerned by the limiting condition of rotation) of the same rotation.
If one launches this case test by replacing "GROT_GDEP" by "PETIT" in operand DEPLACEMENT, the disc "inflates and the" displacements of the nodes tested do not follow more finished rotation.

4 Modelization B

4.1 Characteristic of the modelization

Modelization: C_PLAN .

4.2 Characteristics of the mesh

The mesh (Figure 4.2-a) comprises 722 meshes type TRIA3.

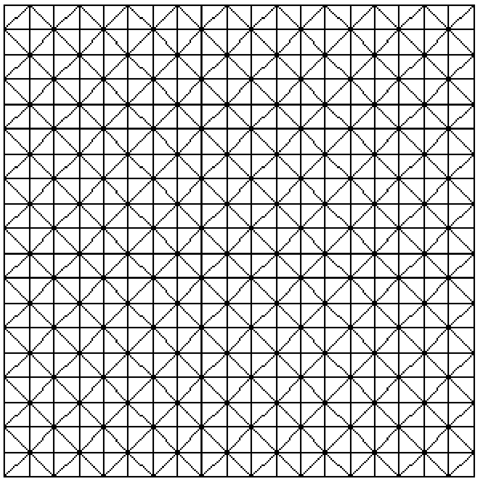


Figure 4.2-a : The mesh of the modelization B.

4.3 Quantities tested and results

One tests the sum of the differences of displacements as for the modelization A .

Not	Standard	Identification	Reference	Tolerance (%)
1	SOMM	0	ANALYTIQUE	1.0e-008
2	SOMM	0	ANALYTIQUE	1.0e-008
3	SOMM	0	ANALYTIQUE	1.0e-008
4	SOMM	0	ANALYTIQUE	1.0e-008

4.4 Comments

Idem modelization A

5 Modelization C

5.1 Characteristic of the modelization

It acts of the same characteristics of modelization as those of the modelization A but in 3D :
- one extrudes of 2 meters in the direction Z (1 element),
- one blocks rigid body motions in this direction,
- the level set norm (defined by equation 3.1) represents a cylinder now.

5.2 Characteristics of the mesh

The mesh (Figure 5.2-a) comprises 361 meshes type HEXA8.

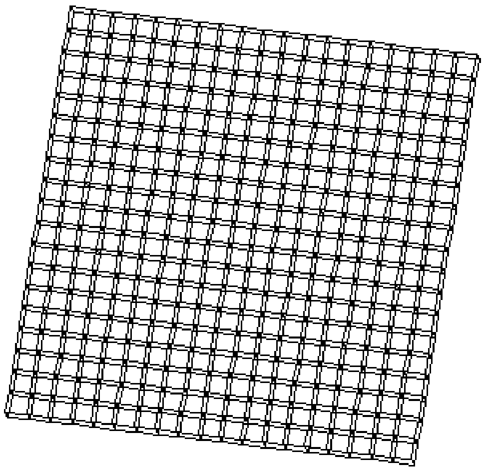


Figure 5.2-a : The mesh of the modelization C.

5.3 Quantities tested and results

One tests the sum of the differences of displacements as for the modelization A .

Not	Standard	Identification	Reference	Tolerance (%)
1	SOMM	0	ANALYTIQUE	1.0e-008
2	SOMM	0	ANALYTIQUE	1.0e-008
3	SOMM	0	ANALYTIQUE	1.0e-008
4	SOMM	0	ANALYTIQUE	1.0e-008

5.4 Comments

Idem modelization A

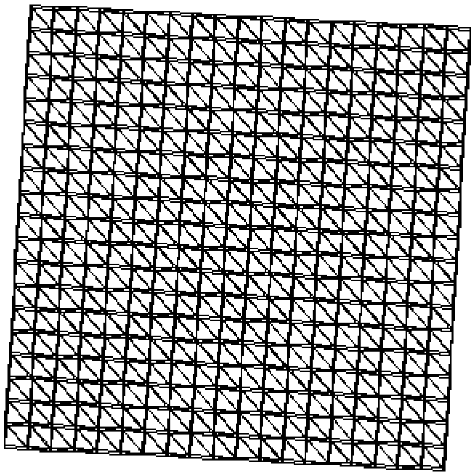
6 Modelization D

6.1 Characteristic of the modelization

It acts of the same characteristics of modelization as those of the modelization C.

6.2 Characteristics of the mesh

The mesh (Figure 6.2-a) comprises 2166 meshes type TETRA4.



Appear 6.2-a: The mesh of modelization D.

6.3 Grandeurs tested and results

One tests the sum of the differences of displacements as for the modelization A.

Not	Standard	Identification	Reference	Tolerance (%)
1	SOMM	0	ANALYTIQUE	1.0e-008
2	SOMM	0	ANALYTIQUE	1.0e-008
3	SOMM	0	ANALYTIQUE	1.0e-008
4	SOMM	0	ANALYTIQUE	1.0e-008

6.4 Comments

Idem modelization A

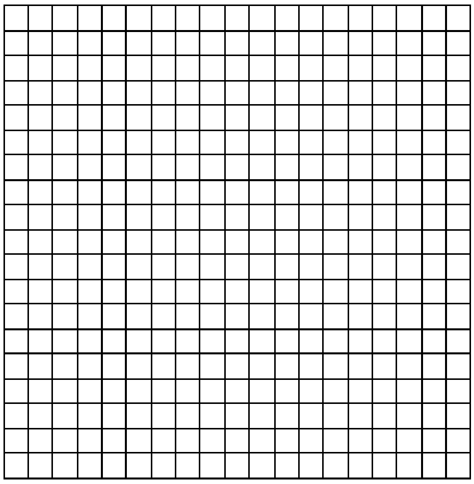
7 Modelization E

7.1 Characteristic of the modelization

Modelization: D_PLAN .

7.2 Characteristics of the mesh

The mesh (Figure 7.2-a) comprises 361 meshes type QUAD8.



Appear 7.2-a: The mesh of the modelization E.

7.3 Quantities tested and results

For each step of load, one calculates for all the nodes inside the disc, the absolute difference between the displacement obtained by computation of Code_Aster and that calculated analytically. For each one of these nodes, the difference must be null. To go more quickly one tests the sum of all these differences which must also be worth zero.

Not	Standard	Identification	Reference	% tolerance
1	SOMM	0	ANALYTIQUE	1E-8
2	SOMM	0	ANALYTIQUE	1E-8
3	SOMM	0	ANALYTIQUE	1E-8
4	SOMM	0	ANALYTIQUE	1E-8

7.4 Comments

This test validates the computation of the stiffness matrix in large rotations with elements QUAD8 .

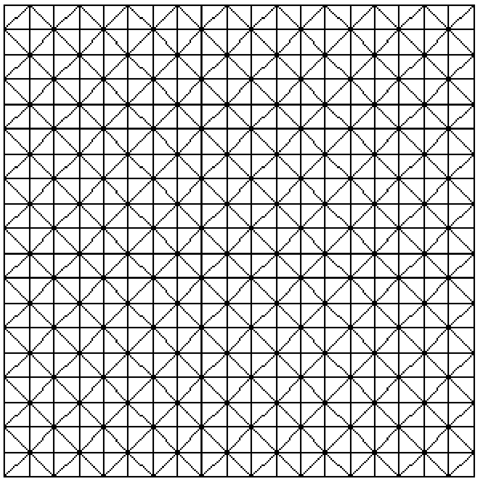
8 Modelization F

8.1 Characteristic of the modelization

Modelization: C_PLAN .

8.2 Characteristics of the mesh

The mesh (Figure 9.2-a) comprises 722 meshes type TRIA6.



Appear 9.2-a: The mesh of the modelization F.

8.3 Quantities tested and results

One tests the sum of the differences of displacements as for the modelization A .

Not	Standard	Identification	Reference	% tolerance
1	SOMM	0	ANALYTIQUE	1E-8
2	SOMM	0	ANALYTIQUE	1E-8
3	SOMM	0	ANALYTIQUE	1E-8
4	SOMM	0	ANALYTIQUE	1E-8

8.4 Comments

This test validates the computation of the stiffness matrix in large rotations with elements TRIA6.

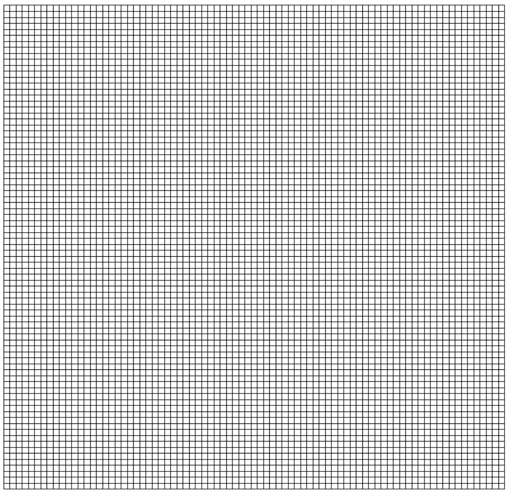
9 Modelization G

9.1 Characteristic of the modelization

Modelization: D_PLAN .
The contact is treated with linear X-FEM elements P1 (displacement) P1 (pressure), i.e. carrying the degrees of freedom of displacement and the Lagrange of contact on all the nodes (contact formulation with the nodes) .

9.2 Characteristics of the mesh

The mesh (Figure 9.2-a) comprises 10201 meshes type QUAD4.



Appear 9.2-a: The mesh of the modelization G.

9.3 Functionalities tested

One uses a diagram of integration Gauss points reduced to 3 per facet of contact. Friction is not taken into account and the contact is active as of the 1st iteration of active stresses. The algorithm aiming at restricting the space of the Lagrange multipliers is the n°2.

9.4 Quantities tested and results

One tests the maximum and minimal value of the normal pressure of contact post-treated with each time step.

Not	Standard	Identification	Reference	% tolerance
1	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	22.0
2	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	22.0
3	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	22.0
4	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	22.0
5	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	22.0
6	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	22.0

	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	22.0
7	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	22.0
8	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	22.0
9	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	22.0
10	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	22.0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	22.0

9.5 Comments

This valid test:

- réappariemment,
- the computation of the stiffness matrix in large rotations,
- the computation of the contact matrixes in great slidings (integration on a SE2 with Gauss points),
- the postprocessing X-FEM of the elements P1P1 .

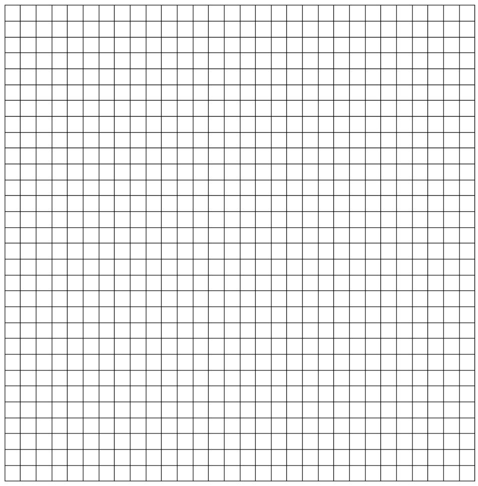
10 Modelization H

10.1 Characteristic of the modelization

Modelization: D_PLAN .
The contact is treated with linear elements X-FEM P2 (displacement) P1 (pressure), i.e. carrying the degrees of freedom of displacement on all the nodes and the Lagrange of contact on the nodes tops (contact formulation with the nodes) .

10.2 Characteristics of the mesh

The mesh (Figure 10.2-a) comprises 2500 meshes type QUAD8.



Appear 10.2-a: The mesh of the modelization I.

10.3 Functionalities tested

One uses a diagram of integration Gauss points reduced to 3 per facet of contact. Friction is not taken into account and the contact is active as of the 1st iteration of active stresses. The algorithm aiming at restricting the space of the Lagrange multipliers is the n°2.

10.4 Quantities tested and results

One tests the maximum and minimal value of the normal pressure of contact post-treated with each time step.

Not	Standard	Identification	Reference	% tolerance
1	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5.5
2	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5.5
3	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5.5
4	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5.5
5	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5.5
6	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5.5

	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5.5
7	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5.5
8	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5.5
9	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5.5
10	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5.5
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5.5

10.5 Comments

This valid test:

- réappariemment (curved interface and elements on right board),
- the computation of the stiffness matrix in large rotations,
- the computation of the contact matrixes in great slidings (integration on a SE3 with Gauss points),
- the postprocessing X-FEM of the elements P2P1 .

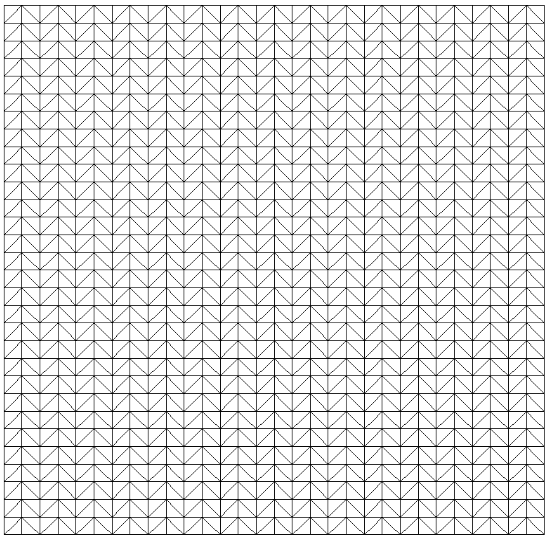
11 Modelization I

11.1 Characteristic of the modelization

Modelization: D_PLAN .
The contact is treated with linear elements X-FEM P2 (displacement) P1 (pressure), i.e. carrying the degrees of freedom of displacement on all the nodes and the Lagrange of contact on the nodes tops (contact formulation with the nodes) .

11.2 Characteristics of the mesh

The mesh (Figure 11.2-a) comprises 2601 meshes type TRIA6.



Appear 11.2-a: The mesh of the modelization J.

11.3 Functionalities tested

One uses a diagram of integration Gauss points reduced to 3 per facet of contact. Friction is not taken into account and the contact is active as of the 1st iteration of active stresses. The algorithm aiming at restricting the space of the Lagrange multipliers is the n°2.

11.4 Quantities tested and results

One tests the maximum and minimal value of the normal pressure of contact post-treated with each time step.

Not	Standard	Identification	Reference	% tolerance
1	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5,0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5,0
2	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5,0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5,0
3	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5,0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5,0
4	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5,0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5,0
5	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5,0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5,0

6	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5,0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5,0
7	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5,0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5,0
8	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5,0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5,0
9	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5,0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5,0
10	LAGS_C (MAX)	-1.11552e6	ANALYTIQUE	5,0
	LAGS_C (MIN)	-1.11552e6	ANALYTIQUE	5,0

11.5 Comments

This valid test:

- réappariemment (curved interface and elements on right board),
- the computation of the stiffness matrix in large rotations,
- the computation of the contact matrixes in great slidings (integration on a SE3 with Gauss points),
- the postprocessing X-FEM of the elements P2P1 .

12 Summary of the results

the goals of this test are achieved. They were to show the feasibility of the taking into account large rotations with X-FEM :

- in 2D (plane strains and plane stresses) for linear elements QUAD4 and TRIA3, and quadratic QUAD8 and TRIA6,
- in 3D for elements HEXA8 and TETRA4.

It was a question of the contact of showing the feasibility of the taking into account on the lips of the interface in great slidings:

- in 2D P1 (displacement) P1 (pressure) on a mesh quadrangle and triangle.
- in 2D P2 (displacement) P1 (pressure) on a mesh quadrangle and triangle.