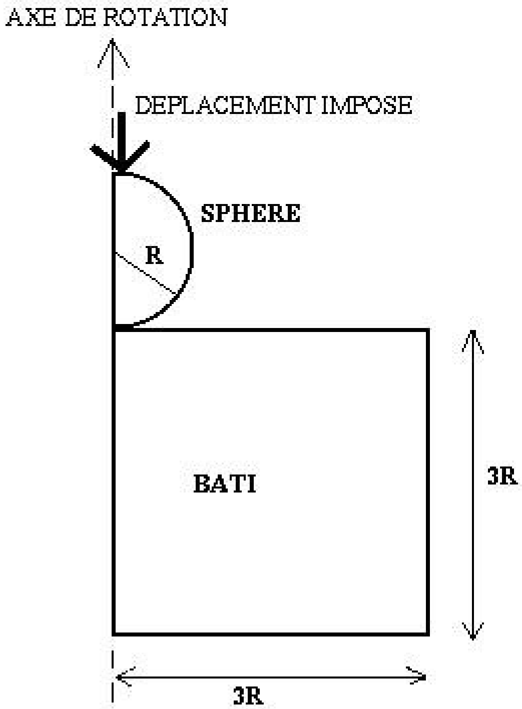
SSNV506 - Elastoplastic indentation of a block by a Summarized elastic spherical

indentor:

This test relates to the modelization of the indentation of an elastic sphere on a half-plane with the elastoplastic behavior. The purpose is to test the features related to the contact on an example comprising nona - linearity material.

1 Problem of reference

1.1 Geometry



Radius of the sphere	$R=500\,mm$
Displacement forced	$100\,mm$

1.2 Properties of the material

Two different modelizations to represent the rigid sphere:

Material rigidification: $E=2{,}1\mathrm{E}9\,Mpa$ and $\nu=0{,}3$
Rigidification by kinematical conditions

Block: Perfect steel, constitutive law élasto - plastic.

Modulate Young	$E=210000\,MPa$
Poisson's ratio	$\nu=0{,}3$
Hardening modulus	$Et=0$
Yield stress	$\sigma_y=50\;MPa$

1.3 Boundary conditions and loadings

the strains are axisymmetric and the block forming the plane is supposed to be embedded on its basis.

An imposed displacement is applied:

- Loading of 0 with $-100\,mm$ on the part higher of the sphere in the models A and D
- Loading of 0 than $-100\,mm$ on the contact surface of the sphere in the models B and C

2 Reference solution

2.1 Méthode de calcul used for the reference solution

the results of reference result from the book quoted below [bib1].

$p_m = 3\sigma_0$ with p_m the contact pressure (page 171).

$Rjohnson = p_m a = 3\sigma_0 a$ if a is contact surface.

However in perfect plasticity $\delta = 0{,}368\, a^2/R$ according to the analysis of Richmond (page 200)

Finally, one obtains:

$Rjohnson = 3\pi R \sigma_0 \delta / 0{,}368$

$Rjohnson$: Normal reaction of contact of the solid mass on the sphere
R : Radius of the sphere
δ : Displacement of the top of the solid mass
σ_0 : Yield stress of the solid mass

This result is valid under the following assumptions:

- axisymmetric problem,
- perfectly plastic material (coefficient 0,368 is resulting from this assumption)
- small strains
- rigid sphere.

2.2 Results of reference

the results of reference are got from the preceding formula. It is valid for the model complete in 3D.

Note:

In our study, Rjohnson depends only on displacement, one can write the relation in the following form thanks to the data of problem: $Rjohnson = 640\,270\,\delta$ *with* $Rjohnson$ *in newton and* δ *millimetre.* δ *is directly connected to the time of computation.*

The value of the normal resultant of contact coming from ASTER is given on a district of 1 radian of opening in 2D axisymmetric and on a district of $\pi/2$ for the model 3D (by symmetry, it is enough to model the quarter of the problem).

Thus, the values of reference are:

in 2D axisymmetric : $Rref = Rjohnson/2\pi = 101902{,}1\,\delta$
in 3D : $Rref = Rjohnson/4 = 160067{,}5\,\delta$

2.3 Uncertainties on the analytical

solution Solution.

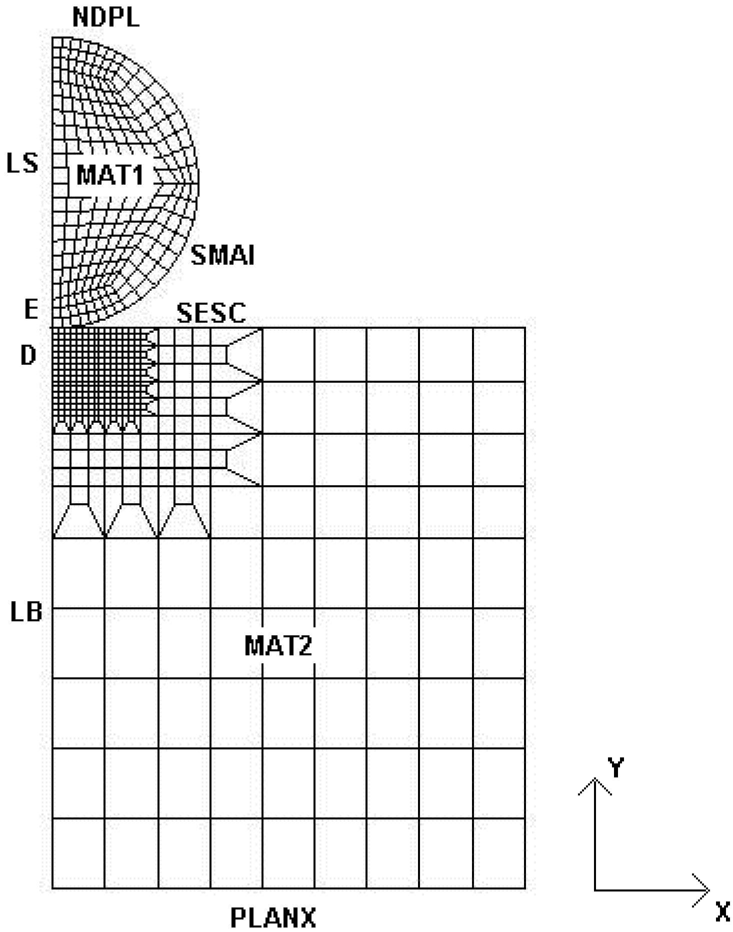
2.4 Bibliographical reference

1 “Contact Mechanics” - K.L. JOHNSON - Cambridge University Close - chapter 6 p. 153 - 201

3 Modelization A

3.1 Characteristic of the modelization

the symmetry of revolution of the problem allows an axisymmetric modelization: The sphere and the block are represented respectively by a half disc and the cut of half of the block, with a grid with elements 2D axisymmetric.
A contact of the node-mesh type is defined between two structures.
A loading in imposed displacement is applied to the upper part of the sphere rigidified by a high Young's modulus.



Boundary condition:

- symmetry of revolution: the nodes located on the axis Y (nodes group *« LB »* and *« LS »*) are blocked according to the direction X (DX = 0),
- fixed support of the base: the nodes of the group *« PLANX »* are blocked according to the directions X and Y (DX = DY = 0),
- motions of rigid bodies are removed by imposing a connection following y between the node E pertaining to the sphere and the node D belonging to massive.

Loadings:

An imposed displacement is applied to the upper part of the sphere (nodes group *« NDPL »*) according to the direction Y : Loading of 0 with $-100.mm$

3.2 Characteristics of the mesh

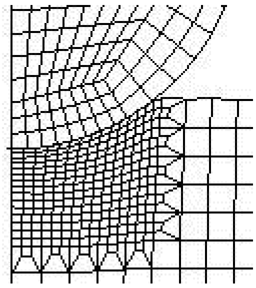
Many nodes: 916
Number of meshes and type: 625 QUAD4 and 289 SEG2

3.3 Values tested

Identification	Displacement (mm)	Reference	*Aster*	% tolerance
Reaction (N)	20	– 2.03804E+06	– 2.06806E+06	5
Reaction (N)	40	– 4.07608E+06	– 4.04698E+06	5
Reaction (N)	60	– 6.11412E+06	– 5.82730E+06	5
Reaction (N)	80	– 8.15217E+06	– 7.66632E+06	10
Reaction (N)	100	– 1.01902E+07	– 9.11899E+06	15

3.4 Remarks

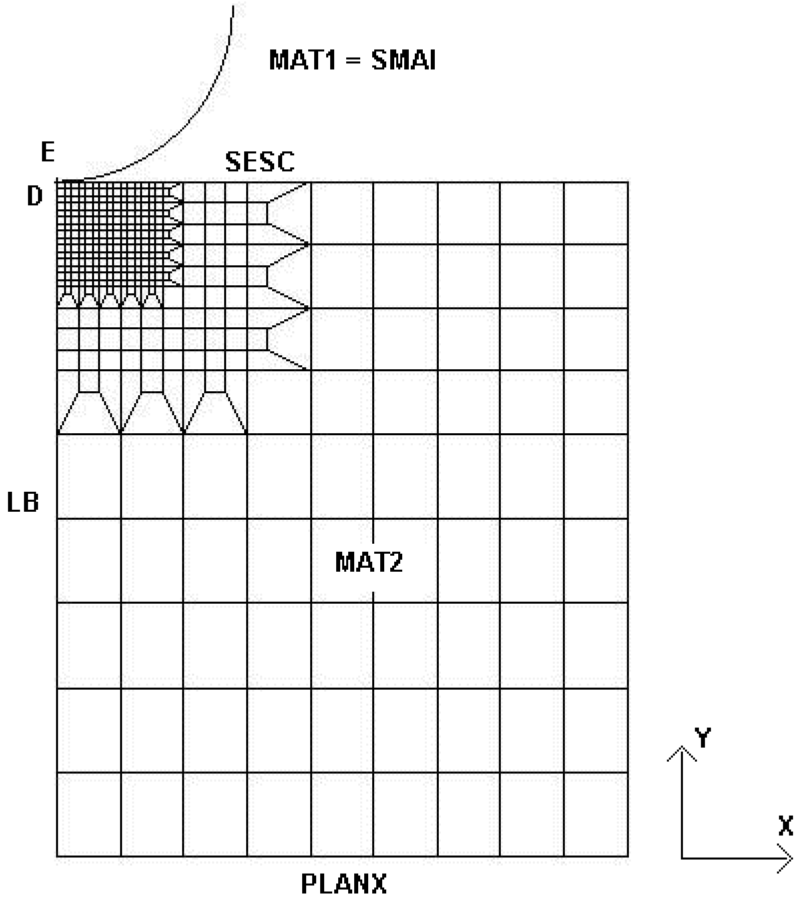
the most important error is for the last result. It remains acceptable nevertheless.
We illustrated the strain of the solid mass to time step final:



4 Modelization B

4.1 Characteristic of the modelization

the symmetry of revolution of the problem allows an axisymmetric modelization: The block is represented by the cut of its half and the sphere is represented by its surface potentially in contact, they are with a grid with elements 2D axisymmetric.
A contact of the node-mesh type is defined between two structures.
A loading in imposed displacement is applied to all meshes representing the sphere, rigidified by kinematical conditions.



Boundary condition:

- Conditions of symmetry: the nodes of the frame located on the axis Y (nodes group *« LB »*) are blocked according to the direction X (`DX = 0`). All the nodes belonging to the sphere (nodes group *« MAT1 »*) are blocked according to the direction X (`DX = 0`).
- Fixed support of the base: the nodes of *« PLANX »* are blocked according to the directions X and Y (`DX = DY = 0`).
- Motions of rigid bodies are removed by imposing a connection rigid, following y , between the node E pertaining to the sphere and the node D belonging to massive.

Loadings:

An imposed displacement is applied to the part representing the sphere (group of node *« MAT1 »*) according to the direction Y : Loading of 0 with $-100.mm$

4.2 Characteristics of the mesh

Many nodes: 458
Number of meshes and type: 419 QUAD4 and 171 SEG2.

4.3 Values tested

Identification	Displacement (mm)	Reference	*Aster*	% difference
Reaction (N)	$d=-20\,mm$	– 2.06771E+06	-2.0677082E+06	10
Reaction (N)	$d=-40\,mm$	– 4.04742E+06	-4.0474212E+06	10
Reaction (N)	$d=-60\,mm$	– 5.82779E+06	-5.8277879E+06	10
Reaction (N)	$d=-80\,mm$	– 7.66673E+06	-7.6667317E+06	10
Reaction (N)	$d=-100\,mm$	– 9.11942E+06	-9.1194226E+06	15

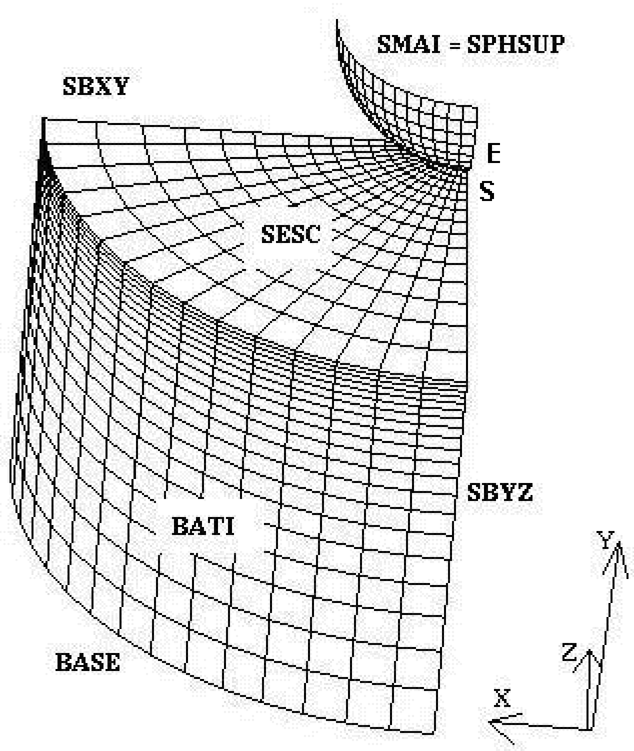
4.4 Remarks

the results are almost identical to those of modelization A.
One notices a computing time reduced by modelling only the contact surface of the sphere rigidified by kinematical conditions.

5 Modelization C

5.1 Characteristic of the modelization

the symmetry of the problem makes it possible not to represent in 3D that a quarter of the model: the sphere and the block are represented respectively by the contact surface of the sphere and a quarter of cylinder, with a grid with solid elements 3D CUB8.
A contact node-mesh is defined between the sphere and the block.
A loading in imposed displacement is applied to all the surface of the sphere rigidified by kinematical conditions.



Boundary condition:

- Conditions of symmetry: the nodes located in the plane (O, y, z) (nodes group $« SBYZ »$) are blocked according to the direction X (DX = 0),
  the nodes located in the plane (O, x, y) (nodes group $« SBXY »$) are blocked according to the direction Z (DZ = 0),
  the nodes of the sphere (nodes group "SPHSUP") are blocked according to the directions X and Z (DX = DZ = 0)
- Fixed support of the base: the nodes of the group $« BASE »$ (plane $Y=0.$) are blocked according to the directions X Y, and Z (DX = DY = DZ = 0).
- Motions of rigid bodies are removed by imposing a connection following there enters the node E pertaining to the sphere and the node S belonging to massive.

Loadings:

An imposed displacement is applied to all surface representing the sphere (nodes group $« SPHSUP »$) according to the direction Y : Loading of 0 with $-100.mm$

5.2 Characteristics of the mesh

Many nodes: 6852
Number of meshes and type: 5326 HEXA8, 387 PENTA6 and 183 QUAD4.

5.3 Values tested

Identification	Displacements	Reference	*Aster*	% tolerance
Reaction (N)	$d=-20mm$	– 3.201351E+06	-3.2100211E+06	1
Reaction (N)	$d=-40\,mm$	– 6.402702E+06	-6.1671049E+06	5
Reaction (N)	$d=-60mm$	– 9.604053E+06	-9.1689400E+06	5
Reaction (N)	$d=-80mm$	– 1.280540E+07	-1.1738899E+07	10
Reaction (N)	$d=-100mm$	– 1.600675E+07	-1.4244367E+07	12

5.4 Remarks

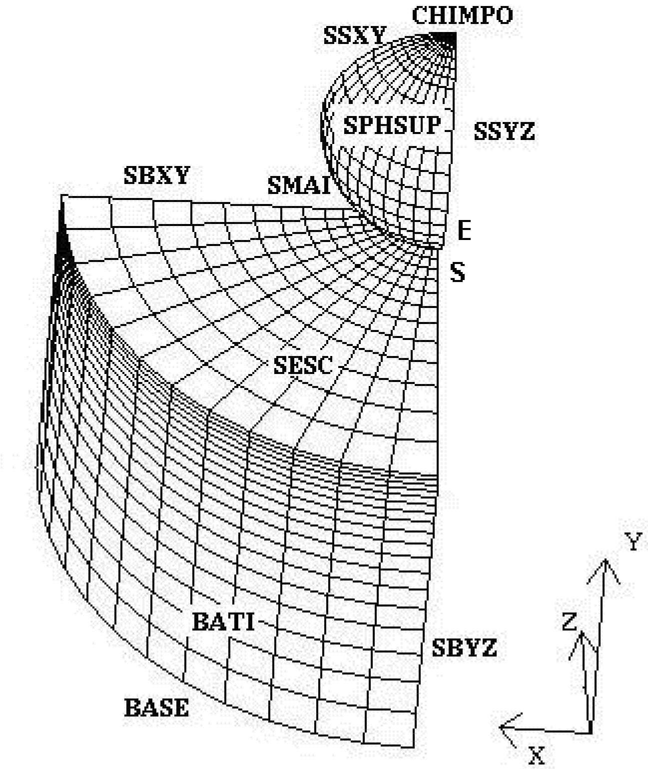
the results are less precise than those resulting from the modelizations 2D. The mesh in 3D made lose the exact character of the axisymmetric case. Moreover, for savings of time of computation and memory capacity, the mesh 3D is refined less than that in 2D.

6 Modelization D

6.1 Characteristic of the modelization

the symmetry of the problem makes it possible not to represent in 3D that a quarter of the model: The sphere and the block are represented respectively by a quarter of sphere and a quarter of cylinder, with a grid with solid elements 3D CUB8.

A contact node-mesh is defined between the sphere and the block.
A loading in imposed displacement is applied to the upper part of the sphere rigidified by a high Young modulus.



Boundary condition:

- Conditions of symmetry: the nodes located in the plane (O, y, z) (nodes groups *« SBYZ »* and *« SSYZ »*) are blocked according to the direction X (`DX = 0`),
  the nodes located in the plane (O, x, y) (nodes groups *« SBXY »* and *« SSXY »*) are blocked according to the direction Z (`DZ = 0`).
- Fixed support of the base: the nodes of *« BASE »* (plane $Y=0.$) are blocked according to the directions X Y, and Z (`DX = DY = DZ = 0`).
- Motions of rigid bodies are removed by imposing a connection following there enters the node E pertaining to the sphere and the node S belonging to massive.

Loadings:

An imposed displacement is applied to the upper part of the sphere (nodes group *« CHIMPO »*) according to the direction Y : Loading of 0 with $-100.\,mm$

6.2 Characteristics of the mesh

Many nodes: 6993
Number of meshes and type: 5544 HEXA8, 407 PENTA6 and 191 QUAD4

6.3 Values tested

Identification	Displacements	Reference	*Aster*	% tolerance
Reaction (N)	$d=-20\,mm$	– 3.201351E+06	-3.82828724E+06	25
Reaction (N)	$d=-40\,mm$	– 6.402702E+06	-7.38942843E+06	20
Reaction (N)	$d=-60\,mm$	– 9.604053E+06	-1.06420713E+07	15
Reaction (N)	$d=-80\,mm$	– 1.280540E+07	-1.28912992E+07	10
Reaction (N)	$d=-100\,mm$	– 1.600675E+07	-1.56376456E+07	5

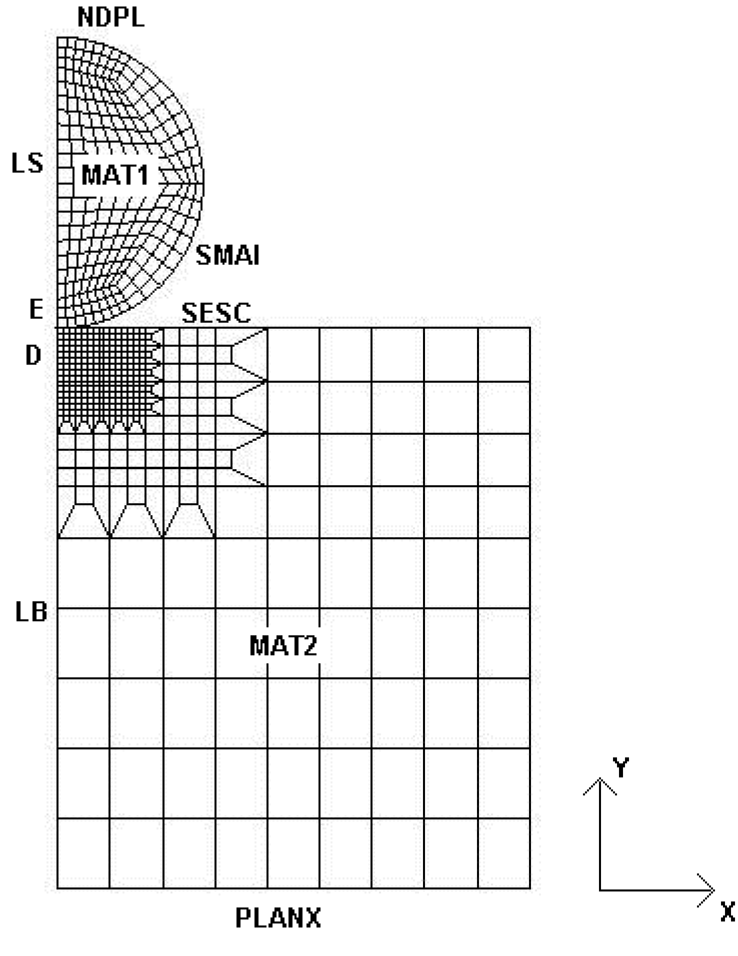
6.4 Remarks

the results are almost identical to those of the modelization C. But computation is even more tiresome because a quarter of the sphere is with a grid.

7 Modelization E

7.1 Characteristic of the modelization

the symmetry of revolution of the problem allows an axisymmetric modelization: The block is represented by the cut of its half and the sphere is represented by its surface potentially in contact, they are with a grid with elements 2D axisymmetric.
A contact of the node-mesh type is defined between two structures.
A loading in imposed displacement is applied to the upper part of the sphere rigidified by a high Young's modulus.



Boundary condition:

- symmetry of revolution: the nodes located on the axis Y (nodes group *« LB »* and *« LS »*) are blocked according to the direction X (`DX = 0`),
- fixed support of the base: the nodes of the group *« PLANX »* are blocked according to the directions X and Y (`DX = DY = 0`),
- motions of rigid bodies are removed by imposing a connection following y between the node E pertaining to the sphere and the node D belonging to massive.

Loadings:

An imposed displacement is applied to the upper part of the sphere (nodes group *« NDPL »*) according to the direction Y : Loading of 0 with $-100.\,mm$

7.2 Characteristics of the mesh

Many nodes: 688
Number of meshes and type: 625 QUAD4 and 241 SEG2.

7.3 Values tested

Identification	Displacement (mm)	Reference	*Aster*	% tolerance
Reaction (N)	20	– 2.03804E+06	-2.0892265E+06	5
Reaction (N)	40	– 4.07608E+06	-4.0928499E+06	5
Reaction (N)	60	– 6.11412E+06	-5.8467590E+06	5
Reaction (N)	80	– 8.15217E+06	-7.6820567E+06	10
Reaction (N)	100	– 1.01902E+07	-9.1299258E+06	15

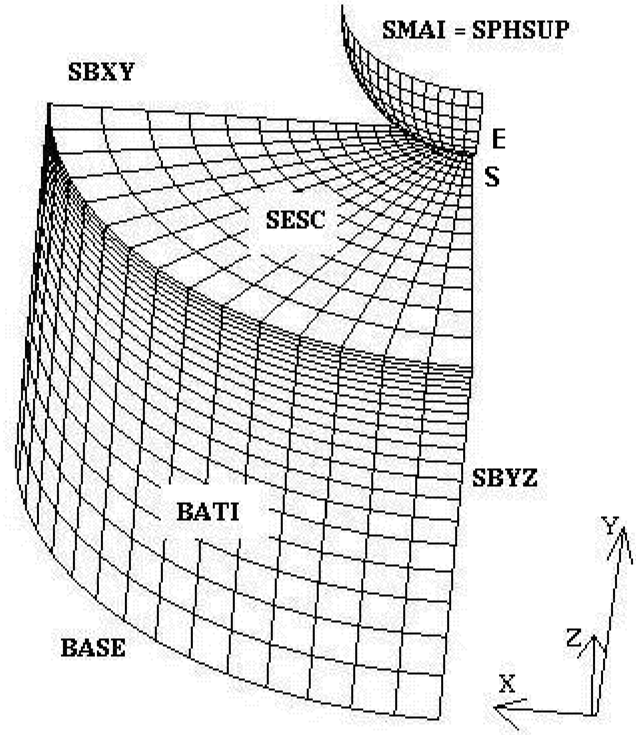
7.4 Remarks

the results are slightly better than those of modelization A.
One higher than notices a computing time 5 times the latter, using the FORCED `method`.

8 Modelization F

8.1 Characteristic of the modelization

the symmetry of the problem makes it possible not to represent in 3D that a quarter of the model: the sphere and the block are represented respectively by the contact surface of the sphere and a quarter of cylinder, with a grid with solid elements 3D CUB8.
A contact node-mesh is defined between the sphere and the block.
A loading in imposed displacement is applied to all the surface of the sphere rigidified by kinematical conditions.



Boundary condition:

- Conditions of symmetry: the nodes located in the plane (O, y, z) (nodes group « $SBYZ$ ») are blocked according to the direction X (DX = 0),
  the nodes located in the plane (O, x, y) (nodes group « $SBXY$ ») are blocked according to the direction Z (DZ = 0),
  the nodes of the sphere (nodes group « $SPHSUP$ ») are blocked according to the directions X and Z (DX = DZ = 0)
- Fixed support of the base: the nodes of group “BASE” (plane $Y=0.$) are blocked according to the directions X, Y, and Z (DX = DY = DZ = 0).
- Motions of rigid bodies are removed by imposing a connection following there enters the node E pertaining to the sphere and the node S belonging to massive.

Loadings:

An imposed displacement is applied to all surface representing the sphere (nodes group « $SPHSUP$ ») according to the direction Y : Loading of 0 with $-100.mm$

8.2 Characteristics of the mesh

Many nodes: 2236
Number of meshes and type: 1638 HEXA8, 126 PENTA6, 725 QUAD4, 27 TRIA3 and 26 SEG2.

8.3 Values tested

Identification	Displacements	Reference	*Aster*	% tolerance
Reaction (N)	$d=-20\,mm$	– 3.201351E+06	-3.6477118E+05	20

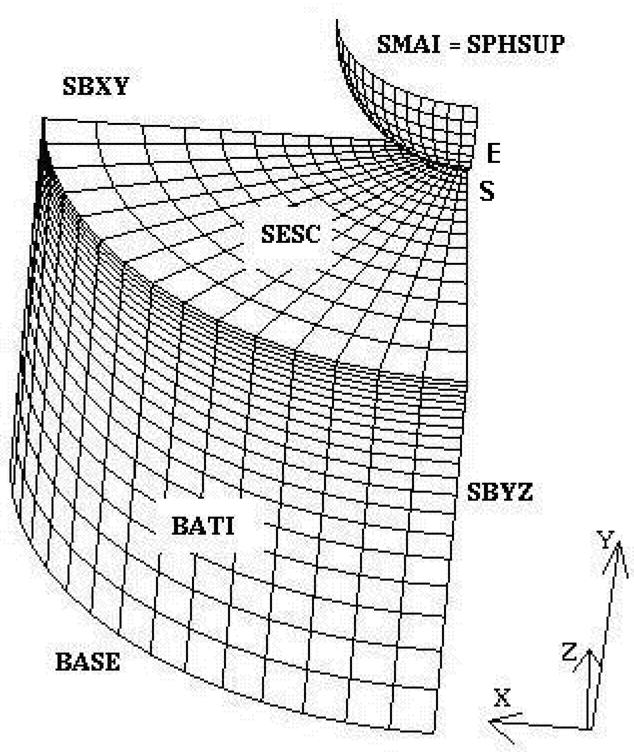
8.4 Remarks

the results are less precise than those resulting from the modelizations 2D. The mesh in 3D made lose the exact character of the axisymmetric case. Moreover, for savings of time of computation and memory capacity, the mesh 3D is refined less than that in 2D.

9 Modelization G

9.1 Characteristic of the modelization

the symmetry of the problem makes it possible not to represent in 3D that a quarter of the model: the sphere and the block are represented respectively by the contact surface of the sphere and a quarter of cylinder, with a grid with solid elements 3D CUB8.
A contact node-mesh is defined between the sphere and the block.
A loading in imposed displacement is applied to all the surface of the sphere rigidified by kinematical conditions.



Boundary condition:

- Conditions of symmetry: the nodes located in the plane (O,y,z) (nodes group *« SBYZ »*) are blocked according to the direction X (DX = 0),
  the nodes located in the plane (O,x,y) (nodes group *« SBXY »*) are blocked according to the direction Z (DZ = 0),
  the nodes of the sphere (nodes group *« SMAI »*) are blocked according to the directions X and Z (DX = DZ = 0)
- Fixed support of the base: the nodes of the group *« BASE »* (plane $Y=0.$) are blocked according to the directions X, Y, and Z (DX = DY = DZ = 0).
- Motions of rigid bodies are removed by imposing a connection following there enters the node E pertaining to the sphere and the node S belonging to massive.

Loadings:

An imposed displacement is applied to all surface representing the sphere (nodes group *« SMAI »*) according to the direction Y: Loading of 0 with $-100.mm$

9.2 Characteristics of the mesh

Many nodes: 2157
Number of meshes and type: 1496 HEXA8, 108 PENTA6, 988 QUAD4, 12 TRIA3 and 8 PYRAM5.

9.3 Values tested

Identification	Displacements	Reference	*Aster*	% tolerance
Reaction (N)	$d=-20mm$	– 3.201351E+05	-3.6492888E+05	20

10 Summary of the results

the got results are good. However, a more important difference between the reference and the results 3D exists. It is possible to fill it by refining even more the mesh but it should be paid out of core memory and computing times.

The size of the elements is very important. If they are too large, one can see appearing on the curve of the reaction according to the displacement of the “waves” (loss of linearity of this curve). Each “vagueness” corresponds to the setting in contact of an element. Moreover, if the mesh is not sufficiently refined, the reaction given by *Aster* moves away appreciably from that of reference.

Not to model that the sphere by its contact surface rigidified by kinematical conditions allows a time-saver.