SSNV223 - Elementary validation of model `ENDO_SCALAIRE` and control `PRED_ELAS` for Summarized `modelization`

GRAD_VARI:

The purpose of this test is validating the algorithm of integration of constitutive law `ENDO_SCALAIRE` to gradient of local variables as well as control `PRED_ELAS` available for this model. The studied problem corresponds to a request with imposed homogeneous strain for which one can obtain an analytical solution.

The various treated modelizations are following:

- **Modelization A** ($2D$): Modelization D_PLAN_GRAD_VARI `is employed`.
- **Modelization B** ($3D$): Modelization 3D_GRAD_VARI `is employed`.

1 Problem of reference

1.1 Geometry

According to the modelization $2D$ or $3D$, one respectively considers a square or a cube on side 2 mm.

1.2 Properties of the material

the material obeys elastic constitutive law brittle ENDO_SCALAIRE with gradient of damage (D_PLAN_GRAD_VARI and 3D_GRAD_VARI). The macroscopic data correspond to:

$E=30\,000\,MPa$	Modulus Young
$\nu=0.2$	Poisson's ratio
$G_f=0.1\,N/mm^2$	Energy of cracking
$p=5$	Limiting Parameter of
$f_t=3\,MPa$	form in Limiting
$f_c=15\,MPa$	tension in Limiting
$\tau=4\,MPa$	compression in shears
$D=50\,mm$	Half-width of the tape of damage

the internal parameters of the model, as described in the booklet [U4.43.01], are obtained by the formulas which are presented there. They lead to the following results:

```
Key key: ELAS            ENDO_SCALAIRE              NON_LOCAL
           E = 3.E4           K = 31.5E-3                C_GRAD_VARI = 1.875
           NU = 0.2           M = 10
                              P = 5
                              C_VOLU = 3.68
                              C_COMP = 1.847520861
```

1.3 Boundary conditions and loadings

displacements are imposed in all the nodes of structure, of kind to correspond to the desired homogeneous strain. More precisely, displacement in a node of coordinates X is worth: $u(x)=\varepsilon\cdot x$

1.4 Initial conditions

None.

2 Reference solution

2.1 Method of calculating used for the reference solution

This problem admits an analytical solution. One determines the relation which with the imposed strain $\boldsymbol{\varepsilon}$ associates the homogeneous level of damage a (reciprocal problem where more generally NON-room does not admit any more a simple analytical solution). The problem being homogeneous, the damage (in load) and the strain are bound by the relation of coherence (function threshold):

$$A'(a)\Gamma(\boldsymbol{\varepsilon})+\omega'(a)=0$$

with

$$\omega(a)=ka, \quad A(a)=\frac{(1-a)^2}{(1-a)^2+ma(1+pa)} \quad \text{et} \quad \Gamma(\boldsymbol{\varepsilon})=\left[c_T tr\,\boldsymbol{\varepsilon}+\sqrt{c_H tr^2\boldsymbol{\varepsilon}+c_S\varepsilon_{eq}^2}\right]$$

where the parameters c_T, c_H et c_S result from the facts of the case by:

$$c_S=\frac{E}{2}\left[(1-2\nu)c_{comp}+(1+\nu)\sqrt{\frac{1-2\nu}{2(1+\nu)}c_{volu}+1}\right] \quad ;c_T=c_{comp}\sqrt{c_S} \quad ;c_H=\frac{1+\nu}{2(1-2\nu)}c_{volu}c_S$$

One adopts a uniaxial strain of the form:

$$\boldsymbol{\varepsilon}=\varepsilon\,\boldsymbol{n}\otimes\boldsymbol{n} \quad \text{où} \quad \|\boldsymbol{n}\|=1 \quad \text{et} \quad \varepsilon>0$$

In this case the function threshold in strain is written simply: $\Gamma(\boldsymbol{\varepsilon})=\left[c_T+\sqrt{c_H+c_S}\right]\varepsilon$

To reach the damage a given, by requesting the strain in the direction $\boldsymbol{n}$, it is thus necessary to force an intensity of strain:

$$\varepsilon=\frac{-k}{A'(a)\left[c_T+\sqrt{c_H+c_S}\right]}$$

For the reference solution we thus adopt a following strategy: one sets the level of damage is one checks by computations EF, that for an estimated theoretical strain one reaches this same level of damage.

2.2 Results of reference

In plane strains, a direction of request is adopted $\boldsymbol{n}=(1/\sqrt{5},\ 2/\sqrt{5})$. In $3D$, it is worth $\boldsymbol{n}=(1/\sqrt{14},\ 2/\sqrt{14},\ 3/\sqrt{14})$. One sets like target a damage $a=0.6$; that corresponds to an intensity of request $\varepsilon=9.574237\times10^{-4}$ according to the reference solution above.

The loading is applied with the help of the technique of control `PRED_ELAS` in which one fixes the maximum limit of kind to reach the level of strain ε above. It will be checked that the damage corresponding reached well 0.6 .

2.3 Uncertainties on the solution

Nothing

2.4 bibliographical References

Without Modelization

3 object A

3.1 Characteristic of the modelization

a modelization D_PLAN_GRAD_VARI with a single mesh, element QUAD8 .
Loading in the direction $\boldsymbol{n}=(1/\sqrt{5},\ 2/\sqrt{5})$.

3.2 Characteristics of the mesh

Many nodes: 8
Number and types of meshes: 1 QUAD8, 4 SEG3

3.3 Quantities tested and results of the modelization A

One tests the damage in three nodes of the mesh, the value with the nodes being obtained by extrapolation (CHAM_NO "VARI_NOEU", component V1).

Standard	identification	Reference	Tolerance
V1 ($X=2$, $Y=0$)	0.6	ANALYTIQUE	RELATIF – 0,1%
V1 ($X=2$, $Y=1$)	0.6	ANALYTIQUE	RELATIF – 0,1%
V1 ($X=2$, $Y=2$)	0.6	ANALYTIQUE	RELATIF – 0,1%

4 Modelization B

4.1 Characteristic of the modelization

a modelization 3D_GRAD_VARI with a single mesh, element HEXA20 .
Loading in the direction $\boldsymbol{n}=(1/\sqrt{14},\ 2/\sqrt{14},\ 3/\sqrt{14})$.

4.2 Characteristics of the mesh

Many nodes: 20
Number and types of meshes: 1 HEXA20 , 6 QUAD8, 8 SEG3

4.3 Quantities tested and results of the modelization B

One tests the damage in three nodes of the mesh, the value with the nodes being obtained by extrapolation (CHAM_NO "VARI_NOEU", component V1).

Standard	identification	Reference	Tolerance
V1 ($X=2$, $Y=0$)	0.6	ANALYTIQUE	RELATIF – 0,1%
V1 ($X=2$, $Y=1$)	0.6	ANALYTIQUE	RELATIF – 0,1%
V1 ($X=2$, $Y=2$)	0.6	ANALYTIQUE	RELATIF – 0,1%

5 Summary of the results

This case-test is carried out on only one mesh, consequently it is the homogeneous response of damage which is found numerically. The reference solution is obtained while being put on the threshold of damage. One notes very a good agreement between the modelization and the reference solution. NON-local part of the model nevertheless is not tested.