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## FORMA04 - Practical works of training “advanced Use”: contact of Hertz

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### Summarized:

This TP deals of the contact with Hertz, one of the rare nontrivial cases of contact for which one has an analytical solution.

The model consists of two half-spheres which one comes to put in contact one against the other.

This simple test will make it possible of the contact to illustrate the setting in data, the various ways solve the problem and finally the tools at disposal post-to treat a computation of contact.

## Contents

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1 Problem of référence3.....	3
1.1 Géométrie3.....	3
1.2 Properties of the matériaux3.....	3
1.3 Boundary conditions and chargements4.....	4
2 Solution of référence5.....	5
2.1 Method of calcul5.....	5
2.2 Quantities and results of référence5.....	5
2.3 Uncertainties on the solution5.....	5
2.4 Références5.....	5
3 Modelization A6.....	6
3.1 Implementation of the TP6.....	6
3.1.1 Géométrie6.....	6
reading Well the statement in integer before commencer6.....	6
3.1.2 Maillage6.....	6
3.1.3 File of commandes7.....	7
3.1.4 Pressure of Hertz7.....	7
3.1.5 Stress of Von Mises8.....	8
3.2 Quantities tested and résultats8.....	8
4 Modelization B9.....	9
4.1 Implementation of the TP9.....	9
4.1.1 continue9 Formulation.....	9
4.2 Quantities tested and résultats9.....	9
5 Modelization C10.....	10
5.1 Implementation of the TP10.....	10
5.1.1 Realization of the géométrie10.....	10
5.1.2 Realization of the maillage10.....	10
5.2 Quantities tested and résultats11.....	11



## 1.3 Boundary conditions and loadings

- following Displacement  $DX$  blocked on the axis  $A1A2$  (this condition is implicit into axisymmetric, one imposes it nevertheless for the points which would not be perfectly on the axis)
- imposed following  $DY$  Displacement:
  - +2 mm on  $A1B1$
  - 2 mm on  $A2B2$

## 2 Reference solution

### 2.1 Method of calculating

In 1881, Hertz established under certain assumptions a solution with the problem which bears its name [1]. Thus by supposing a contact without friction and for which the half-width of contact  $a$  is very small in front of the radius of the spheres  $R$  ( $a \ll R$ ), the contact pressure at the points  $C1$  and  $C2$  is worth [2] :

$$P_0 = - \frac{E}{\pi(1-\nu^2)} \sqrt{\frac{2h}{R}} \quad \text{éq 2.1-1}$$

where  $h$  corresponds to the imposed crushing, which is worth 4 here  $mm$ .

The half-width of contact  $a$  is expressed according to imposed crushing and of the radius of the spheres:

$$a = \sqrt{\frac{Rh}{2}} \quad \text{éq 2.1-2}$$

contact surface is a disc of radius  $a$ , the distribution of pressure according to the radius  $r$  in this zone is the following one:

$$\text{So } r \leq a \text{ then } P(r) = P_0 \sqrt{1 - \left(\frac{r}{a}\right)^2} \quad \text{éq 2.1-3}$$

### 2.2 Quantities and results of reference

One will compare the pressure obtained with the center of the contact zone and the half-width of contact to the analytical solution above.

### 2.3 Uncertainties on the solution

No (under the assumptions given above).

### 2.4 References

[1] HERTZ H. Über die Berührung fester elastischer Körper. *Newspaper für die queen und angewandte Mathematik*, 92:156-171, 1881.

[2] DUMONT G, *method of the active stresses applied to the unilateral contact*. Note technical, EDF-DER, 1993. HI-75/93/016.

## 3 Modelization A

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### 3.1 Implemented of the TP

#### 3.1.1 Géométrie

**to read Well the statement in integer before beginning**

the geometry is carried out with Salome, modulus GEOM. The tools “*Sketcher*” (Finely *New Entity > BASIC > 2D Sketch*) make it possible to build **the contour of the higher disc** (for the arc of circle one can enter an angle and a negative radius). One can then build **a face on this contour** (Finely *New Entity > Build > Face*). The point *D2* will be created during the construction of contour (the arc of a circle is made in 2 times).

By symmetry, one can generate the second sphere (Finely *Operations > Transformation > Mirror image*). **It is necessary to build a horizontal line or a plane to carry out symmetry.**

It remains **to assemble the two spheres** to constitute one object GEOM (Finely *New Entity > Build > Compound*). Finally it is necessary **to create groups** on this geometry (with the names suggested in the figure above).

**One draws attention to the fact that the groups are created in the last.**

To create the points *C1* and *C2*, one uses the functionality allowing to select under element of an entity GEOM (In limps of dialog *Create Group*, notch “*Only Sub-shapes of the Second Shape*” then for *C2* for example, to select like “*Second half Shape*” it higher sphere in the shaft of the objects).

To create the groups for the application of the boundary conditions: along the axis of axisymetry (*A1A2*) and on the upper parts and lower of the half-spheres (*A1B1* and *A2B2*).

For the modelization of contact, one will need for mesh groups representing potential surfaces of contact: to create the groups *CONT1* and *CONT2* with *Create Group*.

#### 3.1.2 Mesh

One uses modulus SMESH. The mesh is realized in triangles. One will use the assumptions of automatic mesh “*Assign has set of assumptions > 2D: Automatic triangulation*” by choosing a maximum length of 2. One then obtains a mesh containing approximately 3000 triangles and 1500 nodes.

One will import then the groups from the geometry (“*Create groups from geometry*”).

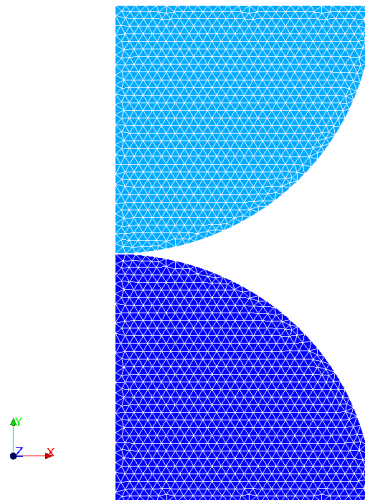


Illustration 3.1: Mesh obtained  
for the modelization A

### 3.1.3 Command file

- By means of nonlinear operator `STAT_NON_LINE`, to build the command file relating to the benchmark by not making the contact counts some initially.

**What does one note on convergence? What would be necessary to make to converge?**

| *Index: how the by default convergence criterion is builds?*

- To add the definition of the contact: **to direct surfaces**, to define the contact load with operator `DEFI_CONTACT` and to adapt the options of the nonlinear solver.

| *An alarm appears computation with contact during. Once understood, it can be removed by informing key word `SYME` of `STAT_NON_LINE` (under `solver`). One will initially preserve the method of contact per default (`FORMULATION=' DISCRETE'` and `"FORCED ALGORITHME_CONT=' "`).*

- Question no-claims bonus: how many y-a it rigid body motions in axisymmetric modelization?

| *Recall: a rigid body motion is a motion with strain null.*

### 3.1.4 Pressure of numerical

- Hertz Application: to calculate with the hand with the facts of the case, the value of the pressure of Hertz and half-width of contact. To compare with the values obtained by computation.

**What does one note? Why this variation?**

| *Note: one will take the component at first approximation `SIYY` `SIEF_NOEU` like contact pressure*

- To trace the contact pressure according to the radius in the contact zone, to compare it with the analytical solution.

| *One will use commands `DEFI_GROUP` (`CREA_GROUP_NO/OPTION=' NOEUD_ORDO'`) in order to create a nodes group with a directed curvilinear abscisse, `POST_RELEVE_T` to extract in an array the component `SIYY` along this group (a column of this array called `ABSC_CURV` defines the curvilinear abscisse) and finally `RECU_FONCTION` (`ARRAY/PARA_X/PARA_Y`) to create the function. One will print it with format `XMGRACE` with `IMPR_FONCTION` (`COURBE/FONCTION`) by selecting a "pilot" `INTERACTIF`.*

Warning : The translation process used on this website is a "Machine Translation". It may be imprecise and inaccurate in whole or in part and is provided as a convenience.

## 3.1.5 Von Mises stress

- To display the von Mises stress on the configuration deformed with Salomé. **How does this stress in the sphere vary? Where is the maximum?**

One will display component `VMIS_SG` of the field of equivalent stresses `SIEQ_NOEU` calculated with operator `CALC_CHAMP`.

## 3.2 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\sigma_{yy}$ not <i>CI</i>	"ANALYTIQUE"	-2798,3 Mpa	1,0%
$\sigma_{yy}$ <i>CI</i>	"NON_REGRESSION"	-2773,5 Mpa	0,1%



## 4 Modelization B

One will set out again of modelization A. The mesh is the same one as that of the modelization A **but it passed into quadratic.**

### 4.1 Implemented of the TP

#### 4.1.1 Formulation continues

•To convert the mesh of the modelization A into quadratic mesh in modulus SMESH (Finely Modification > Convert to/from quadratic)

*In order to benefit fully from the quadratic elements and to obtain a smooth profile of pressure, it is important to place the nodes mediums on the geometry during conversion (to notch for that the box "Medium nodes one geometry" during conversion).*

•To change method of contact and to use the formulation "CONTINUE". **To observe the contact pressure obtained using degree of freedom LAGS\_C of field DEPL.** To compare with the method of contact by default. What does one note? Which of the methods provides the best approximation?

*To obtain a correct profile of pressure, it is necessary to use a diagram of integration of the terms of contact to Gauss points (INTEGRATION=' GAUSS ' in DEFI\_CONTACT).*  
*In order to remove the alarm which appears with the adjustments by default, it is necessary to impose REAC\_ITER=1 under factor key word the NEWTON (in continuous formulation, the tangent matrix is inevitably rebuilt with each iteration).*

### 4.2 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
DEPL LAGS <sub>C</sub> not CI	"ANALYTIQUE"	-2798,3 Mpa	3,0%
DEPL LAGS <sub>C</sub> point CI	"NON_REGRESSION"	-2866,7 Mpa	C

## 5 0,1% Modelization

In this modelization, one carries out a mesh in quadrangles, with a refinement near the contact zone.

### 5.1 Implementation of the TP

#### 5.1.1 Réalisation of the geometry

In this modelization, one must partition the geometry so as to be able to net it in quadrangles. An illustration of the possible partition is given in figure 5.1.

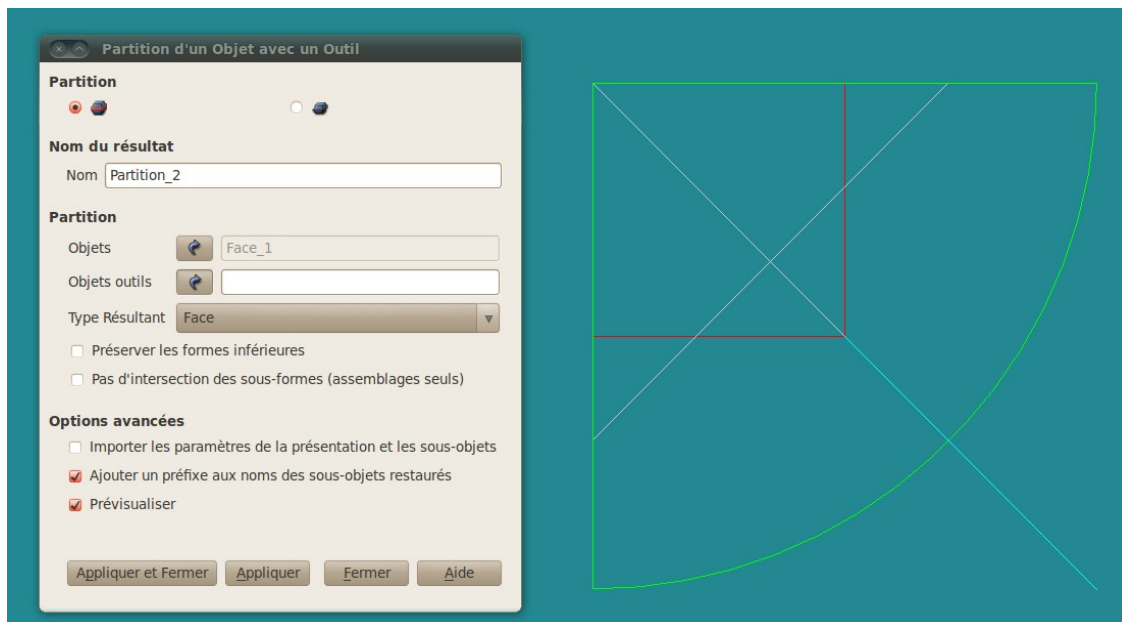
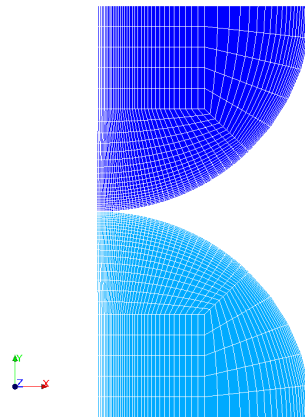


Illustration 5.1: Partition of the higher disc

#### 5.1.2 Realization of the mesh

For the mesh, one chooses to refine by means of around the contact zone a geometric progression on the circumference and the radius. It can be necessary to reorientate certain segments in modulus *SMESH*.



**Illustration 5.2: Mesh in quadrangles obtained for the modelization C**

The mesh comprises 2200 elements for 2300 nodes.

## 5.2 Quantities tested and Standard

Identification	results of reference	Value of reference	Tolerance
$\sigma_{yy}$ <i>CI</i>	"ANALYTIQUE"	-2798,3 <i>Mpa</i>	5,0%
$\sigma_{yy}$ not <i>CI</i>	"NON_REGRESSION"	-2928,8 <i>Mpa</i>	0,1%