SSNV221 – Hydrostatic test with a linear `behavior` DRUCK_PRAGER and parabolic

Abstract:

The case test proposes a purely hydrostatic loading for the associated model Drücker-Prager [R7.01.16]. The formulation of this plastic model, often used for the soils, is made at the same time on the deviatoric and hydrostatic part; nevertheless, surface criterion presents a singularity for a purely hydrostatic stress state. This analytical benchmark is used to check correct hardening in this singularity.

The test is carried out on a material point with command `SIMU_POINT_MAT`. One works with imposed strains.

One makes a test with linear hardening (modelization A) and another with parabolic hardening (modelization B).

1 Problem of reference

1.1 Elastic Properties of

the material :

$E=3000\ MPa$ Young modulus
$\nu=0{,}25$ linear

Poisson's ratio DRUCK_PRAGER (modelization A):

$\alpha=0{,}20$ Coefficient of dependence in pressure
$p_{ultm}=0{,}04$ cumulated Plastic strain ultimate
$\sigma_Y=6\ MPa$ plastic Stress
$h=100\ MPa$ parabolic

Hardening modulus DRUCK_PRAGER (modelization B):

$\alpha=0{,}20$ Coefficient of dependence in pressure
$p_{ultm}=0{,}04$ cumulated Plastic strain ultimate
$\sigma_Y=6\ MPa$ plastic Stress
$\sigma_Y^{ult}=10\ MPa$ ultimate plastic Stress

1.2 Loadings and boundary conditions

One imposes a voluminal strain $\varepsilon_v=\mathrm{tr}(\boldsymbol{\varepsilon})$. The loading is not monotonous: one charges initially until the voluminal strain ε_{v1} , by exceeding the threshold of plasticization, then one discharges on a null level of strain; then one still charges with the strain ε_{v2} by thus exceeding the ultimate cumulated plastic strain p_{ultm} , beyond which one finds a perfect plasticity; one still discharges with stress null (strain equal to the plastic strain ε_{v2}^p) and one reloads while plasticizing later on until the strain ε_{v3} . The time of loading (see Table 1.2-1) is fictitious because the plastic models are independent of time.

t	ε_v
0	0
10	$\varepsilon_{v1}=0{,}018$
14	0
26	$\varepsilon_{v2}=0{,}045$
30	$\varepsilon_{v2}^p=0{,}03667$
40	$\varepsilon_{v3}=0{,}06$

Table 1.2-1: imposed voluminal strain.

1.3 Initial conditions

All the components of the stresses and strains are null at the beginning of the loading.

2 Reference solution

The modelization checks the behavior of the model with linear hardening.

2.1 Method of calculating

the equations which interest us for analytical computation are ($I_1=\mathrm{tr}(\boldsymbol{\sigma})$: trace tensor of the stresses ε_v^p : voluminal plastic strain):

- plastic constitutive law on the voluminal part:

$$I_1=3\mathrm{K}(\varepsilon_v-\varepsilon_v^p) \quad \textbf{(éq 2.1-1)}$$

- surface criterion, by posing null the von Mises stress ($\sigma_{eq}=0$):

$$F(\boldsymbol{\sigma},p)=\alpha I_1-R(p) \quad \textbf{(éq 2.1-2)}$$

- relation between the voluminal plastic strain and the cumulated plastic strain (local variable of the plastic model):

$$\dot{\varepsilon}_v^p=3\alpha\dot{p} \qquad \text{thus while integrating: } \varepsilon_v^p=3\alpha p \quad \textbf{(éq 2.1-3)}$$

- statement of linear
  - hardening:

$$\begin{aligned} R(p)&=\sigma_Y+h\,p && si\ p\leqslant p_{ult} \\ R(p)&=\sigma_Y+h\,p_{ult}=\sigma_Y^{ult} && si\ p>p_{ult} \end{aligned} \quad \textbf{(éq 2.1-4)}$$

  - parabolic:

$$\begin{aligned} R(p)&=\sigma_Y\left(1-\left(1-\sqrt{\frac{\sigma_Y^{ult}}{\sigma_Y}}\right)\frac{p}{p_{ult}}\right)^2 && si\ p\leqslant p_{ult} \\ R(p)&=\sigma_Y^{ult} && si\ p>p_{ult} \end{aligned} \quad \textbf{(éq 2.1-5)}$$

It is observed that, as in the linear case, $R(p)=\sigma_Y$ if $p=0$ and there is perfect plasticity if $p>p_{ult}$.

2.1.1 Strain with the initial yield stress

This strain is obtained for $\varepsilon_v^p=p=0$.

If one poses $F(\boldsymbol{\sigma},p)=0$ (plastic evolution) one ultimate

$$I_1^{el}=\frac{R(p)}{\alpha}=\frac{\sigma_Y}{\alpha}$$

$$\varepsilon_v^{el}=\frac{I_1^{el}}{3K}$$

2.1.2 a: Strain

One calls ultimate strain ε_v^{ult} that obtained for $p=p_{ult}$.

One easily finds the trace of corresponding I_1^{ult} stresses and ε_v^{pult} the plastic strain:

$$I_1^{ult}=\frac{R(p)}{\alpha}=\frac{\sigma_Y^{ult}}{\alpha}$$

$$\varepsilon_v^{pult}=3\,\alpha\,p_{ult}$$
$$\varepsilon_v^{ult}=\frac{I_1^{ult}}{3K}+\varepsilon_v^{pult}$$

2.1.3 Strain between the yield stress and the ultimate strain

One calculates initially the cumulated plastic strain.

- By combining the equations (2.1-1), (2.1-2), (2.1-3) and (2.1-4) with $F(\boldsymbol{\sigma},p)=0$ for **linear hardening** one a:

$$p=\frac{3\,K\,A\,\varepsilon_{vI}-\sigma_Y}{9\,K\,\alpha^2+h}$$ **(éq 2.1-6)**

- By combining the equations (2.1-1), (2.1-2), (2.1-3) and (2.1-5) with $F(\boldsymbol{\sigma},p)=0$ for **parabolic hardening** one arrives at the equation of dismantled 2:

$$A_1\,\bar{p}^2+B_1\,\bar{p}+C_1=0$$

$$\begin{gathered}A_1=\sigma_Y(1-\gamma)^2\\ B_1=9\,K\,\alpha^2\,p_{ult}-2\,\sigma_Y(1-\gamma)\\ C_1=\sigma_Y-3\,K\,\alpha\,\varepsilon_{v1}\\ \gamma=\sqrt{\frac{\sigma_Y^{ult}}{\sigma_Y}}\\ \bar{p}=\frac{p}{p_{ult}}\end{gathered}$$ **(eq 2.1-7)**

the equations (2.1-3)2.1-3 then (2.1-1) to find the plastic strain ε_v^p and the trace of the stresses I_1.

If one makes null discharge the elastic material of way until stress, one finds a residual strain equal to the plastic strain; it is on the other hand necessary to charge the material in compression to obtain a total deflection null. This second branch is also elastic, because the material of Drücker-Prager cannot plasticize in a hydrostatic state of compression. In this last case, the trace of the stresses, negative, is:

$$I_1^c=-3\,K\,\varepsilon_v^p$$ **(éq 2.1-8)**

2.1.4 Strain higher than the ultimate strain

One easily finds all the quantities of interest, because the trace of stresses is known a priori and equal to I_1^{ult}.

$$\varepsilon_v^p=\varepsilon_v-\frac{I_1^{ult}}{3\,K}$$
$$p=\frac{\varepsilon_v^p}{3\alpha}$$

2.2 Quantities and results of reference

the modulus of compressibility K is:

$$K=\frac{E}{3(1-2\nu)}=2000\ MPa$$

2.2.1 Strain with the yield stress

For the two modelizations one finds easily:

$$I_1^{el}=30\ MPa$$

$$\varepsilon_v^{el}=0{,}005$$

2.2.2 Ultimate strain

For the two modelizations one finds:

$$I_1^{ult}=50\ MPa$$

$$\varepsilon_v^{pult}=0{,}024$$

$$\varepsilon_v^{ult}\approx 0{,}03233$$

2.2.3 Strain equal to 0,018 and discharges with strain null

This value from strain $\varepsilon_{v1}=0{,}018$ is higher than the yield stress ε_v^{el} and lower than ε_v^{ult}. One calculates initially the plastic strain cumulated with the equations (2.1-7) and (2.1-8), then the plastic strain and the trace of the stresses:

- linear hardening:

$$p_1=\frac{3K\,A\varepsilon_{vI}-\sigma_Y}{9K\alpha^2+h}\approx 0{,}019$$

$$\varepsilon_{v1}^p=3\alpha\,p_1=0{,}0114$$

$$I_1^1=3K(\varepsilon_{v1}-\varepsilon_{v1}^p)\approx 39{,}51\ MPa$$

- parabolic hardening:

$$p_1\approx 0.0192$$

$$\varepsilon_{v1}^p=3\alpha\,p_1\approx 0.0115$$

$$I_1^1=3K(\varepsilon_{v1}-\varepsilon_{v1}^p)\approx 38.956\ MPa$$

The trace of the stresses with strain null is:

- linear hardening:

$$I_1^{1c}=-3K\varepsilon_{v1}^p\approx -68{,}49\ MPa$$

- parabolic hardening:

$$I_1^{1c}=-3K\varepsilon_{v1}^p\approx -69{,}044\ MPa$$

Indeed, the difference between the parabolic and linear case is very weak.

2.2.4 Loading until strain EGA to 0,045 and 0,06

One reloads the material up to the values of strain $\varepsilon_{v2}=0{,}045$ and $\varepsilon_{v3}=0{,}06$, higher than ε_v^{ult}. The results are the same ones for the two modelizations.

For $\varepsilon_{v2}=0{,}045$:

$$\varepsilon_{v2}^{p}=\varepsilon_{v2}-\frac{I_1^{ult}}{3K}\approx 0{,}03667$$

$$p_2=\frac{\varepsilon_{v2}^{p}}{3\alpha}\approx 0{,}0611$$

Following the elastic discharge (until stress null), one finds $\varepsilon_v=\varepsilon_{v2}^{p}$ $p=p_2$.

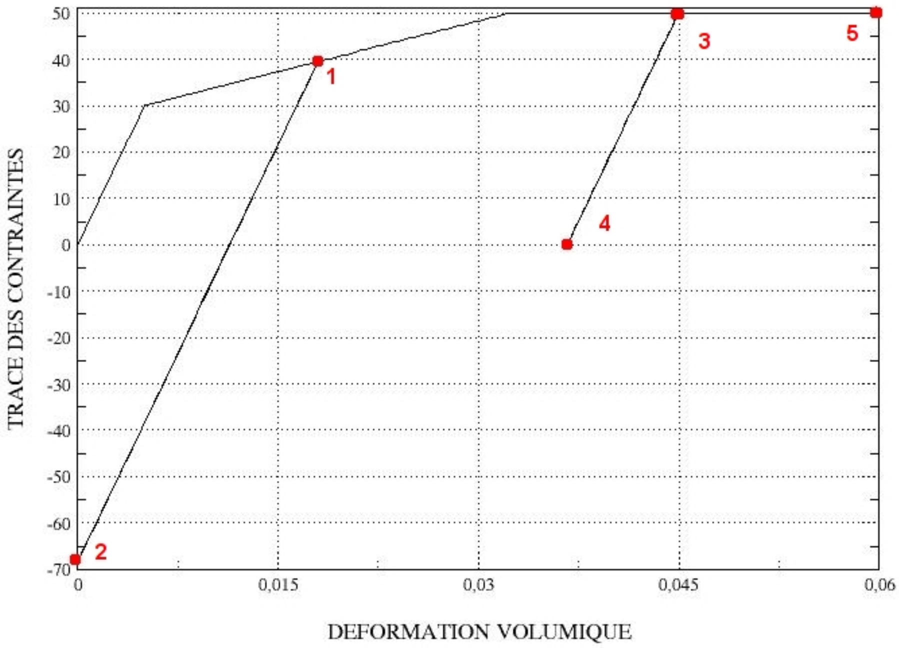
For $\varepsilon_{v3}=0{,}06$:

$$\varepsilon_{v3}^{p}=\varepsilon_{v3}-\frac{I_1^{ult}}{3K}\approx 0{,}051667$$

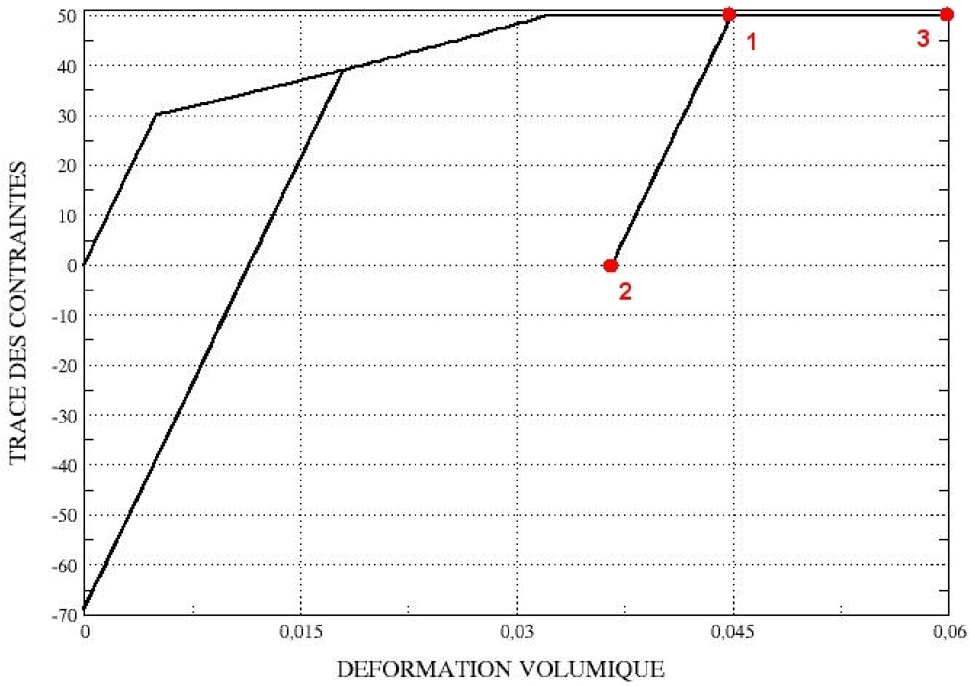
$$p_3=\frac{\varepsilon_{v3}^{p}}{3\alpha}\approx 0{,}0861$$

2.2.5 Stress-strain curves

In the Figures (2.2.5-a) and (2.2.5-b) one represents the curve (ε_v, I_1) for linear and parabolic hardening. In red are the points tested by the benchmark.



Appear 2.2.5-a: stress-strain curves for linear hardening.



Appear 2.2.5-b: stress-strain curves for parabolic hardening.

2.3 Uncertainties on the solution

the solution is analytical.

2.4 Bibliographical references

[1] Document [R3.01.16], Integration of the elastoplastic structural mechanics behavior of Drücker-Prager `DRUCK_PRAGER` and postprocessings. Handbook of reference Code_Aster.

3 Modelization A

3.1 Characteristic of the modelization

the test is carried out on a material point with command SIMU_POINT_MAT . One works with imposed strains.
Hardening is linear.

3.2 Quantities and results of reference

Not on Appear 2.2.5-a	checked Quantity	Standard Value of reference	of reference	Tolerance (relative)
1	Trace of stresses	$I_1^1 = 39,51\ MPa$	ANALYTIQUE	10^{-6} %
2	Traces stresses	$I_1^{1c} = -68,49\ MPa$	ANALYTIQUE	10^{-6} %
3 or 4	spherical Part of plastic strain	$\varepsilon_{v2}^p = 0,03667$	ANALYTIQUE	10^{-6} %
3 or 5	Trace of stresses	$I_1^{ult} = 50\ MPa$	ANALYTIQUE	10^{-6} %
5	Part spherical of plastic strain	$\varepsilon_{v3}^p = 0,051667$	ANALYTIQUE	10^{-6} %

4 Modelization B

4.1 Characteristic of the modelization

the test is carried out on a material point with command SIMU_POINT_MAT . One works with imposed strains.
Hardening is parabolic.

4.2 Quantities and results of reference

Not on Appear 2.2.5-b	checked Quantity	Standard Value of reference	of reference	Tolerance (relative)
1 or 2	spherical Part of plastic strain	$\varepsilon_{v2}^{p}=0{,}03667$	ANALYTIQUE	10^{-6} %
1 or 3	Trace of stresses	$I_1^{ult}=50\ MPa$	ANALYTIQUE	10^{-6} %
3	Part spherical of plastic strain	$\varepsilon_{v3}^{p}=0{,}051667$	ANALYTIQUE	10^{-6} %

5 Summary of the results

the results of the benchmark are satisfactory, *Code_Aster* reproduced the analytical results with a high accuracy.