SSNV219 – Method of the solutions manufactured in contact 3D and large deformations

Summarized:

The purpose of this test is to 3D check the modelization of contact in large deformations thanks to the method of the manufactured solutions [bib1].

1 Problem of reference

1.1 Geometry

One considers a cube of with dimensions $1\,m$.

1.2 Properties of the material

$E=1\text{MPa}$	Modulus Young
$\nu=0.15$	Poisson's ratio

1.3 Boundary conditions and loadings

On edge HAUT , one forces a displacement (see paragraph 2.
On edges BORDX BORDMX BORDY BORDMY , BORDZ and ESCLAVE , one forces a pressure (see paragraph 2.
In all the field, one forces a body force (see paragraph 2.
The surface MAITRE of natural paraboloid is described by the equation:

$$Z=-0.2\times(1.+X^2+Y^2)-0.3 \quad (1)$$

1.4 Initial conditions

Nothing

2 Reference solution

2.1 Method of calculating

the analytical reference solution is given by:

$$\begin{aligned} Ux &= 0.2\times Z^2\times X\times Y \\ Uy &= 0.2\times Z^2\times X\times Y \\ Uz &= -0.2\times(1.+X^2+Y^2)\times(1+0.01\times Z)-0.01*Z-0.3 \end{aligned} \quad (2$$

the conditions of Dirichlet, Neumann and the source term are obtained by the method of the manufactured solutions [bib1].
One starts by determining the gradient of the transformation $\underline{\underline{F}}$:

$$\underline{\underline{F}}=\nabla \underline{U}+\underline{\underline{Id}} \quad (3)$$

Knowing the norm $\underline{N}=[0,0,-1]^T$ on the surface ESCLAVE in the NON-deformed configuration, one obtains his statement in the configuration deformed by the formula of Nanson:

$$\underline{n}=\frac{\underline{\underline{F}}^{-T}\underline{N}}{\|\underline{\underline{F}}^{-T}\underline{N}\|} \quad (4)$$

Knowing the tensor of Hooke $\underline{\underline{\underline{\underline{A}}}}$ and the tensor of Green-Lagrange $\underline{\underline{E}}$, one calculates the second tensor of Piola-Kirchhoff $\underline{\underline{S}}$:

$$\underline{\underline{E}}=\frac{1}{2}(\underline{\underline{F}}^T\cdot\underline{\underline{F}}-\underline{\underline{Id}}) \quad (5)$$

$$\underline{\underline{S}}=\underline{\underline{\underline{\underline{A}}}}:\underline{\underline{E}} \quad (6)$$

It is pointed out that the second tensor of Piola-Kirchhoff $\underline{\underline{S}}$ makes it possible to obtain forces in undistorted configuration per not deformed unit of area:

$$\frac{d\,\underline{f}_0}{dA}=\underline{\underline{S}}\cdot\underline{N} \quad (7)$$

As we seek to determine forces in deformed configuration, we will determine the first tensor of Piola-Kirchhoff $\underline{\underline{\Pi}}$

$$\underline{\underline{\Pi}}=\underline{\underline{F}}\cdot\underline{\underline{S}} \quad (8)$$

One can thus determine the body forces $\underline{f}_{vol}$:

$$\underline{f}_{vol}=\text{-}div\,\underline{\underline{\Pi}} \quad (9)$$

Knowing the norm in initial configuration on the various sides and the first tensor of Piola-Kirchhoff $\underline{\underline{\Pi}}$, one can calculate the forces of surface in deformed configuration:

$$\underline{f}_{surf}=\underline{\underline{\Pi}}\cdot\underline{N} \quad (10)$$

On the surface BAS which is in contact, one needs a particular processing. Indeed, the normal force is taken there into account by the contact:

$$\begin{aligned}\underline{f}_{surf}^{BAS} &= \underline{f}_{surf_n}^{BAS} + \underline{f}_{surf_t}^{BAS}\\ &= \underline{f}_{contact} + \underline{f}_{surf_t}^{BAS}\\ &= p*\underline{n} + \underline{f}_{surf_t}^{BAS}\end{aligned} \quad (11)$$

Where p the contact pressure indicates. It can be given by the statement:

$$p=(\underline{\underline{\Pi}}\cdot\underline{N})\cdot\underline{n} \quad (12)$$

One thus should apply only the tangential stresses to it. One calculates them by the statement:

$$\begin{aligned}\underline{f}_{surf_t}^{BAS} &= \underline{f}_{surf}^{BAS}-\underline{f}_{surf_n}^{BAS}\\ &= \underline{f}_{surf}^{BAS}-(\underline{f}_{surf_n}^{BAS}\cdot\underline{n})\,\underline{n}\end{aligned} \quad (13)$$

Concerning the forces of contact, *it is absolutely essential to build the solution manufactured so that they check the equations of the contact* [bib2], namely:

$$\begin{aligned} gap(\underline{U}) &\geqslant 0 \\ p &\leqslant 0 \\ p \cdot gap(\underline{U}) &= 0 \end{aligned} \quad (14)$$

This checking is done after having calculated in an analytical way the pressure and the jump of displacement associated with the manufactured solution, in general with a formal computational tool (in fact, it is the modulus Python *sympy*). One must then visualize them, in order to check *retrospectively* that the solution which one has contruite checks well (14). In the case of this test, we represented pressure and analytical jump of displacement in figures 2.1-1 and 2.1-2. One notices that they check $p<0$ and $\mathrm{gap}(\underline{U})=0$, which is characteristic of a surface entirely contacting, and conforms to (14).

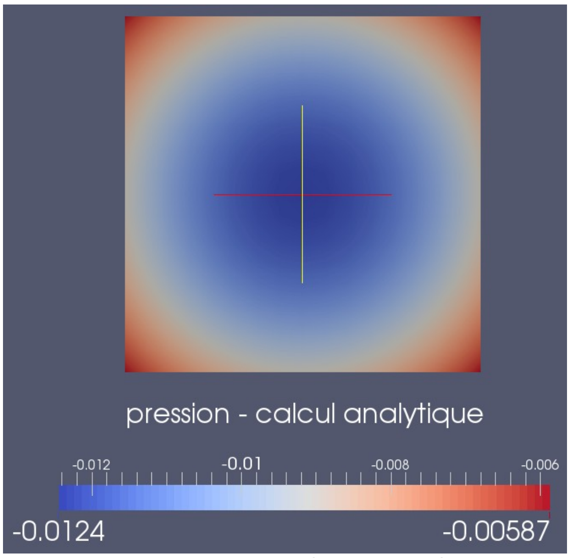

Figure 2.1-1: Validity of the manufactured solution: pressure p

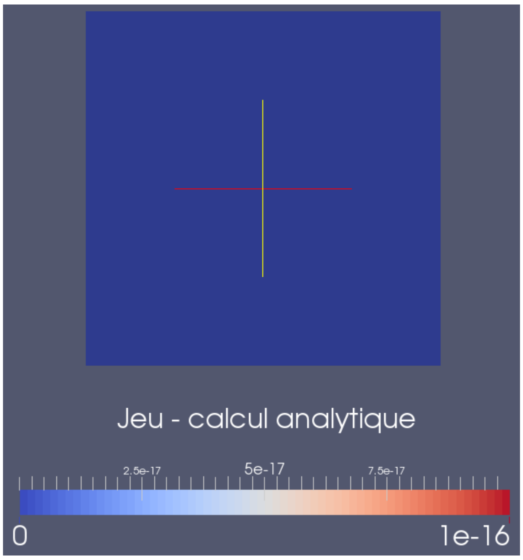

Figure 2.1-2: Validity of the manufactured solution: clearance Gap (U)

2.2 Quantities and results of reference

the value of the difference between solutions analytical and calculated on mesh: $\sum^{\text{noeuds } n} |\underline{U}_{\mathrm{n}}^{\mathrm{calc}} - \underline{U}_{\mathrm{n}}^{\mathrm{ref}}|$ and $\sum^{\text{noeuds } n} |p_{\mathrm{n}}^{\mathrm{calc}} - p_{\mathrm{n}}^{\mathrm{ref}}|$.

In the case of the modelizations which carry out an analysis of convergence with the smoothness of the mesh, velocity of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution in norm L_2 :

–the greatest reality $\alpha_U>0$ such as $\|\underline{U}^{\mathrm{calc}}-\underline{U}^{\mathrm{ref}}\|_{0,\Omega}<C_U\times h^{\alpha_U}$ where C_U is independent of h for displacement;

–the greatest reality $\alpha_p>0$ such as $\|p^{\mathrm{calc}}-p^{\mathrm{ref}}\|_{0,\Gamma_c}<C_p\times h^{\alpha_p}$ where C_p is independent of h for the contact pressure.

2.3 Uncertainties on the solution

No

2.4 bibliographical References

[1] Document U2.08.08, Use of the Method of the Solutions Manufactured for the software validation, Documentation U2 of Code_Aster

[2] R5.03.50 Document, discrete Formulation of contact-friction, Documentation R of Code_Aster

3 Modelization A

3.1 Characteristic of the modelization

One uses a modelization `3D` and the continuous method of processing of the contact.

3.2 Characteristics of the mesh

The mesh contains 1 element of type `QUAD8`, 768 elements of the type `TRIA6` and 3072 elements of the type `TETRA10`.

3.3 Quantities tested and results

One tests the sum of the absolute values of the difference between the calculated solution and the analytical solution.

Standard	identification of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_{\text{n}}^{\text{calc}} - \underline{U}_{\text{n}}^{\text{ref}} \rvert$	"NON_REGRESSION"	0.0410211809958

4 Modelization B

4.1 Characteristic of the modelization

One uses a modelization `3D` and the continuous method of processing of the contact.

4.2 Characteristics of the mesh

One carries out a study of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution. A succession of meshes obtained by uniform refinement using command `MACR_ADAP_MAIL` is used:

- mesh 0: 1 element of type `QUAD8`, 12 elements of the type `TRIA6` and 6 elements of the type `TETRA10`
- mesh 1: 1 element of type `QUAD8`, 48 elements of the type `TRIA6` and 48 elements of the type `TETRA10`
- mesh 2: 1 element of type `QUAD8`, 192 elements of the type `TRIA6` and 384 elements of the type `TETRA10`
- mesh 3: 1 element of type `QUAD8`, 768 elements of the type `TRIA6` and 3072 elements of the type `TETRA10`

4.3 Quantities tested and results

One tests the velocity of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution in norm L_2 :

- the greatest reality $\alpha_U > 0$ such as $\|\underline{U}^{\text{calc}} - \underline{U}^{\text{ref}}\|_{0,\Omega} < C_U \times h^{\alpha_U}$ where C_U is independent of h for displacement;
- the greatest reality $\alpha_p > 0$ such as $\| p^{\text{calc}} - p^{\text{ref}}\|_{0,\Gamma_c} < C_p \times h^{\alpha_p}$ where C_p is independent of h for the contact pressure.

One tests also the sum of the absolute values of the difference between the calculated solution and the analytical solution for displacement.

Standard	identification of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_{\text{n}}^{\text{calc}} - \underline{U}_{\text{n}}^{\text{ref}} \rvert$	"NON_REGRESSION"	4.13935026178E-05
α_U	"ANALYTIQUE"	3.0
α_p	"NON_REGRESSION"	2.534025066720
α_p	"ANALYTIQUE"	2.5

5 Modelization C

5.1 Characteristic of the modelization

One uses a modelization `3D` and method `DISCRETE` of the conjugate gradient (GCP) project of processing of the contact.

5.2 Characteristics of the mesh

One carries out a study of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution. A succession of meshes obtained by uniform refinement using command `MACR_ADAP_MAIL` is used:
–mesh 0: 14 elements of the type `TRIA6` and 6 elements of the type `TETRA10`
–mesh 1: 50 elements of the type `TRIA6` and 48 elements of the type `TETRA10`
–mesh 2: 194 elements of the type `TRIA6` and 384 elements of the type `TETRA10`
–mesh 3: 770 elements of the type `TRIA6` and 3072 elements of the type `TETRA10`

It is noted that, compared to the modelizations A and B, the base is with a grid with 2 `TRIA6` instead of `a QUAD8`. Indeed, the discrete methods are not adapted to the use of these elements (see [R5.03.50]).

5.3 Quantities tested and results

One tests the velocity of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution in norm L_2 :
–the greatest reality $\alpha_U > 0$ such as $\lVert \underline{U}^{\text{calc}} - \underline{U}^{\text{ref}} \rVert_{0,\Omega} < C_U \times h^{\alpha_U}$ where C_U is independent of h for displacement in the field Ω ;
–the greatest reality $\alpha_s > 0$ such as $\lVert U^{\text{calc}} - U^{\text{ref}} \rVert_{0,\Gamma_C} < C_s \times h^{\alpha_s}$ where C_s is independent of h for displacement on surface Γ_C .
One tests also the sum of the absolute values of the difference between the calculated solution and the analytical solution for displacement.

Standard	identification of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_{\text{n}}^{\text{calc}} - \underline{U}_{\text{n}}^{\text{ref}} \rvert$	"NON_REGRESSION"	4.25881911029E-05
α_U	"ANALYTIQUE"	3.0
α_p	"ANALYTIQUE"	3.5

6 Modelization D

6.1 Characteristic of the modelization

One uses a modelization 3D and method DISCRETE of the conjugate gradient (GCP) project of processing of the contact.

6.2 Characteristics of the mesh

One carries out a study of convergence with the smoothness of the mesh of the solution calculated towards the analytical solution. A succession of meshes obtained by uniform refinement using command MACR_ADAP_MAIL is used:
–mesh 0: 2 elements of the type TRIA6, 6 elements of the type QUAD9 and 1 element of type HEXA27
–mesh 1: 2 elements of the type TRIA6, 24 elements of the type QUAD9 and 8 element of type HEXA27
–mesh 2: 2 elements of the type TRIA6, 96 elements of the type QUAD9 and 64 element of type HEXA27
–mesh 3: 2 elements of the type TRIA6, 384 elements of the type QUAD9 and 512 element of type HEXA27

It is noted that, compared to the modelizations A and B, the base is with a grid with 2 TRIA6 instead of a QUAD8. Indeed, the discrete methods are not adapted to the use of these elements (see [R5.03.50]).

6.3 Quantities tested and results

One tests the sum of the absolute values of the difference between the calculated solution and the analytical solution for displacement. The rate of convergence is not tested because element HEXA27 provides the exact solution with a single element.

Standard	identification of reference	Value of reference
$\sum^{\text{noeuds } n} \lvert \underline{U}_{\text{n}}^{\text{calc}} - \underline{U}_{\text{n}}^{\text{ref}} \rvert$	"NON_REGRESSION"	0

7 Summary of the results

the results are in very good agreement with the theory.