SSNV209 – Interface in contact rubbing with X-FEM

Summarized:

This problem corresponds to a quasi-static analysis of a problem of mechanics with contact and friction. A rectangular plate is subjected to horizontal and vertical compressive forces and is compressed on a plane where it undergoes forces of contact and friction.

It is about a case equivalent to the benchmark ssnv128 [V6.04.128], but solved with method XFEM.

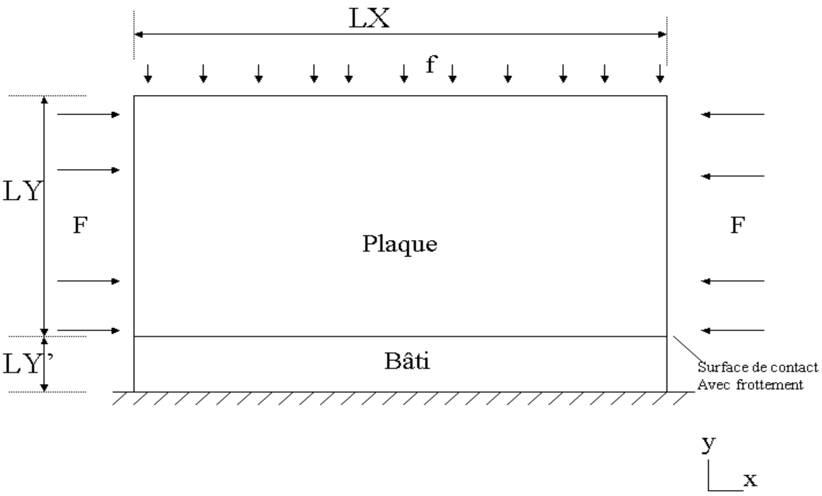
This test brings into play various meshes, in $2D$ and $3D$, in the frame of computations X-FEM [R7.02.12]

It comprises 14 modelizations and makes it possible to test:

- modelizations $2D$ (`QUAD4`, `TRIA3, QUAD8, TRIA6`) and $3D$ (`HEXA8`),
- a crack position to the interface or in the middle of the elements,
- operand `XFEM` of key word `CONTACT`
- integration by subelements (resulting from XFEM) of a quantity depending on a command variable
- the various diagrams of integration for the terms of contact (`NODE, SIMPSON, NCOTES`)
- formulations with lagranges on the edges and those with lagranges on the nodes (of the initial mesh)
- parameters of the generalized formulation (coefficients of regularization and stabilization for the contact and friction),
- the simultaneous computation of the separated zones, in slipping contact, and in contact adherent
- postprocessing, in particular the computation of the normal reaction RN,
- the algorithm of satisfaction of the condition LBB to the transitions taken off/contact and slipping/adherent,
- method of Lagrangian increased and method penalized for the processing of contact-friction.

1 Problem of reference in 2D

1.1 Geometry



the structure is a rectangle made up of two of the same plates material, separated by an interface.

Dimensions of the plate, to which the pressures are applied, are:
$LX=80\text{mm}$, $LY=40\,mm$

The second plate, comparable to a frame, has following dimensions:
$LX=80\text{mm}$, $LY'=10\,mm$

The position of the points of reference under contact surface (mm) is:

	x	y
A	0	0
B	1.25	0
C	5.	0.7.5
D		0
E	11.25	0

1.2 Material properties

Plates:

Poisson's ratio: 0.2
Young modulus: $1.3\,10^{11}\,N/m^2$

Frame:

Poisson's ratio: 0.0
Young modulus: $1.0\,10^{16}\,N/m^2$

The coefficient of kinetic friction under the plane is $\mu=1.0$.

1.3 Boundary conditions and loadings

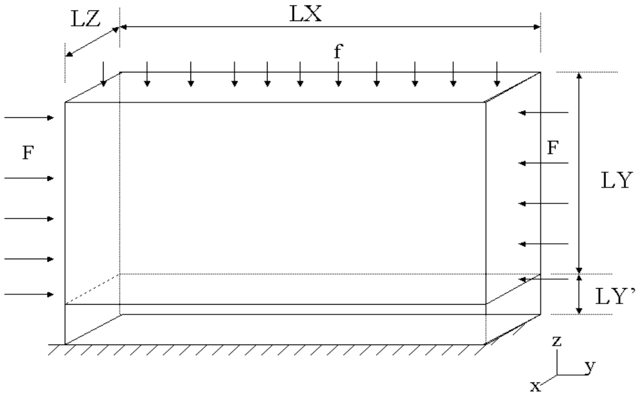
the frame, of the same width than the plate, is blocked by a fixed support of its lower face.

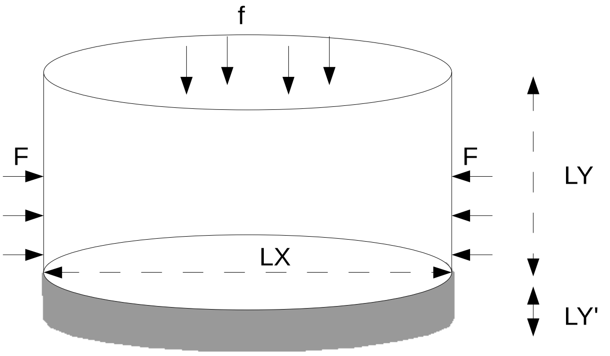
The plate is subjected to two distributed pressures:

- a vertical pressure acting on the face of the top: $f=-5\ daN/mm^2$,
- a horizontal pressure acting on the side sides, for $y>0$, $F=\pm 15\,daN/mm^2$ (according to the principle of compression)

2 Problem of reference in 3D

2.1 Geometry





One takes into account two cases.
The first geometry is obtained by extrusion of the geometry of the problem $2D$, the structure is then a right-angled parallelepiped.

The second geometry is obtained by a revolution around the axis y of the geometry of the problem $2D$, the structure is then a cylinder.

In both cases the structure makes up of two materials, separated by an interface.
Dimensions of the upper part to which the pressures are applied, are fixed:
$LX=80\text{mm}$, $LY=40\,mm$ and $LZ=1\text{mm}$

the comparable lower part with a frame, has following dimensions:
$LX=80\text{mm}$, $LY'=10\,mm$ and $LZ=1\text{mm}$

the position of the points of reference under contact surface (mm) is:

	x	y	z
A	0	0	0
B	1.25	0	0
C	5.	0	0.7.5
D		0	0
E	11.25	0	0

2.2 Material properties

Plates:

Poisson's ratio: 0.2
Young modulus: $1.3\,10^{11}\,N/m^2$

Frame:

Poisson's ratio: 0.0
Young modulus: $1.0\,10^{16}\,N/m^2$
The coefficient of kinetic friction under the plane is $\mu=1.0$.

2.3 Boundary conditions and loadings

the frame, of the same width than the plate, is blocked by a fixed support of its lower face,

In $3D$, to avoid a too heavy computation:

- displacement following the axis Z is blocked only on the upper face,

the plate is subjected to two distributed pressures:

- a vertical pressure acting on the face of the top: $f=-5\ daN/mm^2$,
- horizontal acting on the side sides, for $y>0$, $F=\pm 15\, daN/mm^2$ (according to the principle of compression).

3 Reference solution

3.1 Méthode de calcul used for the reference solution

the reference solution comes from the results got by the modelization A ($2D$) and F ($3D$), namely a modelization `D_PLAN` in $2D$ and `3D` under $3D$ key word `FORMULATION=' CONTINUE'` of operator `DEFI_CONTACT`.

3.2 Results of reference

tangential Displacements (according to X) to the points $A\ B\ C\ D\ E$ of contact surface.
Normal reaction at the point medium.

4 Modelization A

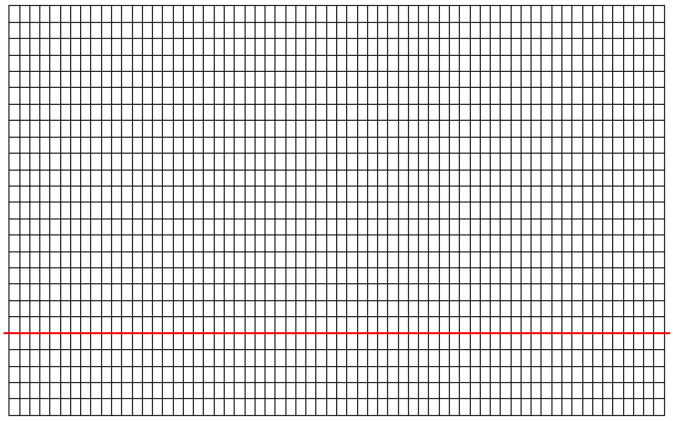
4.1 Characteristic of the modelization

Modelization: D_PLAN to test the operand "CONTINUE" key word FORMULATION under operator DEFI_CONTACT for elements SEG2 .

The plate and the frame are with a grid with elements QUAD4.

4.2 Characteristics of the mesh

Many nodes: 975
Number of meshes and types: 832 QUAD4 for the plate and the frame.



4.3 Quantities tested and results

Identification	***Aster***
RN at the point medium	1.04864 E+5
DX as in point A	2.84595 E-5
DX as in point B	2.70793 E-5
DX as in point C	2.27403 E-5
DX as in point D	1.97271 E-5
DX as in point E	1.53641 E-5

4.4 Notices

These results are used as reference for the four modelizations which follow.

5 Modelization B

5.1 Characteristic of the modelization

Modelization: D_PLAN to test operand "XFEM" of key word FORMULATION under operator DEFI_CONTACT for elements SEG2 .

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of command using operator DEFI_FISS_XFEM [U4.82.08]. The crack is present at a distance $LY=40\,mm$ from higher edge of the plate.

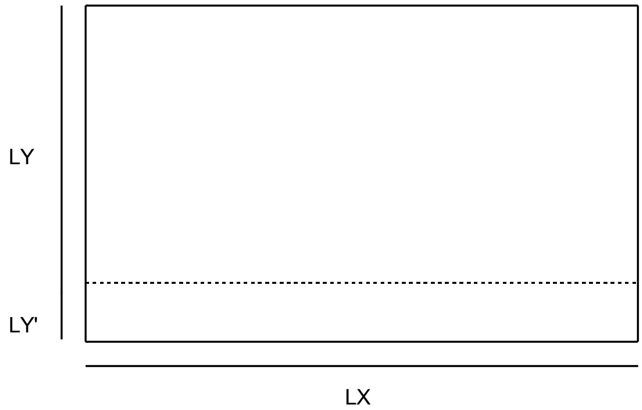
The two materials are introduced thanks to the command variable "NEUT1" under key word AFFE_VARC of the command AFFE_MATERIAU [U4.43.03]. The values taken by this command variable are given by a field at nodes taking the value of the level set norm.

The field of the values of the level set norm is defined using command CREA_CHAM [U4.72.4] and of its key word NOM_CHAM=' LNNO'.

The parameters of the material (E and nu , the Poisson's ratio) are functions of command variable "NEUT1". The material is defined by the key word ELAS_FO of the command DEFI_MATERIAU [U4.43.01].

The crack tip is located apart from structure, which allows a total separation. This modelization is such as the crack is located at the interface of the elements.

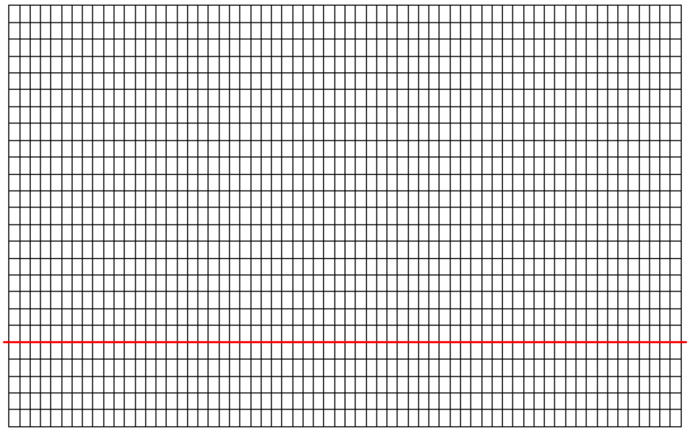
The plate and the frame are with a grid with elements QUAD4 64×25 .



dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\,mm$
Frame:
$LX=80\,mm$ and $LY'=10\,mm$

5.2 Characteristic of the mesh

Many nodes: 1690
Number of meshes and types: 1600 QUAD4 for the plate and the frame and 178 SEG2 for edges.



5.3 Quantities tested and results

Identification	Reference	Tolerance (%)
RN to the point medium	1.04864 E+5	0.1
DX as in point A	2.84595 E-5	2,0
DX as in point B	2.70793 E-5	2,0
DX as in point C	2.27403 E-5	2,0
DX as in point D	1.97271 E-5	2,0
DX as in point E	1.53641 E-5	2,0

6 Modelization C

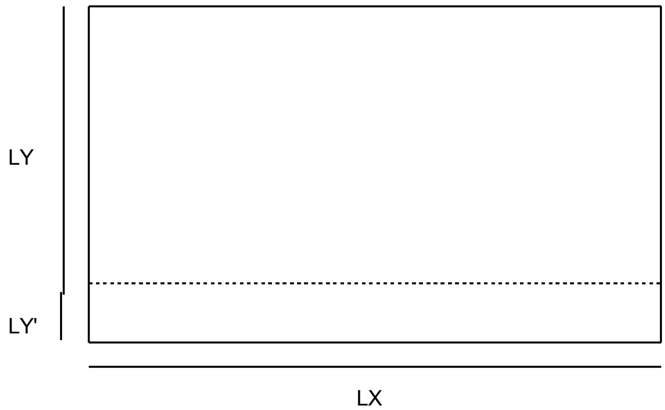
6.1 Characteristic of the modelization

Modelization: D_PLAN to test operand "XFEM" of key word FORMULATION under operator DEFI_CONTACT for elements SEG2 .

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of command using operator DEFI_FISS_XFEM [U4.82.08]. The crack is present at a distance $LY=40\,mm$ from higher edge of the plate.

The two materials are introduced as for modelization A. One tests however that the introduction of the two materials functions with operand COMP_INCR (instead of COMP_ELAS) in operator STAT_NON_LINE.

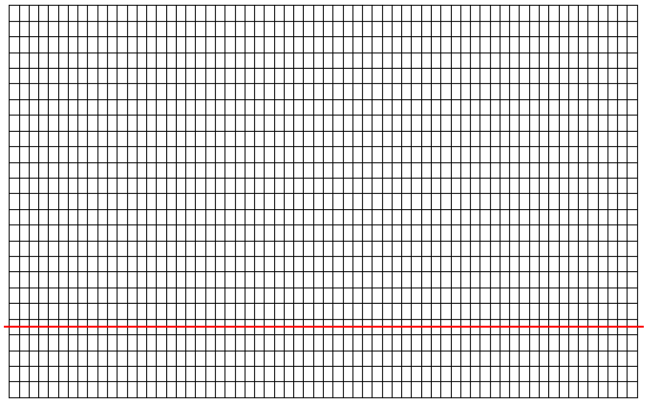
The crack tip is located apart from structure, which allows a total separation. This modelization is such as the crack is located in the middle of the elements.



dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\,mm$
Frame:
$LX=80\,mm$ and $LY'=\frac{3}{325}mm$

6.2 Characteristic of the mesh

Many nodes: 1625
Number of meshes and types: 1536 QUAD4 for the plate and the frame and 176 SEG2 for edges.



Quantities tested and results

Identification	Reference	Tolerance (%)
DX to the item A	2.84595 E-5	2,0
DX as in point B	2.70793 E-5	2,0
DX as in point C	2.27403 E-5	2,0
DX as in point D	1.97271 E-5	2,0
DX as in point E	1.53641 E-5	2,0

7 Modelization D

7.1 Characteristic of the modelization

Modelization: D_PLAN to test operand "XFEM" of key word FORMULATION under operator DEFI_CONTACT for elements SEG2 .

The structure is a rectangle, into which a crack will be introduced directly into card-indexing it of command using operator DEFI_FISS_XFEM [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ from higher edge of the plate.

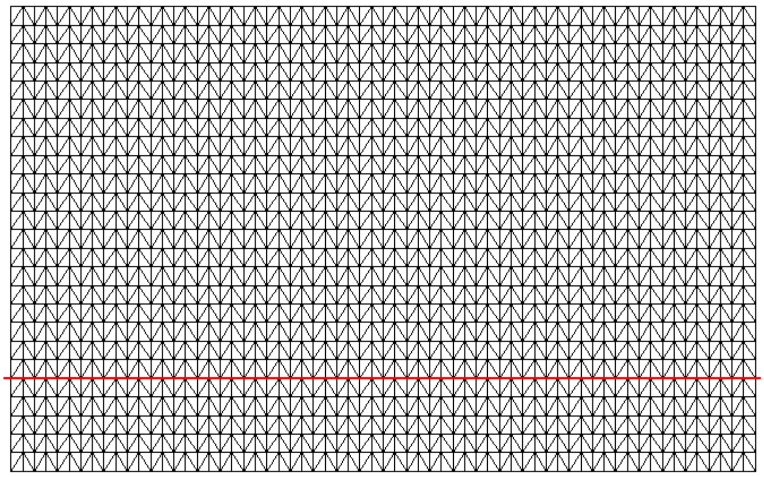
The two materials are introduced as for modelization A.

the crack tip is located apart from structure, which allows a total separation. This modelization is such as the crack is located at the interface of the elements.

Dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\text{mm}$
Frame:
$LX=80\,mm$ and $LY'=10\text{mm}$

7.2 Characteristic of the mesh

Many nodes: 1690
Number of meshes and types: 3200 TRIA3 for the plate and the frame and 178 SEG2 for edges.



7.3 Quantities tested and results

Identification	Reference	Tolerance (%)
RN to the point medium	1.04864 E+5	10.0
DX as in point A	2.84595 E-5	1.0
DX as in point B	2.70793 E-5	1.0
DX as in point C	2.27403 E-5	1.0
DX as in point D	1.97271 E-5	1.0
DX as in point E	1.53641 E-5	1.0

7.4 Remarks

In this case of the oscillations of the coefficient of Lagrange for the pressure appear. The point on which the reaction RN is compared with the reference is a “low” point in the oscillation. The close points have an error of approximately $+8\%$.
Let us note that one can check by decreasing the step of the mesh which the condition LBB is observed: the oscillations decrease in amplitude.

8 Modelization E

8.1 Characteristic of the modelization

Modelization: `D_PLAN` to test operand "`XFEM`" of key word `FORMULATION` under operator `DEFI_CONTACT` for elements `SEG2` .

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of command using operator `DEFI_FISS_XFEM` [U4.82.08]. The crack is present at a distance $LY=40\mathrm{mm}$ from higher edge of the plate.

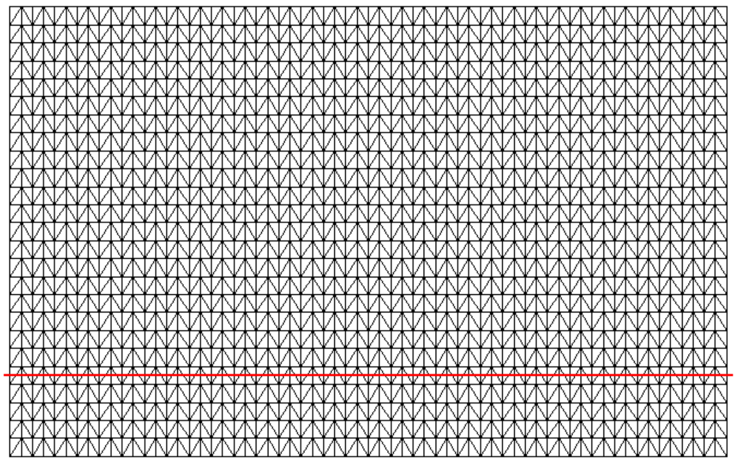
The two materials are introduced as for modelization A.

the crack tip is located apart from structure, which allows a total separation. This modelization is such as the crack is located in the middle of the elements.

Dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\mathrm{mm}$
Frame:
$LX=80\,mm$ and $LY'=\frac{3}{325}mm$

8.2 Characteristic of the mesh

Many nodes: 1625
Number of meshes and types: 3072 `TRIA3` for the plate and the frame and 176 `SEG2` for edges.



8.3 Quantities tested and results

In this test, one validates contact `XFEM` for a crack non-coïncidente with the mesh.

Identification	Reference	Tolerance (%)
RN to the point medium	1.04864 E+5	5.0
DX as in point A	2.84595 E-5	1.0
DX as in point B	2.70793 E-5	1.0
DX as in point C	2.27403 E-5	1.0

DX as in point D	1.97271 E-5	1.0
DX as in point E	1.53641 E-5	1.0

9 Modelization F

9.1 Characteristic of the modelization

Modelization: 3D to test the operand "CONTINUE" key word FORMULATION under operator DEFI_CONTACT for elements HEXA8 .

The geometry, the boundary conditions and the loadings are those of the problem of reference $3\mathrm{D}$ describes paragraph §54 "Problem of reference in $3\mathrm{D}$ ".

The plate and the frame are with a grid with elements HEXA8.

The blocking of rigid body motions is done by blocking displacement along the axis Z, that is to say DZ for the 4 nodes with the corners of the upper face. It is necessary to block these modes for the first iteration of contact, during which friction is not activated. Then, friction prevents these motions.

This case is used as reference for cases $3\mathrm{D}$ XFEM following.

9.2 Characteristics of the mesh

Many nodes: 1950
Number of meshes and types: 832 HEXA8 for the plate and the frame.
1108 QUAD4 for the edge elements.

9.3 Quantities tested and results

Identification	Aster
DX as in point A	3.06970 E-5
DX as in point B	2.92588 E-5
DX as in point C	2.47360 E-5
DX as in point D	2.15871 E-5
DX as in point E	1.70045 E-5

9.4 Remarks

For computations in $3\mathrm{D}$, the boundary conditions $DZ=0$ (to bring back itself to a problem $2\mathrm{D}$) are not respected to avoid a too important memory allocation. One imposes a minimum of it to block rigid body motions.

These results are used as references for the two following modelizations.

10 Modelization G

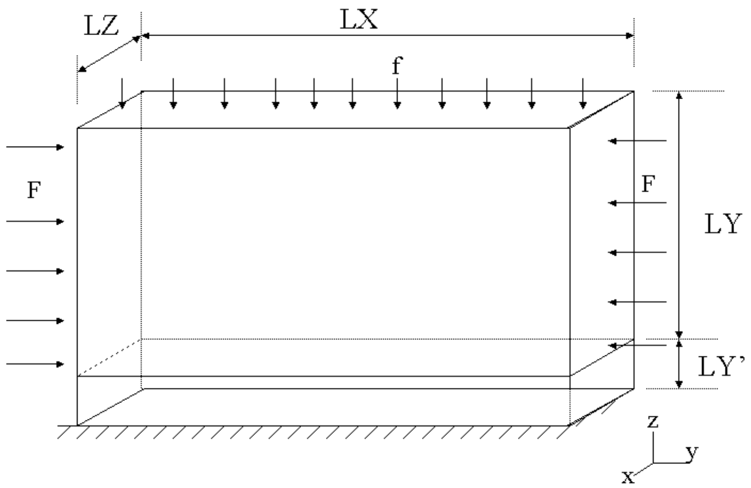
10.1 Characteristic of the modelization

Modelization: `3D` to test operand "`XFEM`" of key word `FORMULATION` under operator `DEFI_CONTACT` for elements `HEXA8` .

The geometry, the boundary conditions and the loadings are those of the problem of reference $3D$ describes paragraph §54 "Problem of reference in $3D$ ".

The two materials are introduced as for modelization A.

10.2 Géométrie



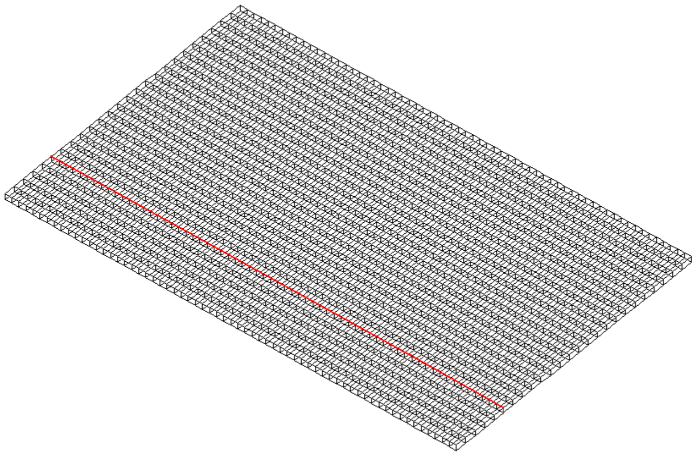
the structure is a healthy parallelepiped, into which a crack will be introduced directly into card-indexing it of command using operator `DEFI_FISS_XFEM` [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ from higher edge of the plate.

The crack tip is located apart from structure, which allows a total separation. This modelization is such as the crack is located at the interface of the elements.

Dimensions are the following ones:
Plate:
$LX=80\text{mm}$, $LY=40\text{mm}$ and $LZ=1\text{mm}$
Built:
$LX=80\text{mm}$, $LY'=10\text{mm}$ and $LZ=1\text{mm}$

10.3 Characteristic of the mesh

Many nodes: 3380
Number of meshes and types: 1600 `HEXA8` for the plate and the frame and
1778 `QUAD4` for the edge sides .



10.4 Quantities tested and results

In this test, one validates contact XFEM for a crack coïncidente with the mesh in $3D$.

Identification	Reference	Tolerance (%)
DX to the item A	3.06970 E-5	1.0
DX as in point B	2.92588 E-5	1.0
DX as in point C	2.47360 E-5	1.0
DX as in point D	2.15872 E-5	1.0
DX as in point E	1.70046 E-5	1.0

11 Modelization H

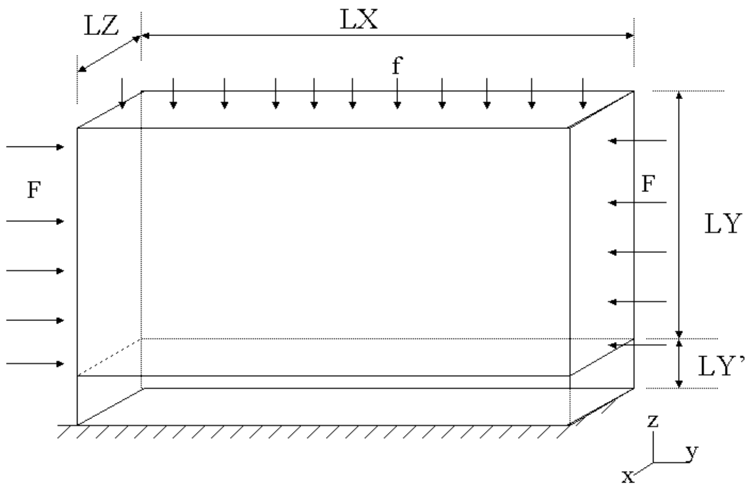
11.1 Characteristic of the modelization

Modelization: `3D` to test operand "`XFEM`" of key word `FORMULATION` under operator `DEFI_CONTACT` for elements `HEXA8` .

The geometry, the boundary conditions and the loadings are those of the problem of reference $3D$ describes paragraph §54 "Problem of reference in $3D$ ".

The two materials are introduced as for modelization A.

11.2 Géométrie



the structure is a healthy parallelepiped, into which a crack will be introduced directly into card-indexing it of command using operator `DEFI_FISS_XFEM` [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ from higher edge of the plate.

The crack tip is located apart from structure, which allows a total separation. This modelization is such as the crack is located in the middle of the elements.

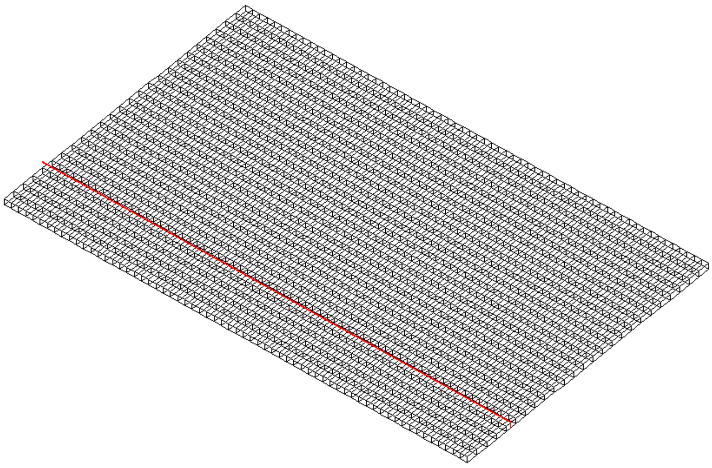
Dimensions are the following ones:
Plate:
$LX=80\text{mm}$, $LY=40\text{mm}$ and $LZ=1\text{mm}$
Built:
$LX=80\text{mm}$, $LY'=\frac{3}{325}mm$ and $LZ=1\text{mm}$

11.3 Characteristic of the mesh

For this case, the mesh was reduced in order to limit the computing time. Consequently, the nodes B and E are not any more in the mesh and the displacements in these points are not calculated.

Many nodes: 858
Number of meshes and types: 384 `HEXA8` for the plate and the frame

472 QUAD4 for the sides of edges



11.4 Quantities tested and results

Identification	Reference	Tolerance (%)
DX to the item A	3.06970 E-5	1.0
DX as in point C	2.47360 E-5	1.0
DX as in point D	2.15872 E-5	1.0

12 Modelization I

12.1 Characteristic of the modelization

Modelization: $2D$ axisymmetric to test the operand "CONTINUE" key word FORMULATION in operator DEFI_CONTACT for elements TRIA3.

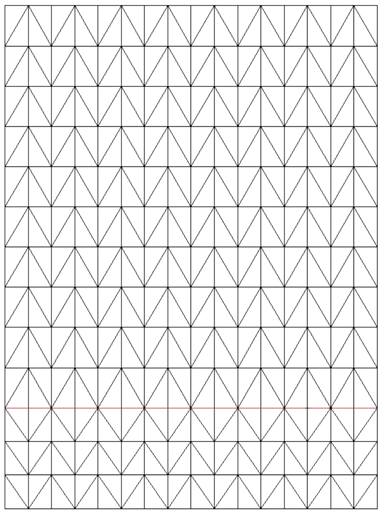
The geometry, the boundary conditions and the loadings are those of the problem of reference $3D$ rolls described paragraph §54 "Problem of reference in $3D$ ".
The cylindrical symmetry of the problem makes it possible to bring back the problem $3D$ of departure to an axisymmetric $2D$ problem.

The plate and the frame are with a grid with elements TRIA3.

This case is used as reference for case $3D$ XFEM of the modelization J.

12.2 Characteristic of the mesh

Many nodes: 255
Number of meshes and types: 416 TRIA3 for the plate and the frame.
58 SEG2 for the edge elements.



12.3 Quantities tested and results

Identification	*Aster*
DX at the point A	-2.3713E-05
DX at the point C	-1.8445E-05
DX at the point D	-1.568E-05

12.4 Remark

These results are used as references for the modelization J.

13 Modelization J

13.1 Characteristic of the modelization

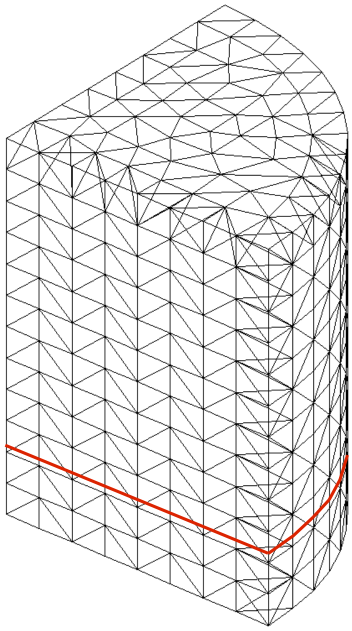
Modelization: $3D$ quarter of cylinder to test operand "XFEM" of key word FORMULATION in operator DEFI_CONTACT for elements TETRA4 .

The geometry, the boundary conditions and the loadings are those of the problem of reference $3D$ rolls described paragraph 4 "Problem of reference in $3D$ ".
However the test is carried out on a quarter of cylinder rather than on the whole cylinder. This simplification makes it possible to save time and memory on the cases test of the base.
Limiting conditions of symmetries are applied within cut in order to simulate the presence of the totality of the cylinder.

The plate and the frame are with a grid with elements TETRA4.

13.2 Characteristics of the mesh

Many nodes: 873
Number of meshes and types: 3613 TETRA4 for the plate and the frame.
918 TRIA3 for the edge elements.



13.3 Limiting conditions

Let us recall that displacement under X-FEM is the sum of a continuous displacement and a discontinuous displacement. In the case of an interface, without crack tip, the approximation of displacement is written in the following way:

$$u^h(x)=\sum_{i\in N_n(x)} a_i \phi_i(x)+\sum_{j\in N_n(x)\cap K} b_j \phi_j(x) H\left(lsn(x)\right)$$

Where:

a_i and b_i of displacement to the node the shape functions associated i

ϕ_i with the node are the degrees of freedom i .

$N_n(x)$ is all the nodes whose support contains the point x

K is all the nodes whose support is entirely cut by crack

$H(x)$ is the Heaviside function generalized defined by $H(x)=\begin{cases}-1 & \text{si } x<0\\ +1 & \text{si } x\geq 0\end{cases}$

$lsn(x)$ is the normal value of the level-set at the point x

For more details, to refer to documentation of reference X-FEM [R7.02.12].

In this test, one imposes conditions of symmetry on the side sides of the quarter of cylinder. With X-FEM, these conditions are not translated by imposing $DX=0$ or $DY=0$ on these sides. Indeed, displacement under X-FEM being the sum of a continuous displacement and a discontinuous displacement, it is possible to have a null displacement in all the nodes of the mesh but a non-zero displacement interpolated in the middle of an enriched mesh.
So that horizontal displacement according to the axis x is null right with the top of the level set (the reasoning is similar for a displacement according to the axis y) while having imposed $DX=0$ on all the nodes, it is necessary and sufficient to add the relation $a_x+b_x=0$ for all the nodes located under the level set in meshes crossed by crack.
One to impose does not add a relation similar to zero displacement right below the level set because the frame is extremely rigid in front of the rest of structure and thus does not become deformed.
This relation is added thanks to key word `LIAISON_GROUP` of operator `AFFE_CHAR_MECA`.

This case is treated in X-FEM great slidings, contrary to all the modelizations X-FEM the preceding ones. It is used to validate the approach and to solve the conflict between the algorithm of LBB the condition changes of status of frictions (member/slipping) proposed in documentation [R4.03.53].
The taking into account of the great slidings is physically not useful but that is used to test the new algorithms for the processing of friction. It is pointed out that in this case there is no reactualization of the thresholds of friction and that the total matrix is not symmetric.

13.4 Quantities tested and results

Identification	Reference	Tolerance (%)
DX to the point A	-2.3713E-05	1.0
DX at the point C	-1.8445E-05	1.0
DX at the point D	-1.568E-05	5.0

13.5 Remarks

the position of the point D for the approach X-FEM is not located exactly in $X=7.5\,mm$, that explains the more important variation for this point compared to the points A and B between the axisymmetric approach FEM and the approach X-FEM quarter of cylinder.

The case X-FEM is cut out in a mesh not structured. There is thus no control on the action of the algorithm of the condition LBB imposing of the relations of equalities on the Lagrangian ones of contact and friction. This test makes it possible to validate the theoretical choices described in Doc. [R4.03.53].

Another difficulty relates to the directions of slidings which are radial and consequently not parallels. This test also makes it possible to validate the strategy of the formulation of Lagrangian to the nodes tops which makes it possible not to impose a linear relation into tough more (the algorithm of the condition LBB imposes nothing any more but relations of equality).

14 Modelization K

14.1 Characteristic of the modelization

Modelization: D_PLAN_XFEM_CONT to test operand "XFEM" of key word FORMULATION of operator DEFI_CONTACT for elements SEG2 .

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of command using operator DEFI_FISS_XFEM [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ from higher edge of the plate.

The two materials are introduced as for modelization A.

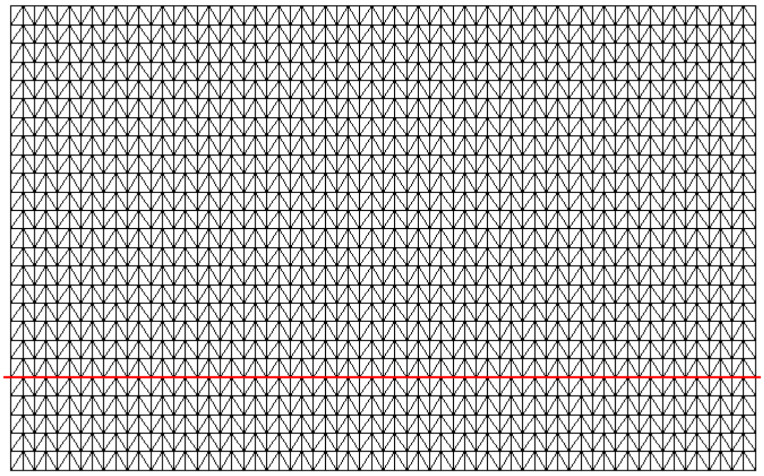
the crack tip is located apart from structure, which allows a total separation. This modelization is such as the crack is located at the interface of the elements.

The penalized method is used for the processing of contact-friction.

Dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\text{mm}$
Frame:
$LX=80\,mm$ and $LY=10\text{mm}$

14.2 Characteristic of the mesh

Many nodes: 1690
Number of meshes and types: 3200 TRIA3 for the plate and the frame and 178 SEG2 for edges.



14.3 Quantities tested and results

Identification	Reference	Tolerance (%)
RN to the point medium	1.04864 E+5	10.0
DX as in point A	2.84595 E-5	1.0
DX as in point B	2.70793 E-5	1.0
DX as in point C	2.27403 E-5	1.0
DX as in point D	1.97271 E-5	1.0

DX as in point E	1.53641 E-5	1.0

15 Modelization L

15.1 Characteristic of the modelization

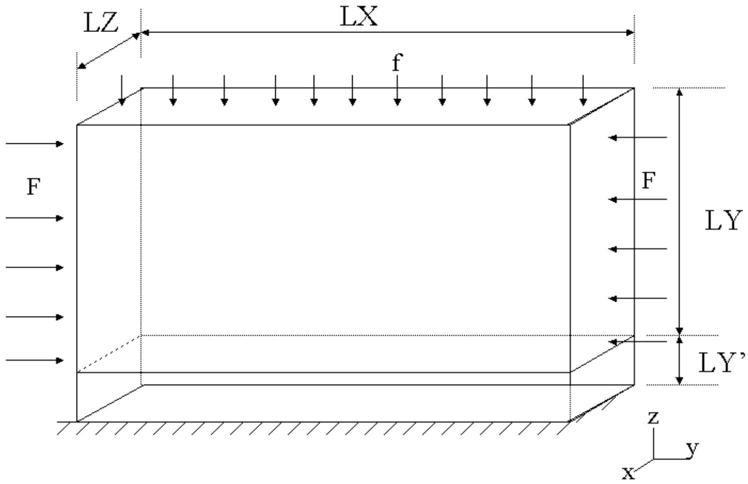
Modelization: 3D to test operand "XFEM" of key word FORMULATION under operator DEFI_CONTACT for elements HEXA8 .

The geometry, the boundary conditions and the loadings are those of the problem of reference $3D$ describes paragraph 4 "Problem of reference in $3D$ ".

The two materials are introduced as for modelization A.

the penalized method is used for the processing of contact-friction.

15.2 Geometry



the structure is a healthy parallelepiped, into which a crack will be introduced directly into card-indexing it of command using operator DEFI_FISS_XFEM [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ from higher edge of the plate.

The crack tip is located apart from structure, which allows a total separation. This modelization is such as the crack is located in the middle of the elements.

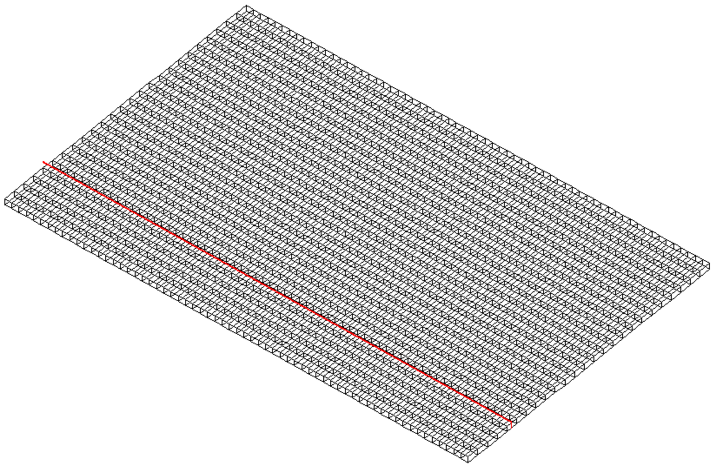
Dimensions are the following ones:
Plate:
$LX=80\,mm$, $LY=40\,mm$ and $LZ=1\,mm$
Built:
$LX=80\,mm$, $LY'=\frac{3}{325}mm$ and $LZ=1\,mm$

15.3 Characteristic of the mesh

For this case, the mesh was reduced in order to limit the computing time. Consequently, the nodes B and E are not any more in the mesh and the displacements in these points are not calculated.

Many nodes: 858

Number of meshes and types: 384 `HEXA8` for the plate and bâti472 `QUAD4` for the edge Quantities sides



15.4 tested and results

Identification	Reference	Tolerance (%)
DX to the item A	3.06970 E-5	1.0
DX as in point C	2.47360 E-5	1.0
DX as in point D	2.15872 E-5	1.0

16 Modelization M

16.1 Characteristic of the modelization

Modelization: `D_PLAN_XFEM_CONT` to test operand "`XFEM`" of key word `FORMULATION` under operator `DEFI_CONTACT` for elements `SEG3` .

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of command using operator `DEFI_FISS_XFEM` [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ from higher edge of the plate.

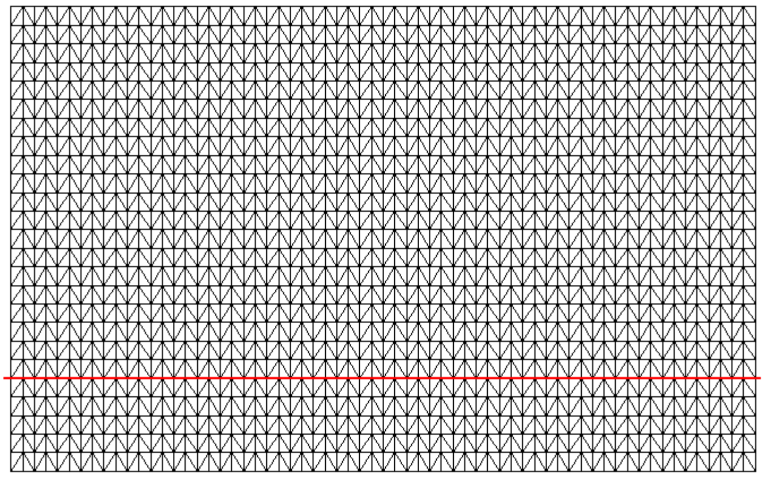
The two materials are introduced as for modelization A.

the crack tip is located apart from structure, which allows a total separation. This modelization is such as the crack is located at the interface of the elements.

Dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\text{mm}$
Frame:
$LX=80\,mm$ and $LY'=10\text{mm}$

16.2 Characteristic of the mesh

Many nodes: 1690
Number of meshes and types: 3200 `TRIA6` for the plate and the frame and 178 `SEG3` for edges.



16.3 Quantities tested and results

Identification	Reference	Tolerance (%)
DX to the item A	2.8617 E-5	1,0
DX as in point B	2.7229 E-5	1,0
DX as in point C	2.2855 E-5	1,0
DX as in point D	1.9805 E-5	1,0
DX as in point E	1.5411 E-5	the 1,0

values of reference result from quadratic modelization FEM.

17 Modelization N

17.1 Characteristic of the modelization

Modelization: `D_PLAN_XFEM_CONT` to test operand "`XFEM`" of key word `FORMULATION` under operator `DEFI_CONTACT` for elements `SEG3`.

The structure is a healthy rectangle, into which a crack will be introduced directly into card-indexing it of command using operator `DEFI_FISS_XFEM` [U4.82.08]. The crack is present at a distance $LY=40\text{mm}$ from higher edge of the plate.

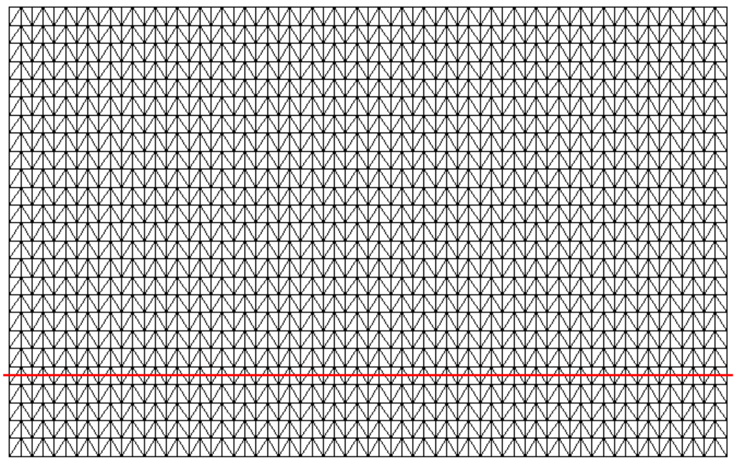
The two materials are introduced as for modelization A.

the crack tip is located apart from structure, which allows a total separation. This modelization is such as the crack is located in the middle of the elements.

Dimensions are the following ones:
Plate:
$LX=80\,mm$ and $LY=40\text{mm}$
Frame:
$LX=80\,mm$ and $LY'=\frac{3}{325}mm$

17.2 Characteristic of the mesh

Many nodes: 1625
Number of meshes and types: 3072 `TRIA6` for the plate and the frame and 176 `SEG3` for edges.



17.3 Quantities tested and results

Identification	Reference	Tolerance (%)
DX to the item A	2.8617 E-5	1,0
DX as in point B	2.7229 E-5	1,0
DX as in point C	2.2855 E-5	1,0
DX as in point D	1.9805 E-5	1,0
DX as in point E	1.5411 E-5	the 1,0

values of reference result from quadratic modelization FEM.

18 Modelization P

18.1 Characteristic of the modelization

It acts of the same modelization that the modelization J, one is always in great slidings, but with a penalized formulation this time. One uses a formulation with the nodes.

18.2 Characteristics of the mesh

It is the same mesh as that of the modelization J.

18.3 Quantities tested and results

the values of reference and the tolerances are identical to the modelization J.

Identification	Reference	Tolerance (%)
DX to the point A	-2.3713E-05	1.0
DX to the point C	-1.8445E-05	1.0
DX to the point D	-1.568E-05	5.0

19 Modelization R

19.1 Characteristic of the modelization

We take again characteristics identical to the modelization H, put except for the mesh which is quadratic. The purpose is thus to test operand " XFEM " of key word FORMULATION under operator DEFI_CONTACT , with friction (FROTTEMENT=' COULOMB') for a modelization 3D , but with a discretization P2P1 (CONTACT=' P2P1' indicated in MODI_MODELE_XFEM), which differentiates this modelization from the modelization H.
For recall, the geometry, the boundary conditions and the loadings those defined in section 21 , corresponding to the problem of reference are 3D described paragraph §5 "Problem of reference in 3D ".

19.2 Characteristics of the mesh

The mesh is that of the modelization H which one made quadratic. As for the modelization H, the nodes B and E are not any more in the mesh and the displacements in these points are not calculated.

19.3 Quantities tested and results

One F tests the results with the references of the modelization. the error is of a few percent (1% max) taking into account the difference in interpolation of the displacement, which is quadratic for this modelization and linear for the reference.

Identification	Reference	Tolerance (%)
DX at the point A	-2.3713E-05	1.0
DX at the point C	-1.8445E-05	1.0
DX at the point D	-1.568E-05	1.0

20 S Modelization

20.1 Characteristic of the modelization

We R take again characteristics identical to the modelization, but by dealing with the problem in great slidings. To recapitulate, the purpose is thus to test operand "XFEM" of key word FORMULATION under operator DEFI_CONTACT, with friction (FROTTEMENT=' COULOMB') for a modelization 3D, with a discretization P2P1 (CONTACT=' P2P1' indicated in MODI_MODELE_XFEM), but in great slidings (REAC_GEOM=' CONTRÔLE' indicated in DEFI_CONTACT) what differentiates this modelization from modelization R.

For recall, the geometry, the boundary conditions and the loadings those defined in section 21, corresponding to the problem of reference are 3D described paragraph §5 "Problem of reference in 3D".

20.2 Characteristics of the mesh

The mesh is that of the modelization H which one made quadratic. As for the modelization H, the nodes B and E are not any more in the mesh and the displacements in these points are not calculated.

20.3 Quantities tested and results

One F tests the results with the references of the modelization. the error is of a few percent (3% max) taking into account the difference in interpolation of the displacement, which is quadratic for this modelization and linear for the reference.

Identification	Reference	Tolerance (%)
DX at the point A	-2.3713E-05	2.0
DX at the point C	-1.8445E-05	3.0
DX at the point D	-1.568E-05	3.0

21 Summary of the results

the case test ssnv209 makes it possible to observe the three states of the conditions of contact, namely separation, the slipping contact and the adhering contact. It puts also concerned modelizations $2D$ and $3D$ with elements of various natures (quadrangles, triangles, parallelepipeds) in the frame of X-FEM.

This case test makes it possible to highlight satisfactory results for modelizations $2D$ and $3D$, in particular the satisfaction of the condition LBB for the modelization of contact-rubbing with X-FEM.

The goals of this test are achieved:

1) with a relative error compared to the reference solution of less 1% for displacements,
2) that the crack is in the middle of an element or with the interface,
3) that the modelization is in $2D$ or in $3D$,
4) that the formulation places the unknowns of contact at the edges of the elements or the nodes,
5) with several diagram of integration for the terms of contact `(NOEUD, SIMPSON, NCOTES)`,
6) by means of the coefficients (friction and pressure) of the generalized formulation `(COEF_REGU_CONT, COEF_STAB_CONT, COEF_REGU_FROT, COEF_STAB_FROT)`,
7) by means of the methods of Lagrangian increased and penalization for the processing of contact-friction.