# SSNV199 – Cracking of a beam DCB with cohesive models

**Summarized:**

This test makes it possible to model the propagation of a plane crack in a beam DCB (Double Beam Cantilever) three-dimensional elastic with the following cohesive models:

1) finite elements of joint (modelization 3D_JOINT)
2) finite elements of interface (modelization 3D_INTERFACE).

Modelization a: Element of joint HEXA8 and cohesive model CZM_EXP_REG
Modelization b: Element of joint PENTA6 and cohesive model CZM_LIN_REG
Modelization C: Element of joint HEXA8 and cohesive model CZM_EXP_REG
Modelization D: Element of joint PENTA6 and cohesive model CZM_LIN_REG
Modelization E: Element of interface HEXA20 and cohesive model CZM_OUV_MIX
Modelization F: Element of interface PENTA15 and cohesive model CZM_OUV_MIX
Modelization G: Element of interface HEXA20 and cohesive model CZM_OUV_MIX
(idem that the modelization E with initial crack with a grid)

the control of the loading by elastic prediction is also tested in all the modelizations. The classification local *ad hoc* of the cohesive elements is assured by the command MODI_MAILLAGE and key word ORIE_FISSURE.

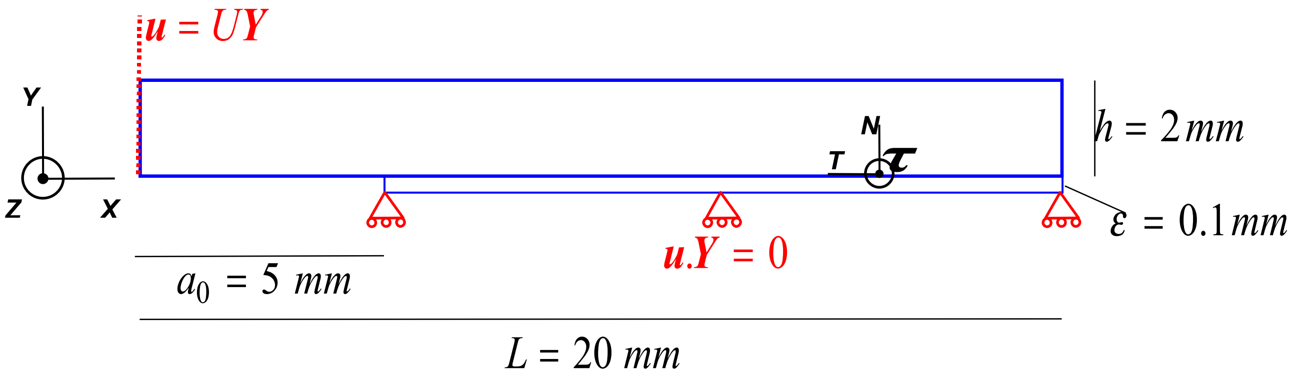
# 1 Problem of reference

## 1.1 Geometry and loading

In the cartesian coordinate system $(X, Y, Z)$ one considers a beam DCB three-dimensional definite on the field $\Omega=[0,L]\times[-h,h]\times[0,b]$ length $L=20\,mm$, height $2h=4mm$ and thickness $b=6mm$, an initial crack $\Gamma_0=[0,a_0]\times\{0\}\times[0,b]$ length $a_0=5\,mm$. One imposes on displacement $u$ the following boundary conditions:

$\boldsymbol{u}=U\,\boldsymbol{Y}$ on the edge $\{0\}\times\{0^+\}\times[0,b]$
$\boldsymbol{u}=-U\,\boldsymbol{Y}$ on the edge $\{0\}\times\{0^-\}\times[0,b]$

Taking into account symmetries of the problem, computation is realized on half of structure (see figure 1.1-a). The mesh of the half-beam is carried out with tetrahedral or hexahedral voluminal elements.




**Appears 1.1-a : Diagram of beam DCB in the plane $(X, Y)$, boundary conditions and loading**

the potential crack way is with a grid by a layer of cohesive elements 3D (pentahedral or hexahedrons) of thickness not nulle1La[1] $\varepsilon=0.1\,mm$ corresponding to the field $[a_{0,}L]\times[-\varepsilon/2,\varepsilon/2]\times[0,b]$. So the voluminal field of the beam is definite par. $[a_{0,}L]\times[\varepsilon/2,h]\times[0,b]$ One imposes the loading $\boldsymbol{u}=U\,\boldsymbol{Y}$ on stops $\{0\}\times\{\varepsilon/2\}\times[0,b]$ and the condition of symmetry $\boldsymbol{u}\cdot\boldsymbol{Y}=0$ on the low part of the layer of joints: $[a_{0,}L]\times[-\varepsilon/2,h]\times[0,b]$.

1 local classification of the elements of this layer is carried out with command ORIE_FISSURE. It is pointed out that this one requires that the elements do not have a thickness null.

# 2 Reference solution

There exists an approximate analytical solution with the mechanical problem presented in the preceding part. This one is based on the theory of the beams, it is valid for a slender structure $h \ll L$. The statement of the kindness $C$ of the DCB is given by:

$$C = U/F = \frac{a^3}{3EI}$$

where $F$ indicates the reaction force corresponding to imposed displacement $U$, $a$ the length of crack, $I = bh^3/12$ the main moment of inertia of the beam and $b$ its thickness. Rate of energy restitution $G$ associated with a crack length $a$ is given by:

$$G = \frac{P^2}{2b}\frac{dC}{da} = \frac{9EI}{ba^4}U^2 \qquad \text{éq 2-1}$$

For a stable crack propagation one supposes the assumption of checked Griffith: $G = G_c$, which leads to the statement length of crack $a$ according to the loading $U$ as well as the total response of beam:

$$a = \left(\frac{9\mathrm{EI}}{bG_c}\right)^{1/4} U^{1/2}, \qquad F = \frac{(EI)^{1/4}(bG_c)^{3/4}}{(3\mathrm{U})^{1/2}} \qquad \text{éq 2-2}$$

Let us note however that the assumption $G = G_c$ is an approximation, owing to the fact that a cohesive model is used. This one is valid if the size of the zone cohésive2Cette[2] is small in front of the length of crack. That returns for cohesive the model taking a sufficiently small $l_c = G_c/\sigma_c$ characteristic length.

# 3 Material parameters

the values of the Young modulus, the Poisson's ratio, the critical stress and the tenacity of the material are in the following way selected:

$E = 100$ MPa , $\nu = 0$ , $\sigma_c = 3$ MPa , $G_c = 0.9$ MPa.mm

(NB: they are values "tests" which do not correspond to any material in particular)

***Note:*** *: The mechanical problem is symmetrized: one models only half of a crack (only one lip). The latter dissipates an energy twice less important than a complete crack. To model a material of tenacity given* $G_c$ *, it is thus necessary to carry out simulation with a value of* $G_c/2$ *.*

2 zone corresponds to the zone of continuous transition between the operational material and the broken material, it does not exist with the approach of Griffith.

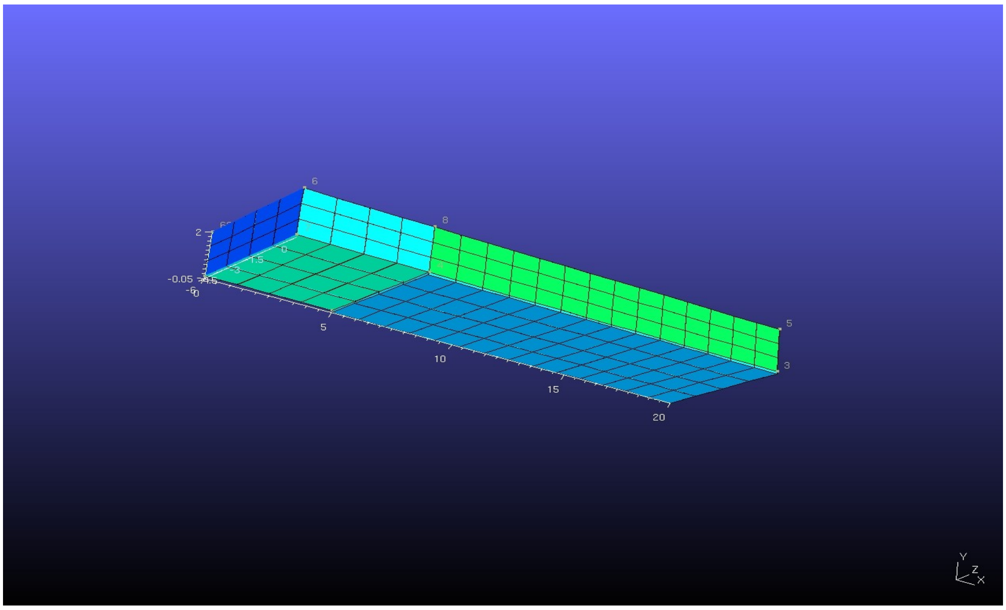
# 4 Modelization A

## 4.1 Characteristic of the modelization

simulation is carried out with modelization 3D_JOINT. The elements are of type HEXA8 for the elements of joint as for the voluminal elements. The adopted cohesive constitutive law is CZM_EXP_REG. The parameter of regularization of model PENA_ADHERENCE is worth 10-5. The voluminal elements are elastic.

## 4.2 Characteristics of the mesh

One carries out a linear structured mesh of the half-beam (figure below).
Voluminal elements (DCB): 216 HEXA8
Elements of joint (crack way): 56 HEXA8



**Appear 4.2-a : Diagram of the mesh of the half beam, (the initial crack is not with a grid).**

## 4.3 Quantities tested and results

***Note** : Works on the approximate comparison of the solutions numerical and analytical were completed in an internal note (cf H-T64-2007-0342). One will be satisfied here to provide the values of non regression obtained with Code_Aster.*

One notes $F^R$ the resultant of the force corresponding to imposed displacement $U$.

| Quantity tested | *Code_Aster* |
|---|---|
| $U$ at time: 2 | 4.6061236901011D+00 |
| $F^R$ time: 2 | 7.0451492319953D+00 |
| $U$ at time: 3 | 6.9693988127164D+00 |
| $F^R$ time: 3 | 5.7661719205232D+00 |
| $U$ at time: 4 | 9.7548271517894D+00 |
| $F^R$ time: 4 | 4.8584218510416D+00 |

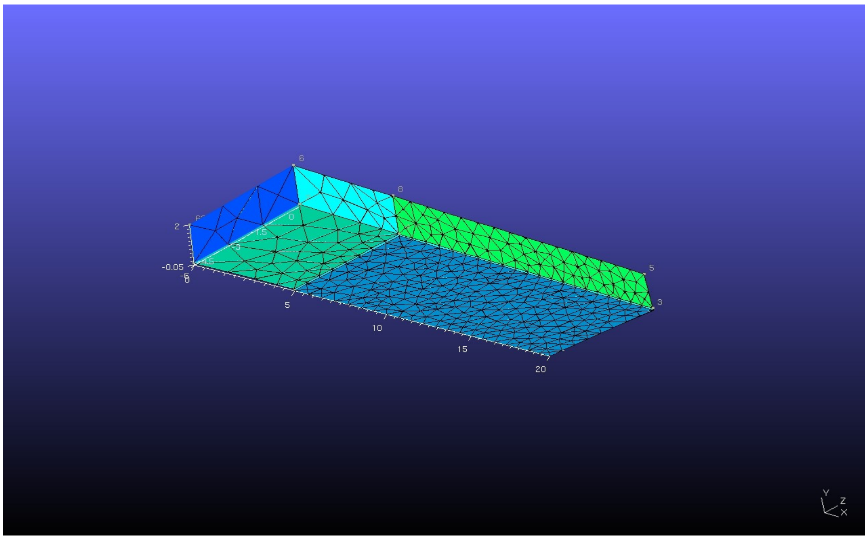
# 5 Modelization B

## 5.1 Characteristic of the modelization

simulation is carried out with modelization 3D_JOINT. The elements are of type TETRA4 for the voluminal elements and PENTA6 for the elements of joint. The adopted cohesive constitutive law is CZM_EXP_REG. The parameter of regularization of model PENA_ADHERENCE is worth 10-5. The voluminal elements are elastic.

## 5.2 Characteristics of the mesh

One carries out a linear mesh not structured of the half-beam (figure below).
Voluminal elements (DCB): 3481 TETRA4
Elements joined (crack way): 462 PENTA6



**Appear 5.2-a : Diagram of the mesh of the half beam, (the initial crack is not with a grid).**

## 5.3 Quantities tested and results

Even notices that in paragraph 4.4.
One notes $F^R$ the resultant of the force corresponding to imposed displacement $U$.

| Quantity tested | *Code_Aster* |
|---|---|
| $U$ at time: 2 | 4.0386002472857D+00 |
| $F^R$ time: 2 | 7.9981249343083D+00 |
| $U$ at time: 3 | 6.1492839708222D+00 |
| $F^R$ time: 3 | 6.5587957007180D+00 |
| $U$ at time: 4 | 8.6763623955462D+00 |
| $F^R$ time: 4 | 5.5595526657741D+00 |

# 6 Modelization C

## 6.1 Characteristic of the modelization

simulation is carried out with modelization 3D_JOINT. The elements are of type HEXA8, for the elements of joint as for the voluminal elements. The adopted cohesive constitutive law is CZM_LIN_REG. The parameter of regularization of model PENA_ADHERENCE is worth 10-5. The voluminal elements are elastic.

## 6.2 Characteristics of the mesh

One carries out a linear structured mesh of the half-beam (even mesh that the modelization A).
Voluminal elements (DCB): 216 HEXA8
Elements of joint (crack way): 56 HEXA8

## 6.3 Quantities tested and results

Even notice that in paragraph 4.4.
One notes $F^R$ the resultant of the force corresponding to imposed displacement $U$.

| Quantity tested | *Code_Aster* |
|---|---|
| $U$ at time: 2 | 4.6186712601876D+00 |
| $F^R$ time: 2 | 7.1316429152946D+00 |
| $U$ at time: 3 | 6.9041423768554D+00 |
| $F^R$ time: 3 | 5.8318660215042D+00 |
| $U$ at time: 4 | 9.6259568305961D+00 |
| $F^R$ time: 4 | 4.9452238152838D+00 |

# 7 Modelization D

## 7.1 Characteristic of the modelization

simulation is carried out with modelization 3D_JOINT. The elements are of type TETRA4 for the voluminal elements and PENTA6 for the elements of joint. The adopted cohesive constitutive law is CZM_LIN_REG. The parameter of regularization of model PENA_ADHERENCE is worth 10-5. The voluminal elements are elastic.

## 7.2 Characteristics of the mesh

One carries out a linear mesh not structured of the half-beam (even mesh that modelization B).
Voluminal elements (DCB): 3481 TETRA4
Elements of joint (crack way): 462 PENTA6

## 7.3 Quantities tested and results

Even notice that in paragraph 4.4.
One notes $F^R$ the resultant of the force corresponding to imposed displacement $U$.

| Quantity tested | *Code_Aster* |
|---|---|
| $U$ at time: 2 | 4.0193719091077D+00 |
| $F^R$ time: 2 | 8.0668656941765D+00 |
| $U$ at time: 3 | 6.0660030864088D+00 |
| $F^R$ time: 3 | 6.6762704371762D+00 |
| $U$ at time: 4 | 8.4416874805246D+00 |
| $F^R$ time: 4 | 5.6476764257501D+00 |

# 8 Modelization E

## 8.1 Characteristic of the modelization

simulation is carried out with modelization 3D_INTERFACE and of the elements of the type HEXA20 for the elements of interfaces as for the voluminal elements. The adopted cohesive constitutive law is CZM_OUV_MIX. The parameter of penalization of Lagrangian the PENA_LAGR is worth 102, the stiffness of sliding RIGI_GLIS is worth 10. The voluminal elements are elastic.

## 8.2 Characteristics of the mesh

One carries out a quadratic structured mesh of the half-beam.
Voluminal elements (DCB): 216 HEXA20
Elements of interface (crack way): 56 HEXA20

## 8.3 Quantities tested and results

Even notice that in paragraph 4.4.
One notes $F^R$ the resultant of the force corresponding to imposed displacement $U$.

| Quantity tested | *Code_Aster* |
|---|---|
| $U$ at time: 2 | 5.00829D+00 |
| $F^R$ time: 2 | 6.78854D+00 |
| $U$ at time: 3 | 6.97298 D+00 |
| $F^R$ time: 3 | 5.75772 D+00 |
| $U$ at time: 4 | 9.12554 D+00 |
| $F^R$ time: 4 | 4.93281 D+00 |

# 9 Modelization F

## 9.1 Characteristic of the modelization

simulation is carried out with modelization 3D_INTERFACE and of the elements of the type PENTA15 for the interfaces and TETRA10 for the voluminal elements. The adopted cohesive constitutive law is CZM_OUV_MIX. The parameter of penalization of Lagrangian the PENA_LAGR is worth 102, the stiffness of sliding RIGI_GLIS is worth 10. The voluminal elements are elastic.

## 9.2 Characteristics of the mesh

One carries out a quadratic mesh not structured of the half-beam.
Voluminal elements (DCB): 3481 TETRA10
Elements of interface (crack way): 462 PENTA15

## 9.3 Quantities tested and results

Even notice that in paragraph 4.4.
One notes $F^R$ the resultant of the force corresponding to displacement imposed $U$.

| Quantity tested | *Code_Aster* |
|---|---|
| $U$ at time: 2 | 4.85116 D+00 |
| $F^R$ time: 2 | 6.89090 D+00 |
| $U$ at time: 3 | 6.74636 D+00 |
| $F^R$ time: 3 | 5.83393 D+00 |
| $U$ at time: 4 | 8.92820 D+00 |
| $F^R$ time: 4 | 5.06475 D+00 |

# 10 Modelization G

## 10.1 Characteristic of the modelization

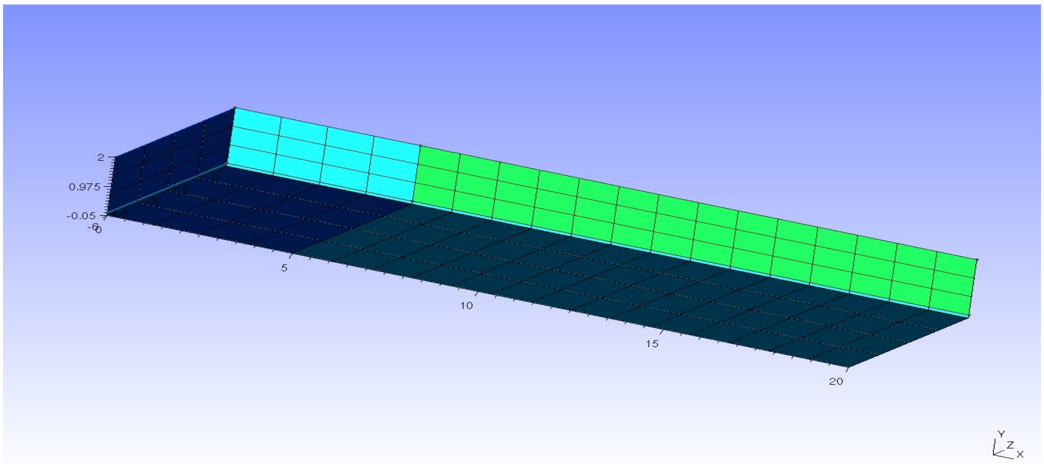
simulation is carried out with modelization 3D_INTERFACE and of the elements of the type HEXA20 for the elements of interfaces as for the voluminal elements.

The mechanical test is identical to the modelization E. One only changes mesh: the initial crack is with a grid (see figure 10.2-a) and one initialise3à[3] to "broken" the local variables the elements of interface located in this zone.
This modelization is used as example to the users. That can for example be useful when one wishes to define a crack front which is not right (here it is right). In addition that makes it possible to take into account the contact on the initial crack what is not the case if this one is not with a grid with cohesive elements.

The adopted cohesive constitutive law is CZM_OUV_MIX. The parameter of penalization of Lagrangian the PENA_LAGR is worth $10^2$, the stiffness of sliding RIGI_GLIS is worth 10. The voluminal elements are elastic.

## 10.2 Characteristics of the mesh

One carries out a quadratic structured mesh of the half-beam.
Voluminal elements (DCB): 216 HEXA20
Elements of interface, initial crack : 16 HEXA20, fissures potential : 56 HEXA20




**Appear 10.2-a : Diagram of the mesh of the half beam, (the initial crack is with a grid).**

## 10.3 Quantities tested and results

Even notices that in paragraph 4.4.
One notes $F^R$ the resultant of the force corresponding to displacement imposed $U$. The same tests exactly are carried out that the modelization E, the results are identical.

| Quantity tested | *Code_Aster* |
|---|---|
| $U$ at time: 2 | 5.01006 D+00 |
| $F^R$ at time: 2 | 6.78363 D+00 |

3 to leave of the command CREA_CHAMP

| | |
|---|---|
| $U$ at time: 3 | 6.97518 D+00 |
| $F^R$ at time: 3 | 5.75485 D+00 |
| $U$ at time: 4 | 9.13210 D+00 |
| $F^R$ at time: 4 | 4.93322 D+00 |

# 11 Summary of the results

the cohesive models of joint and interface predict a propagation of cracking all made correct taking into account the approximate analytical solution. That makes it possible to validate, partly, these types of models in 3D. For more details on the numerical results, one can refer to internal note H-T64-2007-03420-FR.