SSNV195 - The purpose of bar in multi-cracking with X-FEM

Summarized

This test is in the case of testing and validating the features of the method X-FEM the structures 2D and 3D multi-fissured. Broadly it is a question of remaking essential tests SSNV173 [V6.04.173] and SSNV182 [V6.04.182] for a multi-fissured structure this time.

This test brings into play a parallelepipedic bar, whose median section is embedded, presenting two cracks the beam completely (one will speak then about interfaces), symmetrically placed coast and other of the clamped section. The bar is subjected to imposed displacements, which has as a consequence the total opening of the interfaces and the separation of structure in three parts when she is requested in tension, or the appearance of contact pressure when the request is of compression.

Two types of interfaces are considered, horizontal (only one stage of elements is cut) and inclined (several stages of elements are cut, in order to test the good management of the under-cutting of the elements at the time of the presence of several cracks in structure.

The case of a plate 2D presenting two interfaces is also studied.

1 Problem of reference

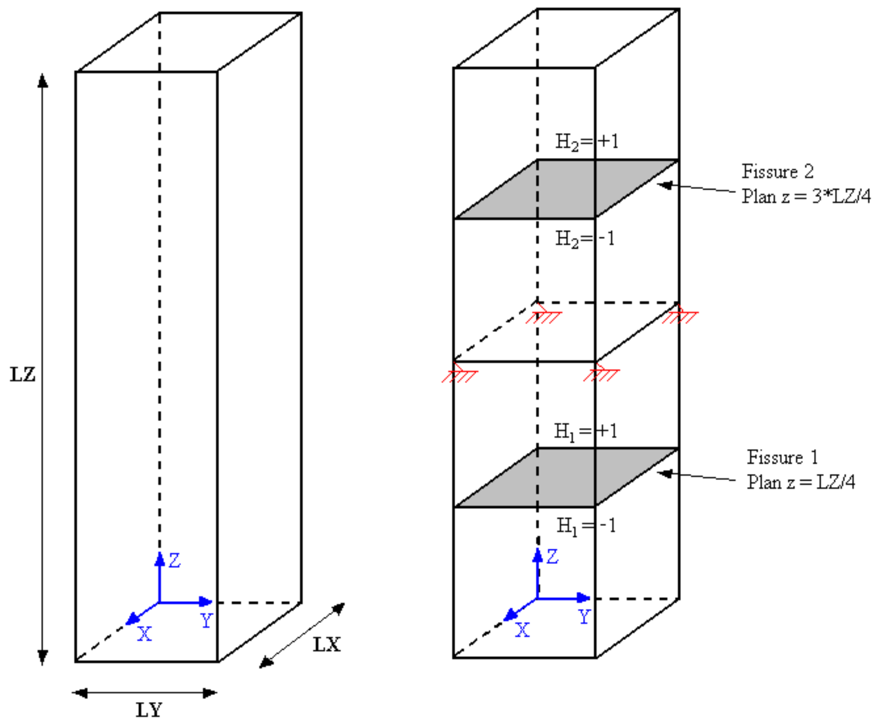
1.1 Geometry 3D

the structure is a right at square base and healthy parallelepiped. Dimensions of the bar (see [Figure 1.1-1]) are: $LX=5\,m$, $LY=5\,m$ and $LZ=50\,m$. It does not comprise any crack.

The interfaces will be introduced by functions of levels (level sets noted LN for level set norm) directly into the command file using operator `DEFI_FISS_XFEM` [U4.82.08]. Initially, the interfaces horizontal, and will be introduced each one in the middle of the blocks inferior and superior compared to the clamped section (see [Figure 1.1-2]). The equations of the functions of levels for the two horizontal interfaces are thus the following ones:

$$\mathrm{LN1}=Z-LZ/4 \qquad \textbf{éq 1.1-1}$$
$$\mathrm{LN2}=Z-3\cdot LZ/4 \qquad \textbf{éq 1.1-2}$$

No level set tangential is necessary since one used key word `TYPE_DISCONTINUITE=' INTERFACE'`, which makes it possible to have a structure completely cut in three parts.



1) 2)

Figure 1.1. Geometry of the bar and positioning of crack

For the modelization C the two interfaces will be tilted undergoing rotations along the axis OX (see [Figure 4.1-1]). A this occasion one tests also the correct operation of the operators of post processing in X-FEMs which underwent modifications during the transition with multi-cracking.

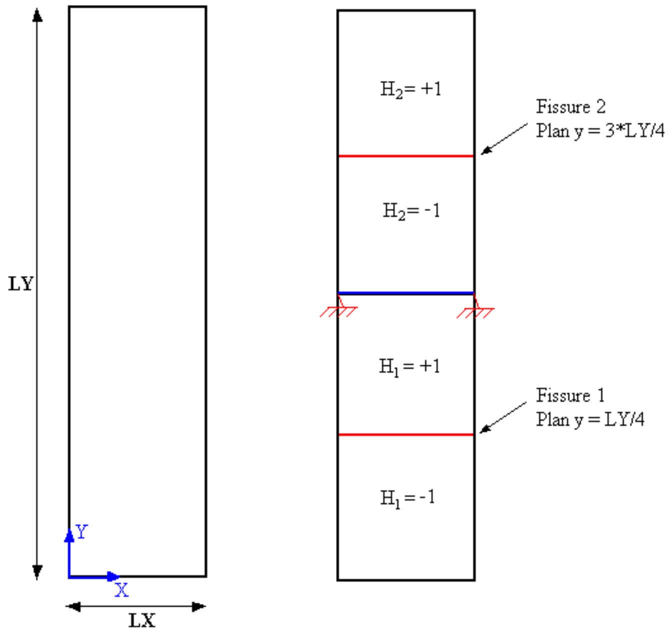
1.2 Geometry 2D

the structure is a healthy rectangle. Dimensions of the bar (see [Figure 1.2-1]) are: $LX=5\,m$ and $LY=50\,m$. It does not comprise any crack.

The interfaces will be introduced by functions of levels (level sets) directly into the file orders using operator `DEFI_FISS_XFEM` [U4.82.08]. By analogy with the case $3\mathrm{D}$, the interfaces are present at the mediums of the two parts of the plate, separated by line from fixed support (see Figure 1.2-a). The corresponding equations of the functions of levels are:

$$\mathrm{LN1}=\mathrm{Y}-LY/4 \quad \textbf{éq 1.2-1}$$
$$\mathrm{LN2}=Y-3\cdot LY/4 \quad \textbf{éq 1.2-3}$$

No level set tangential is necessary since one used key word `TYPE_DISCONTINUITE='INTERFACE'`, which makes it possible to have a structure completely cut in three parts.



1) **2)**

Figure 1.2. Geometry of the plate and positioning of the crack

1.3 Properties of the material

Modulus Young: $E=100\,MPa$

Poisson's ratio: $\nu=0.0$

1.4 Boundary conditions and loadings

the nodes of the median surface of the bar are clamped (see them [Figure 1.1-2] and [Figure 1.2-2]) while displacements are imposed on those of surfaces lower and higher. One wishes to initially show the possibility of separating a structure in 3 parts following the introduction of two interfaces (imposed displacements will open the interfaces) and in the second time one of the contact will prove the taking into account on the lips of two interfaces (imposed displacements will close the interfaces).

2 Modelization a: opening horizontal Interfaces

2.1 Characteristic of the mesh

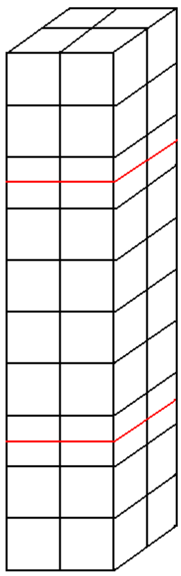


Figure 2.1-1: The mesh of the modelization A

One discretizes structure using finite elements HEXA8. According to the three directions of the reference system chosen, there are $2\times2\times10$ elements thus a total of 40 finite elements (see [Figure 2.1-1])

the number of stages of elements according to the direction Z was selected to avoid the enrichment of the same element by the two interfaces. Indeed, in the implementation of multi-cracking (see manual) one avoids approaching two cracks with less than 4 healthy elements in order to avoiding the conflicts during the management of the degrees of freedom nouveau riches.
Thus, by choosing 10 stages, one will be able at the same time to impose boundary conditions on the meshes extreme ones in the usual way and to prevent that enrichments characteristic of the X-FEM are touched. The first interface will be introduced in the middle of the third stage while the second in the middle of the eighth.

2.2 Boundary conditions

In order to open the two interfaces and to prevent rigid body motions, one embeds the nodes belonging to median surface:

```
GROUP_NO=SURFMED: DX = 0, DY = 0 and DZ = 0
```

For the nodes belonging to extreme surfaces (lower and higher) one imposes displacements as follows:

```
GROUP_NO=SURFINF: DX = 0, DY = 0 and DZ = - DEPZ
GROUP_NO=SURFSUP: DX = 0, DY = 0 and DZ = DEPZ
```

the value of imposed displacement is $DEPZ=3.10^{-3}m$. One will be able in makes move the two extreme blocks of the bar following the three directions by simple assignment of a non-zero value for the degrees of freedom corresponding to DX or DY .

2.3 Analytical resolution

the solution of such a problem is of course obvious. As explained for the case test SSNV173 [V6.04.173], the solution is the following one: all displacements according to x and y are null, all displacements following z below the level-set lower are equal to the displacement imposed u_z on the base of structure, all displacements following z between the two level-set are null and all displacements following z to the top of the level set higher are equal to the displacement imposed u_z on the top of structure.

2.4 Quantities tested and results

One tests the values of displacement after the convergence of the iterations of operator STAT_NON_LINE. It is checked that one finds well the values determined with [§2.32.3].

IdentificationRéférenceTolérance DX
for all the nodes right below the interface inférieure0.001.0E-15
DX for all the nodes right with the top of the interface inférieure0.001.0E-15
DY for all the nodes right below the interface inférieure0.001.0E-15
DY for all the nodes right with the top of the interface inférieure0.001.0E-15
DZ for all the nodes right below the interface lower-3.E-31.0E-09%
DZ for all the nodes right with the top of the interface inférieure0.001.0E-15
DX for all the nodes right below the interface supérieure0.001.0E-15
DX for all the nodes right with the top of the interface supérieure0.001.0E-15
DY for all the nodes right below the interface supérieure0.001.0E-15
DY for all the nodes right with the top of the interface supérieure0.001.0E-15
DZ for all the nodes right belowinterface supérieure0.001.0E-15
DZ for all the nodes right with the top of the interface supérieure3.E-31.0E-09%

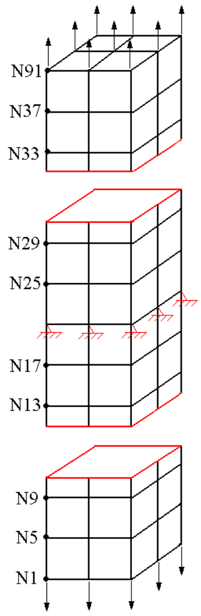


Figure 2.4-1: Final mesh and positioning of the nodes tested

to test all the only one nodes times, one tests the minimum and the maximum of column.

2.5 Remarks

It is noticed that the values of the degrees of freedom tested are those expected, the interfaces open thus separating the bar in three parts. Result is visualized using the operators of postprocessing.

3 Modelization b: horizontal Interfaces in closing

3.1 Characteristics of the mesh and the interface

The mesh is the same one as that of modelization A. the purpose being of the contact to test management on the interfaces represented with X-FEM, only the boundary conditions changed.

3.2 Boundary conditions

the boundary conditions will allow this time of the contact to test the taking into account without friction on the lips of the interfaces. One thus keeps the fixed support of the nodes belonging to median surface but the displacements imposed on the nodes of the two extremes (lower and higher) will close the interfaces. One will thus have:

```
GROUP_NO=SURFMED: DX = 0, DY = 0 and DZ = 0
GROUP_NO=SURFINF: DX = 0, DY = 0 and DZ = DEPZ
GROUP_NO=SURFSUP: DX = 0, DY = 0 and DZ = - analytical
```

3.3 DEPZ Resolution

the interfaces being horizontal and the state of uniaxial pressing and normal to the interface, it does not have there possible sliding. One checks the value of the contact pressure and the solution of the problem is that of the same problem without interface, for each block of a side and other of clamped surface. As shown for the case test SSNV182 [V6.04.182] and taking into account the geometry and of the boundary conditions described with the §2.2, the contact pressure is given by:

$$\lambda = E\frac{DEPZ}{LZ/2}$$ **éq 4.3-1**

With the numerical values previously introduced $\lambda = -12.10^3$ Pa .

3.4 Quantities tested and results

the good progress of computation makes it possible *a priori* of the contact to validate good management for the multi cracking. The components of `SD FISS_XFEM` concerning the contact for each crack are managed correctly on the level of operators `MODI_MODELE_XFEM` and `STAT_NON_LINE`. One tests the values of the contact pressure for the medium nodes belonging to the elements cut by cracks.

Identification	Reference
`LAGS_C` for all the nodes of the two cracks	-1.2E4
`LAGS_F1` for all the nodes of two cracks	0.00
`LAGS_F2` for all the nodes of two cracks	0.00

4 Modelization C – tilted Interfaces

the purpose of This modelization is proving the taking into account of the contact rubbing for tilted interfaces as well as the correct operation of the algorithms of cutting and postprocessing.
One chose an angle of inclination $\theta=30°$ according to rotation around the axis OX, for the two interfaces. The norm with the interfaces thus create is noted $\mathbf{n}$ and the tangent vector is noted $\boldsymbol{\tau}$:

$$\mathbf{n}=\begin{pmatrix}0\\-1/2\\\sqrt{3}/2\end{pmatrix},\quad \boldsymbol{\tau}=\begin{pmatrix}0\\\sqrt{3}/2\\1/2\end{pmatrix}$$ **éq 6-1**

the new functions of level introducing two cracks into the command file are:

$$\mathrm{LN1}=\mathrm{Z}-\tan\theta\cdot Y-\left(LZ/4-\tan\theta\cdot LX/2\right)$$ **éq 6-2**

$$\mathrm{LN2}=\mathrm{Z}-\tan\theta\cdot Y-\left(3\cdot LZ/4-\tan\theta\cdot LX/2\right)$$ **éq 6-3**

4.1 of the mesh

Compared to the modelizations A and B, the number of elements according to the direction Z (height of the bar) was multiplied by 2 in order to avoid the adjacency of the elements nouveau riches by the two tilted interfaces. There is thus a mesh $2\times2\times20$ elements HEXA8 (see [Figure 4.1-1]).

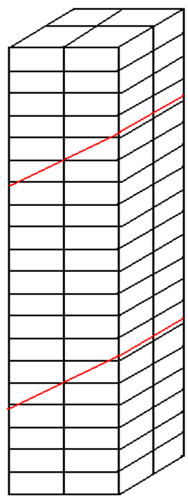


Figure 4.1-1: Mesh for the CCaractéristiques modelization

4.2 Boundary conditions

the boundary conditions make it possible to carry out a mixed request: compression on the level of the first interface and tension for the second.
Thus, besides the fixed support of the nodes belonging to median surface (as for the first two modelizations), one imposes positive displacements according to the direction Z for the nodes belonging to extreme surfaces (lower and higher). One will thus have:

```
GROUP_NO=SURFMED: DX = 0, DY = 0 and DZ = 0
GROUP_NO=SURFINF: DX = 0, DY = 0 and DZ = DEPZ
GROUP_NO=SURFSUP: DX = 0, DY = 0 and DZ = analytical
```

4.3 DEPZ Resolution

the analytical solution concerns, for the lower block (interface 1), the values of the contact pressure and the semi-multipliers of friction. For the higher block (the interface 2), the analytical solution relates

to the values of displacements of the nodes. The solution for this last is identical to that of the modelization A and thus one will detail only the analytical solution for the lower block.
The interfaces being inclined, one will be able to have sliding. To avoid that, one forces the dependancy by choosing a coefficient of kinetic friction of sufficiently high Coulomb. Theoretically, it is enough to take:

$$\mu > \tan(\theta) \quad \textbf{éq 6.3-1}$$

Thus, the solution of the problem remains identical to that of the same problem without interface. The analytical resolution for this problem is presented in [V604182 - §4.1]. One finds for the contact pressure:

$$\lambda = n_z \cdot E \frac{DEPZ}{LZ/2} \cdot n_z \quad \textbf{éq 6.3-2}$$

and for the semi-multipliers of friction:

$$\Lambda = \left(\frac{1}{\mu} \frac{\tau_z}{n_z} \right) \tau \quad \textbf{éq 6.3-3}$$

With the numerical values previously introduced and considering $\mu = 1$, one obtains for the contact pressure $\lambda = -9.10^3 Pa$ and the semi-multiplier of friction following the direction τ $\Lambda \cdot \tau = 1/\sqrt{3}$.

4.4 Quantities tested and results

One tests the values of the contact pressure and the semi-multipliers of friction for interface 1. For interface 2 one tests the values of displacements to the top and below the level-set.

IdentificationRéférenceDX	
for all the nodes right below the interface inférieure0.00DX	
for all the nodes right with the top of the interface inférieure0.00DY	
for all the nodes right below the interface inférieure0.00DY	
for all the nodes right with the top of the interface inférieure0.00LAGS_C	
for all the nodes of the interface inférieure-9E3LAGS_F1	
for all the nodes of the interface inférieure0LAGS_F2	
for all the nodes of the interface inférieureDX	$1/\sqrt{3}$
for all the nodes right below the interface supérieure0.00DX	
for all the nodes right with the top of the interface supérieure0.00DY	
for all the nodes right below the interface supérieure0.00DY	
for all the nodes right with the top of the interface supérieure0.00DZ	
for all the nodes right below the interface supérieure0.00DZ	
for all the nodes Juste with the top of the interface supérieure3.0E-3	

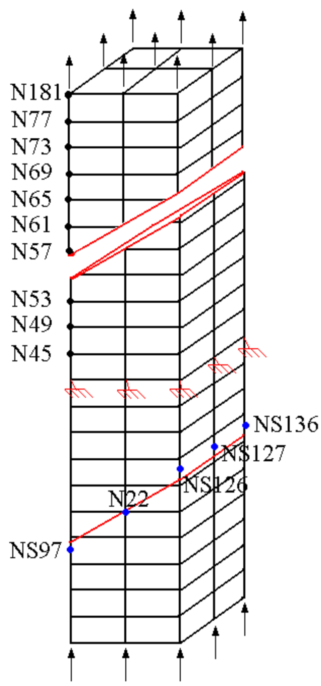


Figure 4.4-1: Final mesh and positioning of the nodes tested

to test all the nodes in only once, one tests the minimum and the maximum of column.

4.5 Remarks

It is checked that the numerical values obtained for the degrees of freedom tested, following the analysis with several interfaces X-FEM, are quite identical with the values of the analytical solution for the two interfaces.

5 Modelization D – multiple the purpose of Interfaces in 2D

This modelization is testing the operation of the X-FEM multi-cracking for structures $2D$.

5.1 Characteristics of the mesh

One chooses to model structure $2D$ by a mesh made up of 2×10 elements `QUAD4`, as shown on [Figure 5.4-1].

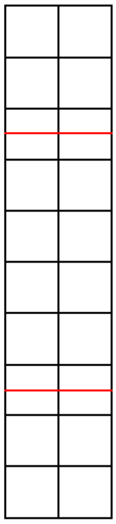


Figure 5.1-1: Mesh for the modelization D

5.2 Boundary conditions

the boundary conditions are similar to those imposed for the modelization A, obviously adapted for the case $2D$. One embeds the nodes belonging line of centers and one imposes displacements on the nodes belonging to the extreme lines (lower and higher). One thus has:

```
GROUP_NO=LIGMED: DX = 0 and DY = 0
GROUP_NO=LIGINF: DX = 0 and DY = - DEPY
GROUP_NO=LIGSUP: DX = 0 and  DY = DEPY
```

One considers $DEPY=2.E-3$.

5.3 Analytical resolution

the solution of such a problem is of course still obvious: all displacements according to x are null, all displacements following y below the level-set lower are equal to the displacement imposed u_y on the base of structure, all displacements following y between the two level set are null and all displacements following y to the top of the level-set higher are equal to the displacement imposed u_y on the top of structure.

5.4 Quantities tested and results

One tests the values of displacement after convergence of the iterations of operator STAT_NON_LINE. One does not test with each time only one node of each stage and one obtains the following table or the position of each node tested is indicated on [Figure 5.4-1].

Identification	Référence	DX
for all the nodes right below the interface inférieure	0.00	DX
for all the nodes right with the top of the interface inférieure	0.00	DY
for all the nodes right below the interface lower	-2.00E-3	DY
for all the nodes right than the top of the interface inférieure	0.00	DX
for all the nodes right below the interface supérieure	0.00	DX
for all the nodes right with the top of the interface supérieure	0.00	DY
for all the nodes right below the interface supérieure	0.00	DY
for all the nodes right with the top of the interface supérieure	2.00E-3	

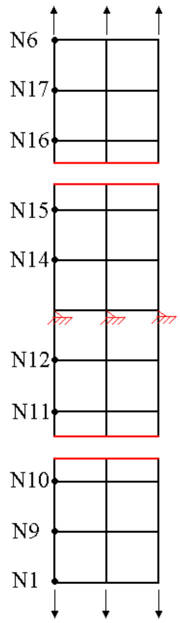


Figure 5.4-1: Final mesh and positioning of the nodes tested

6 Summaries of the results

the purposes of this test are reached. It was a question of validating the operation of the evolution of the X-FEM towards multi-cracking. One could note the good taking into account of two interfaces in several situations: models $3D$ and $2D$, horizontal interfaces, inclined interfaces, contact without or with friction, etc

All the operators who underwent modifications for the transition with multi-cracking function correctly.