SSNV191 – The purpose of validation of the conditions of Neumann with X-FEM in 2D and 3D

Summarized

This test is validating the taking into account of boundary conditions of the Neumann type on elements of skin X-FEM in 2D and 3D.

This test 3D brings into play a structure or an equivalent in 2D comprising a horizontal plane interface cutting structure in two parts (higher part a “above the interface, and “a lower” part below the interface). The interface is represented by of level sets.

The loadings applied are several types: a loading with constant pressure on the side sides of structure, a loading with a positive pressure on the higher side sides and a negative pressure on the lower side sides. One deals also with the same problem by replacing the pressures by surface forces in 3D and linear forces in 2D.

1 Problem of reference

1.1 Geometry

the structure 3D of dimensions $LX=1\text{m}$, $LY=2\text{m}$ and $LZ=3\text{m}$ comprises a horizontal plane interface being at middle height (see [Figure 1.1-1]).

The interface is not with a grid, and the geometry is in fact a healthy structure without interface. The interface is then introduced by functions of levels (level sets) directly into the command file using operator DEFI_FISS_XFEM [U4.82.08].

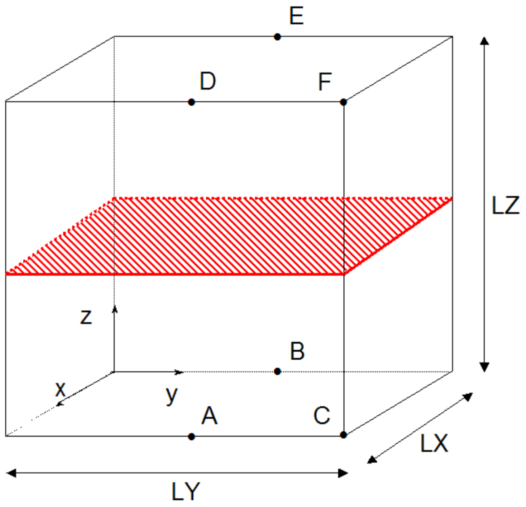

Figure 1.1-1 : Geometry of structure 3D

One defines the points A B C D, E and F which will be used to block the rigid modes.

	x	y	z
A	LX	$LY/2$	0
B	0	$LY/2$	0
C	LX	LY	0
D	LX	$LY/2$	LZ
E	0	$LY/2$	LZ
F	LX	LY	LZ

Tableau 1.1-1 : Coordinated cut-off points in 3D

One defines also equivalent structure in 2D, of dimensions $LX=2\text{m}$, $LY=3\text{m}$ comprising a horizontal plane interface being located at middle height (see [Figure 1.1-2]).

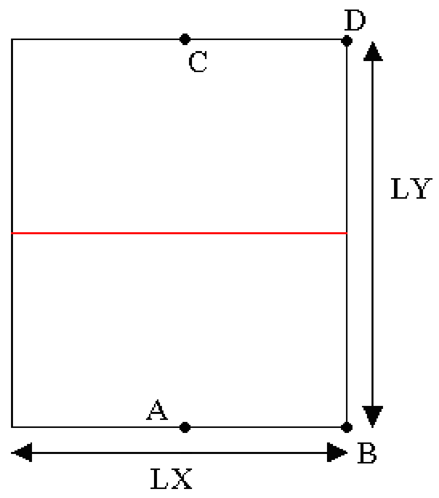

Figure 1.1-2 : Geometry of structure 2D

One defines the points A B , C and D which will be used to block the rigid modes.

	x	y
A	$LX/2$	0
B	LX	0
C	$LX/2$	LY
D	LX	LY

Table 1.1-2 : Coordinates of the cut-off points in 2D

1.2 Properties of the material

Modulus Young: $E=10000\,MPa$
Poisson's ratio: $\nu=0$

1.3 Boundary conditions and loadings

In 3D, the rigid modes are blocked in the following way:

- The points A and D are blocked according to the 3 directions: $\begin{cases} DX^A=DX^D=0 \\ DY^A=DY^D=0 \\ DZ^A=DZ^D=0 \end{cases}$
- The points B and E are blocked along the axis Oz : $DZ^B=DZ^E=0$
- The points C and F are blocked along the axes Ox and Oz : $\begin{cases} DX^C=DX^F=0 \\ DZ^C=DZ^F=0 \end{cases}$

In 2D, the rigid modes are blocked in the following way:

- The points A and C are blocked according to the 2 directions: $\begin{cases} DX^A=DX^C=0 \\ DY^A=DY^C=0 \end{cases}$
- The points B and D are blocked along the axis Oy : $DY^B=DY^D=0$

In 3D, two loadings are considered:

- Constant distributed pressure ($p=10000\,Pa$) on the side sides (planes $y=0$ and $y=LY$), corresponding to a case of compression according to the axis Oy .
- Positive pressure ($p=10000\,Pa$) on the higher side sides and a negative pressure ($p=-10000\,Pa$) on the lower side sides, corresponding to a case of compression on higher solid and a case of tension on lower solid

In 2D, one considers two loadings:

- Constant distributed pressure ($p=10000\,Pa$) on the side segments ($x=0$ and $x=LX$), corresponding to a case of compression according to the axis Ox .
- Positive pressure ($p=10000\,Pa$) on the higher side edges and a negative pressure ($p=-10000\,Pa$) on the lower side edges, corresponding to a case of compression on higher solid and a case of tension on lower solid

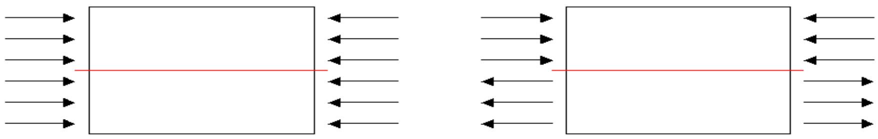

Figure 1.3-1 : Loadings in pressure

1.4 Solutions of the problem

One presents the solution of the problem 3D. That of the problem 2D is obtained by replacing y par. x

1.4.1 Loading in pure compression

the structure is in pure compression, the solution in displacement is commonplace. For the nodes of the side face of left, solution displacement is:

$$u_y=-\frac{LY}{2}\varepsilon_{yy}=-\frac{LY}{2}\frac{\sigma_{yy}}{E}=\frac{LY}{2}\frac{p}{E}=\frac{2.10^4}{2.10^{10}}=10^{-6}m$$

For the nodes of the side face of right, solution displacement is:

$$u_y=\frac{LY}{2}\varepsilon_{yy}=\frac{LY}{2}\frac{\sigma_{yy}}{E}=\frac{-LY}{2}\frac{p}{E}=-\frac{2.10^4}{2.10^{10}}=-10^{-6}m$$

When a node of edge is enriched by the Heaviside function, its displacement is written like a combination of a continuous term and a discontinuous term. For this case of loading, there is no discontinuity through the interface, therefore the degrees of freedom all nouveau riches are null.

1.4.2 Loading in compression/tension

the higher structure is in pure compression and the displacements of the nodes of the higher side face are the same ones as those of the case of preceding loading.

The lower structure is in pure tension. Only the signs of the values of displacement change. For the nodes of the lower side face of left, solution displacement is:

$$u_y = -\frac{LY}{2}\varepsilon_{yy} = \frac{LY}{2}\frac{p}{E} = -\frac{2.10^4}{2.10^{10}} = -10^{-6} m$$

For the nodes of the lower side face of right, solution displacement is:

$$u_y = \frac{LY}{2}\varepsilon_{yy} = \frac{LY}{2}\frac{p}{E} = \frac{2.10^4}{2.10^{10}} = 10^{-6} m$$

In this case, displacement is discontinuous through the interface. The values of the discontinuous degrees of freedom can be easily given (see similar case treaty in [V6.04.173]).

2 Modelization A

In this modelization, for the loading in pressure, the loading is applied using a constant distributed pressure or constant surface forces for the loading n°1, and of a distributed pressure or distributed forces functions of z for the loading n°2.
In this modelization, for the loading in surface force, the loading is applied using constant surface forces for the loading n°1, and of surface forces function of z for the loading n°2.

2.1 Characteristics of the mesh

the structure is with a grid with hexahedrons with 8 nodes. The number of elements is smallest possible, that is to say an element following the axis Ox , 2 elements along the axis Oy (in order to be able to define the nodes in medium plane in $y=LY/2$), and 5 elements along the axis Oz . Along the axis Oz , the number of elements is odd so that the interface does not coincide with the sides of the elements; the 3 layers of central elements use elements X-FEM, and the 2 layers of elements in top and in bottom use classical elements.

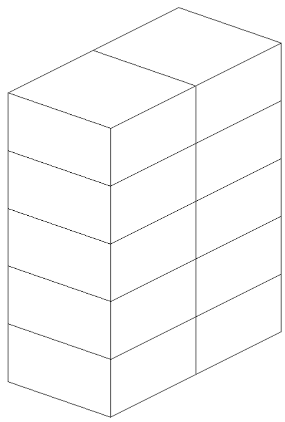

Figure 2.1-1 : Mesh 3D

2.2 Functionalities tested

key word `PRES_REP` of operator `AFFE_CHAR_MECA` [U4.44.01] makes it possible to apply a constant distributed pressure to meshes of skin. When the pressure is a function or a formula, one uses key word `PRES_REP` of `AFFE_CHAR_MECA_F` [U4.44.01]). This functionality is tested with the loading n°2. Indeed, with X-FEM, one cannot define a higher and lower side surface as a mesh group. In this case, only one mesh group comprising all meshes the surface side ones is defined, and one applies a pressure function of z to this mesh group.

Key word `FORCE_FACE` of operator `AFFE_CHAR_MECA` [U4.44.01] makes it possible to apply a constant surface force to meshes of skin. When the surface force is a function or a formula, one uses key word `FORCE_FACE` of `AFFE_CHAR_MECA_F` [U4.44.01]).

2.3 Quantities tested and results

operator `POST_MAIL_XFEM` allows to net cracks represented by the method X-FEM. Operator `POST_CHAM_XFEM`, then allows to export the X-FEM results on this new mesh. These two operators are to be used only in a posterior way with computation at sights of postprocessing. They make it possible to generate nodes right in lower part and with the top of the interface and to display their displacements.

For each side face of structure ($y=0$ and $y=LY$), one tests displacements of the nodes located just at the top and right below the level set.

2.3.1 Loading in compression (loading in pressure)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	10^{-6}
DY for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DY for all the nodes of the surface of right located just below the interface	-10^{-6}
DY for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

2.3.2 Loading in compression/tension (loading in pressure)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	-10^{-6}
DY for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DY for all the nodes of surface of right located just below the interface	10^{-6}
DY for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

2.3.3 Loading in compression (loading in surface force)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	10^{-6}
DY for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DY for all the nodes of the surface of right located just below the interface	-10^{-6}
DY for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

2.3.4 Loading in compression/tension (loading in surface force)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	-10^{-6}
DY for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DY for all the nodes of the surface of right located just below the interface	10^{-6}
DY for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

to test all the nodes in only once, one tests the minimum and the maximum of column.

3 Modelization B

In this modelization, for the loading in pressure, the loading is applied using a constant distributed pressure for the loading n°1, and of a distributed pressure function of y for the loading n°2.
In this modelization, for the loading in linear force, the loading is applied using constant linear forces for the loading n°1, and of linear forces functions of y for the loading n°2.

3.1 Characteristics of the mesh

In 2D, the structure is with a grid with quadrangles with 4 nodes. The number of elements is smallest possible, are 2 elements along the axis Ox (in order to be able to define the nodes in medium plane in $x=LX/2$), 5 elements along the axis Oy . Along the axis Oy , the number of elements is odd so that the interface does not coincide with the sides of the elements; the 3 layers of central elements use elements X-FEM, and the 2 layers of elements in top and in bottom use classical elements.

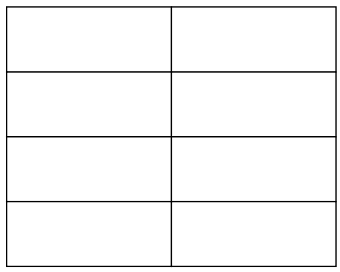

Figure 3.1-1 : Mesh 2D

3.2 Functionalities tested

key word PRES_REP of operator AFFE_CHAR_MECA [U4.44.01] makes it possible to apply a constant distributed pressure to meshes of skin. When the pressure is a function or a formula, one uses key word PRES_REP of AFFE_CHAR_MECA_F [U4.44.01]). This functionality is tested with the loading n°2. Indeed, with X-FEM, one cannot define a higher and lower side edge in so much of mesh group. In this case, only one mesh group comprising all meshes the 1D side ones is defined, and one applies a pressure function of y to this mesh group.

Key word FORCE_CONTOUR of operator AFFE_CHAR_MECA [U4.44.01] makes it possible to apply a constant linear force to meshes of skin. When the linear force is a function or a formula, one uses key word FORCE_CONTOUR of AFFE_CHAR_MECA_F [U4.44.01]).

3.3 Quantities tested and results

For each side face of structure ($x=0$ and $x=LX$), one tests displacements of the nodes located just at the top and right below the level set.

3.3.1 Loading in compression (loading in pressure)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	10^{-6}

DX for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DX for all the nodes of the surface of right located just below the interface	-10^{-6}
DX for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

3.3.2 Loading in compression/tension (loading in pressure)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	-10^{-6}
DX for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DX for all the nodes of surface of right located just below the interface	10^{-6}
DX for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

3.3.3 Loading in compression (loading in surface force)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	10^{-6}
DX for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DX for all the nodes of the surface of right located just below the interface	-10^{-6}
DX for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

3.3.4 Loading in compression/tension (loading in surface force)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	-10^{-6}
DX for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DX for all the nodes of the surface of right located just below the interface	10^{-6}
DX for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

to test all the nodes in only once, one tests the minimum and the maximum of column.

4 Modelization C

the only difference compared to the modelization B is the fact that one chooses a modelization C_PLAN and either D_PLAN.

4.1 Quantities tested and results

For each side face of structure ($x=0$ and $x=LX$), one tests displacements of the nodes located just at the top and right below the level set.

4.1.1 Loading in compression (loading in pressure)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	10^{-6}
DX for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DX for all the nodes of the surface of right located just below the interface	-10^{-6}
DX for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

4.1.2 Loading in compression/tension (loading in pressure)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	-10^{-6}
DX for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DX for all the nodes of surface of right located just below the interface	10^{-6}
DX for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

4.1.3 Loading in compression (loading in surface force)

Identification	Reference
DX for all the nodes of the surface of left located just below the interface	10^{-6}
DX for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DX for all the nodes of the surface of right located just below the interface	-10^{-6}
DX for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

4.1.4 Loading in compression/tension (loading in surface force)

Identification	Reference

DX for all the nodes of the surface of left located just below the interface	-10^{-6}
DX for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DX for all the nodes of the surface of right located just below the interface	10^{-6}
DX for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

to test all the nodes in only once, one tests the minimum and the maximum of column.

5 Modelization D

In this modelization, one replaced the linear elements of the modelization A by quadratic elements. All the rest is unchanged.

5.1 Characteristics of the mesh

They is the same characteristics of modelization that for the modelization A but one transformed the mesh by linear elements into mesh by quadratic elements by means of command `CREA_MAILLAGE/LINE_QUAD`. The structure is thus with a grid with hexahedrons with 20 nodes.

5.2 Features tested

They is the same features tested as those of modelization A.

5.3 Grandeurs tested and results

operator `POST_MAIL_XFEM` allows to net cracks represented by the method X-FEM. Operator `POST_CHAM_XFEM`, then allows to export the X-FEM results on this new mesh. These two operators are to be used only in a posterior way with computation at sights of postprocessing. They make it possible to generate nodes right in lower part and with the top of the interface and to display their displacements.

For each side face of structure ($y=0$ and $y=LY$), one tests displacements of the nodes located just at the top and right below the level set.

5.3.1 Loading in compression (loading in pressure)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	10^{-6}
DY for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DY for all the nodes of the surface of right located just below the interface	-10^{-6}
DY for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

5.3.2 Loading in compression/tension (loading in pressure)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	-10^{-6}
DY for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DY for all the nodes of surface of right located just below the interface	10^{-6}
DY for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

5.3.3 Loading in compression (loading in surface force)

Identification	Reference

DY for all the nodes of the surface of left located just below the interface	10^{-6}
DY for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DY for all the nodes of the surface of right located just below the interface	-10^{-6}
DY for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

5.3.4 Loading in compression/tension (loading in surface force)

Identification	Reference
DY for all the nodes of the surface of left located just below the interface	-10^{-6}
DY for all the nodes of the surface of left located just at the top of the interface	10^{-6}
DY for all the nodes of the surface of right located just below the interface	10^{-6}
DY for all the nodes of the surface of right located just at the top of the interface	-10^{-6}

to test all the only one nodes times, one tests the MIN and the MAX of the column.

6 Summaries of the results

the purposes of this test are reached:

- To validate on a simple case the taking into account of the conditions of Neumann on edge elements X-FEM nouveau riches with the Heaviside function.
- To validate various loadings: constant pressure, pressure function, constant surface force and surface force function in 3D, constant linear force and linear force function in 2D.