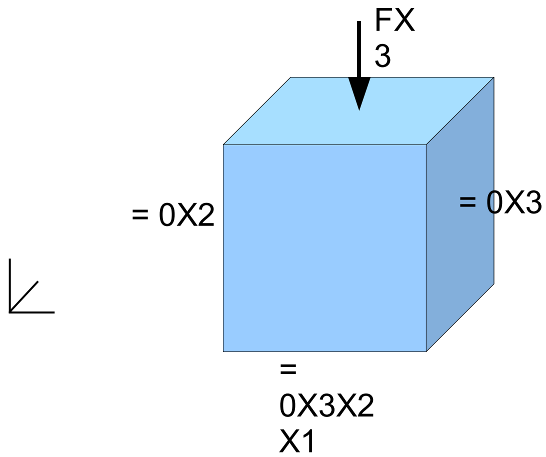
SSNV187 - Validation of model ELAS_HYPER on a Summarized

cube:

This test makes it possible to validate the behavior very-elastic of the type Signorini (material ELAS_HYPER). One leans on an elementary test in plane strains and in $3D$, compared to an analytical reference.

1 Problem of reference

1.1 Geometry



One considers a cube of with dimensions 1m which rests on a level ($x_2=0$ on the lower face), subjected to a pressure F on the upper face and in situation of plane strain according to x_3 ($x_3=0$ on the sides right-hand side and left). The cube can thus only be stretched along the axis x_1).

1.2 Properties of the materials

One tests on three different materials, corresponding to three standard models in very-elasticity.

Behavior `ELAS_HYPER`	Mooney-Rivlin	Néo-Hookéen	Signorini
`C10`	0.709	1.2345	0.1234
`C01`	2.3456	0	1.2345
`C20`	0	0	0.456
`NU`	0.499	0.499	0.499

1.3 Boundary conditions and loadings

- lower Face : $DY=0$
- Upper face : $F=0.876\,\text{Pa}$
- Left face and right : $DZ=0$ in 3D, nothing in `D_PLAN`

the loading is increasing of $F=0$ with $F=0.876\text{Pa}$, in 20 increments.

2 Reference solution

2.1 Method of calculating

One rests on result [bib1]. The strain state plane allows to very easily write the uniform field of displacement in the cube:

$$\begin{cases} u_1 = a_1 . x_1 \\ u_2 = w . x_2 \\ u_3 = 0 \end{cases} \quad (1)$$

with w the vertical displacement (negative) of the upper face and a_1 an arbitrary constant.
The condition of incompressibility makes it possible to write:

$$a_1 = \frac{-w}{1+w} \quad (2$$

And one finds the relation between the applied force F and the displacement w of the upper face:

$$F = 2\mathrm{S}. \frac{w.(2+w).(1+(1+w)^2)}{(1+w)^3} . \left(\frac{\partial \Psi}{\partial J_1} + \frac{\partial \Psi}{\partial J_2} \right) \quad (3)$$

S is surface, Ψ is the potential of strain and J_1, J_2 are the invariants of the tensor of Green-Lagrange. The potential of strain used by ELAS_HYPER is the following:

$$\Psi = C_{10} . (J_1 - 3) + C_{01} . (J_2 - 3) + C_{20} . (J_1 - 3)^2 + \Psi_{vol} \quad (4)$$

Ψ_{vol} is the potential corresponding to the incompressibility. It depends on the invariants J_1 and J_2 on C_{10}, C_{01} and C_{20} which are the characteristic materials. As moreover $S=1$ one obtains:

$$F = 2. \frac{w.(2+w).(1+(1+w)^2)}{(1+w)^3} . \left[\left(C_{10} + \frac{C_{01}}{1+w} \right) + 2.C_{20} . \frac{w^3.(3+w)}{1+w} \right] \quad (5)$$

the solution of this nonlinear equation in w is done simply by dichotomy for $w<0$.

3 Bibliographical references

1 G.A. HOLZAPFEL: Nonlinear solid mechanics, 2001, Wiley.

4 Modelization A

4.1 Characteristic of the modelization

It is a modelization in $2D$ with plane strains `D_PLAN`, by means of meshes linear.

4.2 Characteristics of the mesh

Many linear elements: 207 including 132 triangles and 47 quadrangles (the rest being meshes of edge).
Many nodes: 132

4.3 Quantities tested and results

the First computation (MOONEY-RIVLIN)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-3{,}40091\text{E-}2$	Analytical	0,20%

the Second computation (NEO-HOOKEAN)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-7{,}8175\text{E-}2$	Analytical	0,20%

the Third computation (SIGNORINI)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-6{,}62\text{E-}2$	Analytical	7,5%

5 Modelization B

5.1 Characteristic of the modelization

It is a modelization in $2D$ with plane strains D_PLAN, by means of meshes quadratic.

5.2 Characteristics of the mesh.

Many quadratic elements: 207 including 132 triangles and 47 quadrangles (the rest being meshes of edge).
Many nodes: 132

5.3 Quantities tested and results

the First computation (MOONEY-RIVLIN)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-3{,}40091E\text{-}2$	Analytical	0,20%

the Second computation (NEO-HOOKEAN)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-7{,}8175E\text{-}2$	Analytical	0,20%

the Third computation (SIGNORINI)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-6{,}62E\text{-}2$	Analytical	7,5%

6 Modelization C

6.1 Characteristic of the modelization

It is a modelization 3D.

6.2 Characteristics of the mesh

Many elements: 8734 tetrahedrons and 1728 nodes.

6.3 Quantities tested and results

the First computation (MOONEY-RIVLIN)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-3,40091E-2$	Analytical	0,20%

the Second computation (NEO-HOOKEAN)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-7,8175E-2$	Analytical	0,20%

the Third computation (SIGNORINI)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-6,62E-2$	Analytical	7,5%

7 Modelization D

7.1 Characteristic of the modelization

It is a modelization 3D_SI (under-integrated elements TETRA10).

7.2 Characteristics of the mesh

Many elements: 271 tetrahedrons and 514 nodes.

7.3 Quantities tested and results

the First computation (MOONEY-RIVLIN)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-3{,}40091\text{E-}2$	Analytical	0,20%

the Second computation (NEO-HOOKEAN)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-7{,}8175\text{E-}2$	Analytical	0,20%

the Third computation (SIGNORINI)

Value tested	Urgent	Standard	Reference	*Tolerance*
Displacement w	1,0	$-6{,}62\text{E-}2$	Analytical	7,5%

8 Summary of the results

the got results are in concord with the reference solution, except in the case of Signorini. This difference of about 7,5 % compared to the analytical solution can be explained by the processing of the incompressibility in penalization in model ELAS_HYPER but also by the use of linear elements which do not make it possible to take account of the condition of incompressibility well.