SSNV185 – the purpose of Crack emerging in a plate 3D of width finished with X-FEM

Summarized

This test is validating the method X-FEM [bib1] on an academic case 3D, in the frame of the linear elastic fracture mechanics.

This test brings into play a plate 3D comprising an emerging crack plane at right bottom. The computation complete as well as the extraction of the stress intensity factors is realized in the frame of the method X-FEM. The mesh is healthy, the crack being represented virtually with of level sets.

Several configurations of mesh are tested and compared with the analytical solution. With the same dealt problem in a classical way (with a cracked mesh) is used as reference in order to compare the precise details of the two methods.

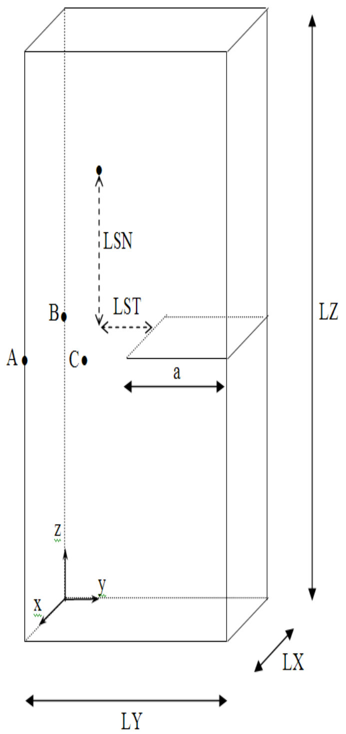
The commands of postprocessing for the visualization of the field of solution displacement on a cracked mesh are tested for all the modelizations calling on X-FEM.

1 Problem of reference

1.1 Geometry

the structure is a plate 3D of dimensions $LX=1\,m$, $LY=10\,m$ and $LZ=30\,m$, comprising an emerging plane crack length $a=5\,m$, being at middle height (see [Figure 1.1 - has]).

If with the problem is dealt by a classical method, the crack is with a grid. On the other hand, if the method X-FEM is employed, the crack is not with a grid, and the geometry is in fact an operational plate without crack. The crack will then be introduced by functions of levels (level sets) directly into the file orders using operator DEFI_FISS_XFEM [U4.82.08]. The level set norm ($LSN=$ distance to the plane of cracking) makes it possible to define the plane of crack and the level set tangent ($LST=$ distance to the crack tip) makes it possible to define the position of the crack tip.



Appear 1.1-a : Geometry of the fissured plate

One defines the points $A(1,0,15)$, $B(0,0,15)$ and $C(1,3,15)$ which will be used to block the rigid modes.

1.2 Properties of the material

Modulus Young: $E=205\,000\,MPa$ (except contrary mention)
Poisson's ratio: $\nu=0.$

1.3 Bibliographical references

1 MASSIN P., GENIAUT S.: Method X-FEM, Handbook of reference of *the Code_Aster*, [R7.02.12]

2 SCREWS E.: Computation of the coefficients of intensity of stresses, Handbook of reference of *the Code_Aster*, [R7.02.05]

3 BARTHELEMY B.: Practical notions of the fracture mechanics, Eyrolles, 1980.

4 LABORDE P., APPLE TREE J., FOX Y., SALÜN MR.: “High-order extended finite element method for cracked domains”, International Newspaper for Numerical Methods in Engineering, 64 (3), 354-381, 2005.

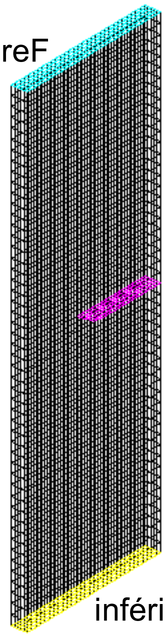
5 G. Erdogan, G.C. Sih, “One the ace extension in punts under planes loading and transverse shear”, Newspaper of BASIC Engineering, 85,519-27, 1963.

2 Modelization a: fissures with a grid in tension

In this modelization, the crack is with a grid, and one uses the standard method of the finite elements to carry out computation. This modelization will be used as reference and will allow the comparison with the method X-FEM .

2.1 Characteristics of the mesh

the structure is modelled by a regular mesh composed of $5\times30\times50$ HEXA8, respectively along the axes x, y, z (see [Figure 2.1 - has]). Two superimposed surfaces are the lips of the crack



Appears 2.1-a : Cracked mesh

2.2 Boundary conditions and loadings

Two types of loading will be studied: a loading of tension on the sides lower and higher of structure, then a loading which consists in imposing a field of displacement in any node, identical to the field of asymptotic displacement in mode I (solution of Westergaard for an infinite medium [bib2]).

2.2.1 Loading of tension

a distributed pressure is imposed on the sides lower and higher of structure (see [Figure 2.1 - has]). The pressure is $p=10^{-6} Pa (\sigma_{zz}=-p)$, which makes it possible to request crack in pure mode of I opening.
The rigid modes are blocked in the following way:

- The point A is blocked according to the 3 directions: $DX^{N4265}=0$, $DY^{N4265}=0$, $DZ^{N4265}=0$
- The point B is blocked along the axis Oz : $DZ^{N3751}=0$
- The point C is blocked along the axes Ox and Oz : $DX^{N4256}=0$, $DZ^{N4256}=0$

2.2.2 Loading with the asymptotic field in mode I

the asymptotic field in pure I mode, solution of a problem of elastic fracture linear is known in an analytical way [bib2]. In the defined reference, this field takes the following shape:

$$u_x=0 \quad \text{éq 2.2.2-1}$$

$$u_y=-\frac{1+\nu}{E}\sqrt{\frac{r}{2\pi}}\cos\frac{\theta}{2}(3-4\nu-\cos\theta) \quad \text{éq 2.2.2-2}$$

$$u_z=\frac{1+\nu}{E}\sqrt{\frac{r}{2\pi}}\sin\frac{\theta}{2}(3-4\nu-\cos\theta) \quad \text{éq 2.2.2-3}$$

This field is imposed on all the nodes of structure by the means of formulas in operator `AFFE_CHAR_MECA_F` [U4.44.01]. These formulas utilize the polar coordinates (r,θ) in the local base with the crack tip:

$$r=\sqrt{(5-y)^2+(z-15)^2}, \qquad \theta=\arctan\left(\frac{z-15}{5-y}\right) \quad \text{éq 2.2.2-4}$$

However, it is appropriate to treat except for the nodes belonging to the lips of crack. Indeed, for the nodes of the lower lip, the formula being used to calculate the angle θ is not valid (it would give π whereas theoretically, θ is worth $-\pi$). For the nodes of the lower lip, the value of the angle is thus not calculated by the equation [éq 2.2.2-4] but is directly put at $-\pi$. For the nodes of the upper lip, the formula is nevertheless valid.

2.3 Solutions of the problem

2.3.1 Loading of tension

the stress intensity factor in mode I is given [bib3] by:

$$K_I=\sigma_{zz}\sqrt{\pi a}\, f\left(\frac{a}{LY}\right) \quad \text{éq 2.3.1-1}$$

where

$$f\left(\frac{a}{b}\right)=\left(\frac{2\mathrm{b}}{\pi a}\tan\frac{\pi a}{2\mathrm{b}}\right)^{1/2}\frac{0.752+0.37\left(1-\sin\frac{\pi a}{2\mathrm{b}}\right)^3+2.02\frac{a}{b}}{\cos\frac{\pi a}{2\mathrm{b}}} \quad \text{éq 2.3.1-2}$$

the accuracy of this formula reaches 0.5% whatever the ratio $\frac{a}{b}$.

2.3.2 Loading with the asymptotic field in mode I

In the presence of such a loading, the theoretical value is

$$K_I=1 \quad \text{éq 2.3 .2 - 1}$$

2.4 Quantities tested and results

option `CALC_K_G` of operator `CALC_G` [U4.82.04] allows the computation of the stress intensity factors by the energy method "G-theta". This functionality is tested with the loading n°1. This case of loading is used as a basis of comparison for the method X-FEM . When one of the loads is a function or a formula (coming from `AFFE_CHAR_MECA_F` [U4.44.01]), the option becomes `CALC_K_G_F` . This functionality is tested with the loading n°2.

The loading n°1 characterizes a loading of tension, which will be either constant on the sides higher and lower of structure, or constant or variable on the level of the lips of crack.
One tests the values of K_I along the crack tip, for various contours of fields theta.
The values of radius $\inf$ and $\sup$ the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	2.5 Contour
Rinf	6	2	0.666	1	1	
Rsup	1.2.1	4	1.666	2	3	

4.3.9 Table - has

to test all the nodes of the crack tip in only once, one tests the values min and max of K_I on all the nodes of the crack tip.

2.4.1 Loading of tension

2.4.1.1 constant Pressure on the sides higher and lower

Identification	Standard of reference	Value of reference	% Tolerance
Crowns 1: `MAX (KI)`	`"ANALYTIQUE"`	1.1202664 107	6.5%
Contour 1: `MIN (KI)`	`"ANALYTIQUE"`	1.1202664 107	6.5%
Contour 2: `MAX (KI)`	`"ANALYTIQUE"`	1.1202664 107	6.5%
Contour 2: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	6.5%
Contour 3: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	6.5%
Contour 3: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	6.5%
Contour 4: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	6.5%
Contour 4: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	6.5%
Contour 5: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	6.5%
Contour 5: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	6.5%
Contour 6: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	6.5%
Contour 6: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	6.5%

2.4.1.2 constant Pressure on the lips

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: `MAX (KI)`	`"ANALYTIQUE"`	1.1202664 107	6.5%
Contour 1: `MIN (KI)`	`"ANALYTIQUE"`	1.1202664 107	6.5%
Contour 2: `MAX (KI)`	`"ANALYTIQUE"`	1.1202664 107	6.5%
Contour 2: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	6.5%
Contour 3: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	6.5%

Contour 3: MIN (KI)	"ANALYTIQUE"	1.1202664 10 ^{7}	6.5%
Contour 4: MAX (KI)	"ANALYTIQUE"	1.1202664 10 ^{7}	6.5%
Contour 4: MIN (KI)	"ANALYTIQUE"	1.1202664 10 ^{7}	6.5%
Contour 5: MAX (KI)	"ANALYTIQUE"	1.1202664 10 ^{7}	6.5%
Contour 5: MIN (KI)	"ANALYTIQUE"	1.1202664 10 ^{7}	6.5%
Contour 6: MAX (KI)	"ANALYTIQUE"	1.1202664 10 ^{7}	6.5%
Contour 6: MIN (KI)	"ANALYTIQUE"	1.1202664 10 ^{7}	6.5%

2.4.1.3 variable Pressure on the lips

Standard	Identification of reference	Value of reference	% Tolerance
Crowns 1, not initial: KI	"NON_REGRESSION"	5.99 106	0.2%
Contour 1, full stop: KI	"NON_REGRESSION"	4.52 106	0.2%
Contour 2, not initial: KI	"NON_REGRESSION"	5.99 106	0.2%
Contour 2, full stop: KI	"NON_REGRESSION"	4.52 106	0.2%
Contour 3, not initial: KI	"NON_REGRESSION"	5.99 106	0.2%
Contour 3, full stop: KI	"NON_REGRESSION"	4.52 106	0.2%
Contour 4, not initial: KI	"NON_REGRESSION"	5.99 106	0.2%
Contour 4, full stop: KI	"NON_REGRESSION"	4.52 106	0.2%
Contour 5, not initial: KI	"NON_REGRESSION"	5.99 106	0.2%
Contour 5, full stop: KI	"NON_REGRESSION"	4.52 106	0.2%
Contour 6, not initial: KI	"NON_REGRESSION"	5.99 106	0.2%
Contour 6, full stop: KI	"NON_REGRESSION"	4.52 106	0.2%

2.4.2 Loading with the asymptotic field in Standard mode

I	Identification of reference	Value of reference	% Tolerance
Crown 1: MAX (KI)	"ANALYTIQUE"	1.0	0.2%
Contour 1: MIN (KI)	"ANALYTIQUE"	1.0	0.2%
Contour 2: MAX (KI)	"ANALYTIQUE"	1.0	0.2%
Contour 2: MIN (KI)	"ANALYTIQUE"	1.0	0.2%
Contour 3: MAX (KI)	"ANALYTIQUE"	1.0	0.2%
Contour 3: MIN (KI)	"ANALYTIQUE"	1.0	0.2%
Contour 4: MAX (KI)	"ANALYTIQUE"	1.0	0.2%
Contour 4: MIN (KI)	"ANALYTIQUE"	1.0	0.2%
Contour 5: MAX (KI)	"ANALYTIQUE"	1.0	0.2%
Contour 5: MIN (KI)	"ANALYTIQUE"	1.0	0.2%
Contour 6: MAX (KI)	"ANALYTIQUE"	1.0	0.2%
Contour 6: MIN (KI)	"ANALYTIQUE"	1.0	0.2%

2.5 Comments

the 1st loading of this modelization is used as a basis of comparison for the method X-FEM . The 2nd case of loading makes it possible to validate option CALC_K_G_F for the elements 3D .

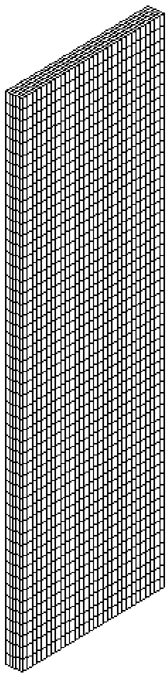
3 Modelization b: fissures X-FEM coïncidente in tension

In this modelization, the crack is not with a grid, but it is represented by of the level sets:

$$LSN = z - 15 \quad \textbf{éq 3-1}$$
$$LST = LY - a - y \quad \textbf{éq 3-2}$$

3.1 Characteristics of the mesh

the structure is modelled by a sane mesh, regular composed of $5\times30\times50$ HEXA8, respectively along the axes X, Y, Z in order to have the same number of elements as for the mesh of the modelization A (see [Figure 4.1 - has]). Thus, the plane of crack is in correspondence with sides d'HEXA8 and the crack tip with edges d'HEXA8. It is said that it is coïncidente with the mesh.



Appear 3.1-a : Sane mesh

3.2 Boundary conditions and loadings

Only one type of loading is studied here: it is about a distributed pressure imposed on the sides lower and higher of structure (identical to the 1st case of loading of the modelization A).
The rigid modes are blocked in the following way:

- The point A is blocked according to the 3 directions: $DX^{N3751}=0$, $DY^{N3751}=0$, $DZ^{N3751}=0$
- The point B is blocked along the axis Oz : $DZ^{N9276}=0$
- The point C is blocked along the axes Ox and Oz : $DX^{N3760}=0$, $DZ^{N3760}=0$

3.3 Quantities tested and results

One tests the values of K_I along the crack tip, for various contours of fields theta.
The values of radius inf and sup the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
R_{inf}	2	0.666	1	1	1.2.1	
R_{sup}	4	1.666	2	3	4.3.9	

Table 3.4 - B

to test all the nodes of the crack tip in only once, one tests the values min and max of K_I on all the nodes of the crack tip.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1: `MAX (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%
Contour 1: `MIN (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%
Contour 2: `MAX (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%
Contour 2: `MIN (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%
Contour 3: `MAX (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%
Contour 3: `MIN (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%
Contour 4: `MAX (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%
Contour 4: `MIN (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%
Contour 5: `MAX (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%
Contour 5: `MIN (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%
Contour 6: `MAX (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%
Contour 6: `MIN (KI)`	`"ANALYTIQUE"`	1.1202664 10^{7}	1.0%

One also tests the min and max values of the Standard G_IRWIN

parameter	Identification of reference	Value of reference	Tolerance
Crowns 1: `MAX (G_IRWIN)`	`"ANALYTIQUE"`	612.193573558	2.5%
Contour 1: `MIN (G_IRWIN)`	`"ANALYTIQUE"`	612.193573558	2.5%
Contour 2: `MAX (G_IRWIN)`	`"ANALYTIQUE"`	612.193573558	2.5%
Contour 2: `MIN (G_IRWIN)`	`"ANALYTIQUE"`	612.193573558	2.5%
Contour 3: `MAX (G_IRWIN)`	`"ANALYTIQUE"`	612.193573558	2.5%
Contour 3: `MIN (G_IRWIN)`	`"ANALYTIQUE"`	612.193573558	2.5%
Contour 4: `MAX (G_IRWIN)`	`"ANALYTIQUE"`	612.193573558	2.5%
Contour 4: `MIN (G_IRWIN)`	`"ANALYTIQUE"`	612.193573558	2.5%
Contour 5: `MAX (G_IRWIN)`	`"ANALYTIQUE"`	612.193573558	2.5%
Contour 5: `MIN (G_IRWIN)`	`"ANALYTIQUE"`	612.193573558	2.5%
Contour 6: `MAX (G_IRWIN)`	`"ANALYTIQUE"`	612.193573558	2.5%

Contour 6 : MIN (G_IRWIN)	"ANALYTIQUE"	612.193573558	2.5%

One by the operator tests also the value K_I of product POST_K1_K2_K3 at the first point of the crack tip:

Standard	identification of reference	Value of reference	Tolerance
Crowns 1, not initial : KI	"ANALYTIQUE"	1.1202664 10^{7}	5.0%

Moreover, in order to test the commands of postprocessing POST_MAIL_XFEM and POST_CHAM_XFEM one carries out a test to check the exactitude of results files containing the cracked mesh (resulting of the command POST_MAIL_XFEM) and the field of displacement solution (resulting of the command POST_CHAM_XFEM).

One makes a test of non regression on displacements, which relates to the sum of the absolute values of displacements according to X, Y and Z. The tests of non regression are made compared to the version 10.2.17.

Standard	identification of reference	Value of reference	Tolerance
DY Somme absolute values on	DX "NON_REGRESSION	" 5.91783	10-4 ^{10}
3% Somme of the absolute values on	"NON_REGRESSION"	1.694474	10 $^{3\%}$
Somme of the absolute values on DZ	"NON_REGRESSION"	1.253088	10 $^{3\%}$

One makes a test of non regression on the coordinates of the nodes of the mesh X-FEM, which relates to the sum of the absolute values of the Y-coordinates according to X, Y and Z. The tests of non regression are made compared to the version 10.2.17.

Standard	identification of reference	Value of reference	Tolerance
Somme absolute values on COOR_X	"NON_REGRESSION"	4.743 103	1.00E-008
Somme of the absolute values on COOR_Y	"NON_REGRESSION"	4.743 104	1.00E-008
Somme of the absolute values on COOR_Z	"NON_REGRESSION"	1.4229 105	1.00E-008

3.4 Comments

the results are stable for any selected contour.
With same number of elements, the accuracy of the results got with X-FEM is much better than that obtained in the classical case (less 1% for X-FEM against 6% for a classical method).

4 Modelization C: fissure X-FEM non-coïncidente in tension

Modelization identical to the modelization B , but the crack tip is in the middle of the elements. The crack is thus not coïncidente with the mesh.

4.1 Characteristics of the mesh

the structure is modelled by a sane mesh, regular composed of $5\times31\times51$ HEXA8, respectively along the axes x, y, z . Of this way, the crack tip is in the center of elements and the plane of crack does not correspond any more to sides of elements. Figure 6.1 - has represents out of cut Oyz enrichment in a zone near crack tip.

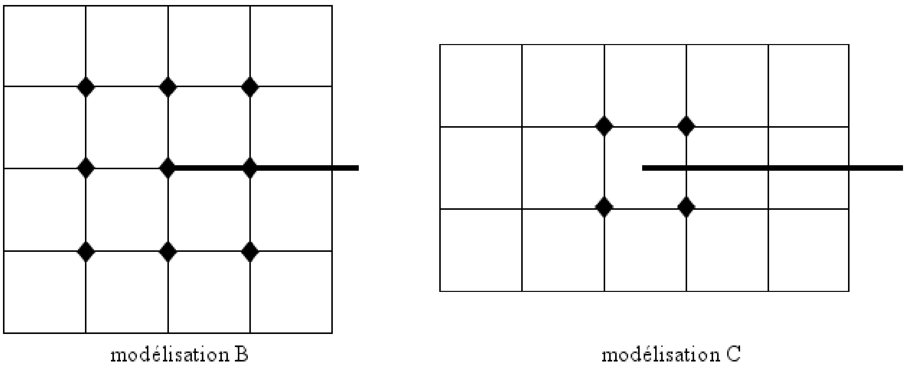


Figure 4.1 -4.1-a : Various enrichments in crack tip

4.2 Boundary conditions and loadings

the loading is identical to that of the modelization A :

- either there is a distributed pressure imposed on the sides lower and higher of structure,
- or one has a constant pressure imposed on the lips of crack,
- or one has a variable pressure according to X imposed on the lips of crack.

In order to reproduce the preceding cases, it is necessary to block the same points A , B and C . However, here, there are no nodes in the median plane. To block the rigid modes, it is then necessary to just force relations between the degrees of freedom of the nodes above and below median plane [Figure 6.2 - has]:

- the point A is blocked according to the 3 directions: $$\begin{aligned} DX^{N4031}+DX^{N3876} &= 0 \\ DY^{N4031}+DY^{N3876} &= 0 \\ DZ^{N4031}+DZ^{N3876} &= 0 \end{aligned}$$
- the point B is blocked along the axis Oz : $$DZ^{N3886}+DZ^{N4041} = 0$$
- the point C is blocked along the axes Ox and Oz : $$\begin{aligned} DX^{N9768}+DX^{N9767} &= 0 \\ DZ^{N9768}+DZ^{N9767} &= 0 \end{aligned}$$

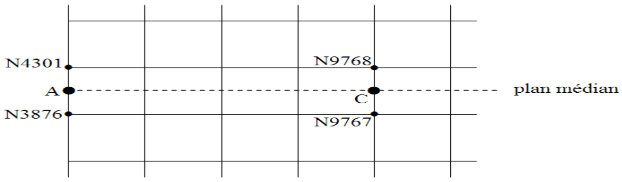


Figure 4.2-a : 4.2-a Conditions of Dirichlet around the median plane

It [Figure 6.2 - has] is a schematic sight of the plane Oyz, on which the number of finite elements is not respected. It is simply used to understand the linear relations forced in order to block displacements of the points A and C. For the point B, one acts in the same way.

4.3 Quantities tested and results

One tests the values of K_I along the crack tip, for various loadings, various contours of fields theta. The values of radius lower and superior of the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
$Rinf$	2	0.666	1	1	1.2.1	
Rsup	4	1.666	2	3	4.3.9	

Table 4.4 - C

One tests also 3 different lissages for K_I: a lissage of the type `"LAGRANGE"`, a lissage of the type `"LAGRANGE_REGU"`, and a lissage of the type `"LAGRANGE_NO_NO"` (for this lissage, only contour 1 is tested).

To test all the nodes of the crack tip in only once, one tests the min and max values of K_I on all the nodes of the crack tip.

4.3.1 Lissage `"LAGRANGE"`

4.3.1.1 Pressure constant on the sides higher and lower

Identification	Standard of reference	Value of reference	Tolerance
Crowns 1: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 1: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 2: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 2: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 3: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 3: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 4: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 4: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 5: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 5: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 6: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 6: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%

4.3.1.2 constant Pressure on the lips

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 1: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 2: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 2: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 3: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 3: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 4: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 4: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 5: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 5: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 6: `MAX (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%
Contour 6: `MIN (KI)`	`"ANALYTIQUE"`	$1.1202664\ 10^{7}$	4.0%

One tests also computation from G the command `CALC_G`, option `CALC_G` only for the first contour.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1: `MAX (G)`	`"ANALYTIQUE"`	612.19	8.00%
Contour 1: `MIN (G)`	`"ANALYTIQUE"`	612.19	8.00%

4.3.1.3 variable Pressure on the lips

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1, point 1 of the crack tip: `KI`	`"AUTRE_ASTER"`	5.99 106	4.0%
Contour 1, point 6 of the crack tip: `KI`	`"AUTRE_ASTER"`	4.52 106	5.0%
Contour 2, point 1 of the crack tip: `KI`	`"AUTRE_ASTER"`	5.99 106	4.0%
Contour 2, point 6 of the crack tip: `KI`	`"AUTRE_ASTER"`	4.52 106	5.0%
Contour 3, point 1 of the crack tip: `KI`	`"AUTRE_ASTER"`	5.99 106	4.0%
Contour 3, point 6 of the crack tip: `KI`	`"AUTRE_ASTER"`	4.52 106	5.0%
Contour 4, point 1 of the crack tip: `KI`	`"AUTRE_ASTER"`	5.99 106	4.0%
Contour 4, point 6 of the crack tip: `KI`	`"AUTRE_ASTER"`	4.52 106	5.0%
Contour 5, point 1 of the crack tip: `KI`	`"AUTRE_ASTER"`	5.99 106	4.0%
Contour 5, point 6 of the crack tip: `KI`	`"AUTRE_ASTER"`	4.52 106	5.0%
Contour 6, point 1 of the crack tip: `KI`	`"AUTRE_ASTER"`	5.99 106	4.0%
Contour 6, point 6 of the crack tip: `KI`	`"AUTRE_ASTER"`	4.52 106	5.0%

4.3.2 Lissage `"LAGRANGE_REGU"`

4.3.2.1 Pressure constant on the sides higher and lower

Identification	Standard of reference	Value of reference	% Tolerance
Crown 1: MAX (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%
Contour 1: MIN (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%
Contour 2: MAX (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%
Contour 2: MIN (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%
Contour 3: MAX (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%
Contour 3: MIN (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%
Contour 4: MAX (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%
Contour 4: MIN (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%
Contour 5: MAX (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%
Contour 5: MIN (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%
Contour 6: MAX (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%
Contour 6: MIN (KI)	"NON_REGRESSION"	1.1202664 10^{7}	4.0%

4.3.3 Lissage "LAGRANGE_NO_NO"

Identification	Standard of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	1.1202664 10^{7}	4.0%
Contour 1: MIN (KI)	"ANALYTIQUE"	1.1202664 10^{7}	4.0%

One by the operator tests also the value K_I of product POST_K1_K2_K3 at the first point of the crack tip from result produced with the loading of type constant pressure on the sides higher and lower of structure.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1, not initial: KI	"ANALYTIQUE"	1.1202664 10^{7}	6.0%

Moreover, in order to test the commands of postprocessing POST_MAIL_XFEM and POST_CHAM_XFEM one carries out a test to check the exactitude of results files containing the cracked mesh (resulting of the command POST_MAIL_XFEM) and the field of displacement solution (resulting of the command POST_CHAM_XFEM). For the mesh, one chooses a test of non regression on the sum of the absolute values of the coordinates of the nodes. For displacement, the test of non regression door on the sum of the absolute values of displacements according to X, Y and Z. The tests of non regression are made compared to the version 8.2.14 for the mesh and 8.2.8 for displacement.

Standard	identification of reference	Value of reference	Tolerance
Somme absolute values on COOR_X	"NON_REGRESSION"	4.8900 103	0.0%
Somme of the absolute values on COOR_Y	"NON_REGRESSION"	4.8406 104	0.0%
Somme of the absolute values on COOR_Z	"NON_REGRESSION"	1.4670 105	0.0%

Standard	Identification of reference	Value of reference	Tolerance
DY Somme absolute values on	DX "NON_REGRESSION	" 1.25083	10-3 [10]
4% Somme of the absolute values on	"NON_REGRESSION"	1.79347	10-4%
Somme of the absolute values on DZ	"NON_REGRESSION"	1.53478	10 [4%]

4.4 Comments

the results are stable for any selected contour.

The accuracy of the got results is less good than for the modelization B. That can be explained by the fact that the zone of enrichment is less wide here.

However, the results remain better than in the classical case.

5 Modelization D: fissure semi-coïncidente X-FEM in tension

This modelization is exactly the same one as the modelization B, except that the length of crack is: $a=4.8333$, so that the crack tip does not coincide with edges of the elements. On the other hand, the surface of crack coincides with the sides of the elements (semi-coïncidente crack).

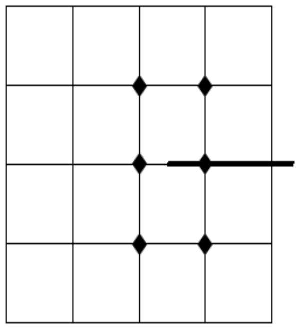


Figure 4.5 -5-a : Enrichment in a zone close to the crack tip

5.1 Quantities tested and results

One tests the values of K_I along the crack tip, for various contours of fields theta.
The values of radius the inf and sup of the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
Rinf	2	0.666	1	1	1.2.1	
Rsup	4	1.666	2	3	4.3.9	

Table 5.1 - D

to test all the nodes of the only one crack tip times, one KI tests the values minimum and maximum of on all the nodes of the crack tip.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1: `MAX (KI)`	"ANALYTIQUE"	1.044774 107	3.0%
Contour 1: `MIN (KI)`	"ANALYTIQUE"	1.044774 107	3.0%
Contour 2: `MAX (KI)`	"ANALYTIQUE"	1.044774 107	3.0%
Contour 2: `MIN (KI)`	"ANALYTIQUE"	1.044774 107	3.0%
Contour 3: `MAX (KI)`	"ANALYTIQUE"	1.044774 107	3.0%
Contour 3: `MIN (KI)`	"ANALYTIQUE"	1.044774 107	3.0%
Contour 4: `MAX (KI)`	"ANALYTIQUE"	1.044774 107	3.0%
Contour 4: `MIN (KI)`	"ANALYTIQUE"	1.044774 107	3.0%
Contour 5: `MAX (KI)`	"ANALYTIQUE"	1.044774 107	3.0%
Contour 5: `MIN (KI)`	"ANALYTIQUE"	1.044774 107	3.0%
Contour 6: `MAX (KI)`	"ANALYTIQUE"	1.044774 107	3.0%
Contour 6: `MIN (KI)`	"ANALYTIQUE"	1.044774 107	3.0%

One by the operator tests also the value K_I of product POST_K1_K2_K3 at the first point of the crack tip from result produced with the loading of type constant pressure on the sides higher and lower of structure.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1, not initial : KI	"ANALYTIQUE"	1.044774 10 7	10.0%

Moreover, in order to test the commands of postprocessing POST_MAIL_XFEM and POST_CHAM_XFEM one carries out a test to check the exactitude of results files containing the cracked mesh (resulting of the command POST_MAIL_XFEM) and the field of displacement solution (resulting of the command POST_CHAM_XFEM). One makes a test of non regression on displacements, which relates to the sum of the absolute values of displacements according to x, y and z. The tests of non regression are made compared to the version 10.2.17.

Standard	identification of reference	Value of reference	Tolerance
DY Somme absolute values on	DX "NON_REGRESSION	" 4.1238	10-3 10
4% Somme of the absolute values on	"NON_REGRESSION"	1.480480	10 4%
Somme of the absolute values on DZ	"NON_REGRESSION"	1.120438	10 4%

One makes a test of non regression on the coordinates of the nodes of the mesh XFEM , which relates to the sum of the absolute values of the Y-coordinates according to X, Y and Z. The tests of non regression are made compared to the version 10.2.17.

Standard	identification of reference	Value of reference	Tolerance
Somme absolute values on COOR_X	"NON_REGRESSION"	4.743 103	0.0%
Somme of the absolute values on COOR_Y	"NON_REGRESSION"	4.743 104	0.0%
Somme of the absolute values on COOR_Z	"NON_REGRESSION"	1.4229 105	0.0%

5.2 Comments

These results confirm that the size of the zone of enrichment influences the accuracy of the results. Here, the zone of enrichment is intermediate between of the same case B and case C, and accuracy.

6 Modelization E: fissure X-FEM in tension – conditions of Dirichlet in mode I

Modelization identical to the modelization C, but the loading of tension is the loading n°2 of the modelization A

In this modelization, the modulus Young is equal to $100\,MPa$.

6.1 Characteristics of the mesh

the structure is modelled by a sane mesh, regular, composed of $3\times11\times31$ HEXA8, respectively along the axes X, Y, Z (see [Figure 10.1 - has]). Such a discretization C leads to a configuration of enrichment similar to that of the modelization.

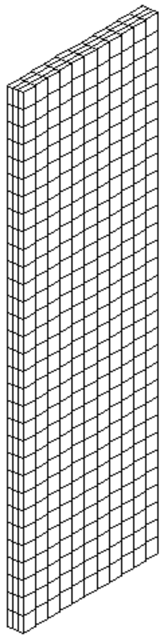


Figure 6.1 -6.1-a : Mesh

6.2 Boundary conditions and loadings

One wishes to apply the same loading as the loading n°2 of the modelization A, i.e. to impose nodes of the mesh on all the asymptotic field of displacement in pure I mode.

For all the classical nodes (not nouveau riches), one then imposes the fields previously definite. For the nodes nouveau riches in crack tip, one seeks to impose each enriched degree of freedom.
With this intention, one rewrites the analytical statements of the fields of displacements to impose on the nodes in the base functions of enrichment:

$$u_x=0 \quad \textbf{éq 6.2-1}$$

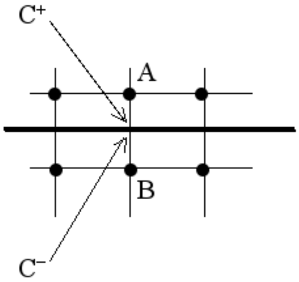
$$u_y=-\frac{1+\nu}{E}\sqrt{\frac{1}{2\pi}}\left(\sqrt{r}\cos\frac{\theta}{2}(2-4\nu)+\sqrt{r}\sin\frac{\theta}{2}\sin\theta\right) \quad \textbf{éq 6.2-2}$$

$$u_z=\frac{1+\nu}{E}\sqrt{\frac{1}{2\pi}}\left(\sqrt{r}\sin\frac{\theta}{2}(4-4\nu)-\sqrt{r}\cos\frac{\theta}{2}\sin\theta\right)$$ **éq 6.2-3**

One recalls that the base of the functions of enrichment is the following one:

$$\sqrt{r}\sin\frac{\theta}{2},\sqrt{r}\cos\frac{\theta}{2},\sqrt{r}\sin\frac{\theta}{2}\sin\theta,\sqrt{r}\cos\frac{\theta}{2}\sin\theta$$ **éq 6.2-4**

For the nodes nouveau riches by the function Heaviside, it is also necessary to carry out a first computation. Let us consider a couple of nodes A and B , and the C^+ note C^- points located on the upper lips and lower of crack, this one cutting a symmetric element of way. One is in the following configuration:



Appear 6.2-a : Heaviside enrichment

the nodes represented by rounds [Figure 10.2 - has] carry classical degrees of freedom a and Heaviside degrees of freedom h .
According to the approximation X-FEM , the crack passing in the middle of the elements, displacements are written:

$$\begin{cases} u(A)=a^A+h^A \\ u(B)=a^B-h^B \\ u(C^+)=\frac{a^A+h^A}{2}+\frac{a^B+h^B}{2} \\ u(C^-)=\frac{a^A-h^A}{2}+\frac{a^B-h^B}{2} \end{cases}$$ **éq 6.2-5**

By reversing this linear system, one obtains the statements of the nodal unknowns according to analytically known displacements:

$$\begin{cases} a^A=\frac{u(A)-u(B)}{2}+u(C^-) \\ h^A=\frac{u(A)+u(B)}{2}-u(C^-) \\ a^B=\frac{u(B)-u(A)}{2}+u(C^+) \\ h^B=\frac{-u(B)-u(A)}{2}+u(C^+) \end{cases}$$ **éq 6.2-6**

6.3 Quantities tested and results

One tests the values of K_I along the crack tip, for various contours of fields theta.
The values of radius inf and $\sup$ the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
$Rinf$	2	0.666	1	1	1.2.1	
Rsup	4	1.666	2	3	4.3.9	

Table 6.4 - E

to test all the nodes of the only one crack tip times, one KI tests the values minimum and maximum of on all the nodes of the crack tip.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 1: MIN (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 2: MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 2: MIN (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 3: MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 3: MIN (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 4: MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 4: MIN (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 5: MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 5: MIN (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 6: MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 6: MIN (KI)	"ANALYTIQUE"	1.0	3.0%

6.4 Comments

the results are stable for any selected contour.
They make it possible to validate option CALC_K_G_F for the elements 3D X-FEM .

7 Modelization F: fissure X-FEM in tension – geometrical enrichment

This modelization is exactly the same one as the modelization C. the only difference is that the zone of enrichment in crack tip now has a size fixed by the user, it is not thus more restricted with only one layer of elements in crack tip.

7.1 Enrichment in crack tip

the nodes being at a distance from the crack tip equal or lower than a certain criterion are enriched by the singular functions. This criterion is selected as in [bib4], equal to a tenth of size of structure. Here, it is worth $1m$ since LY is worth $10m$.

7.2 Quantities tested and results

One tests the values of K_I along the crack tip, for various contours of fields theta.
The values of radius inf and $\sup$ the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
$Rinf$	2	0.666	1	1	1.2.1	
Rsup	4	1.666	2	3	4.3.9	

Table 7.3 - F

to test all the nodes of the crack tip in only once, one on all the tests the values minimum and maximum K_I of nodes of the crack tip.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%
Contour 1: MIN (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%
Contour 2: MAX (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%
Contour 2: MIN (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%
Contour 3: MAX (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%
Contour 3: MIN (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%
Contour 4: MAX (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%
Contour 4: MIN (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%
Contour 5: MAX (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%
Contour 5: MIN (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%
Contour 6: MAX (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%
Contour 6: MIN (KI)	"ANALYTIQUE"	1.1202664 10^7	1.8%

One by the operator tests also the value K_I of product POST_K1_K2_K3 at the first point of the crack tip.

Standard	identification of reference	Value of reference	Tolerance

Crowns 1, not initial :	KI	"ANALYTIQUE"	1.1202664 10^{7}	4.0%

7.3 Comments

the results are stable for any selected contour.
The accuracy of the got results is better than for the modelization C. That proves the beneficial influence of the increase the size of the zone of enrichment, with identical mesh.
However, if one compares with the accuracy of the modelization B (lower than 1%) one could be astonished not to find better results with the fixed zone. The explanation is in [bib4]. Indeed, on the modelization B, the approximation of displacement is exactly in $\sqrt{r}$ on a layer of element around the bottom. On the other hand, in the modelization F, the approximation is $\Sigma_{\text{élémentsenrichis}} \sqrt{r}$ on the zone of enrichment. In this case (relatively coarse mesh), the approximation by a sum of square roots on a wide zone is less good than the approximation by only one square root on a more restricted zone.

However, when one refines sufficiently the mesh, the accuracy obtained with an enrichment on a fixed zone becomes better than that obtained with an enrichment on only one elements layer.

8 Modelization G: fissure X-FEM compression

This modelization is of the same type as the modelization C. the only difference is at the level of the sign of the pressure applied to the upper face. In order to request structure in compression, the pressure is $p=10^6 Pa$.

8.1 Quantities tested and Contact pressures

8.1.1 results

to avoid an iteration of the loop of contact on the active stresses, activation of the contact is ensured as of the 1st iteration thanks to key word CONTACT_INIT=' OUI'. The elementary terms of contact are integrated by a numerical diagram of Gauss into 12 points per facet of contact.
In order to the contact validate the taking into account on the lips of crack, and in particular on the zone of the lips of crack close to the crack tip, the values of contact pressures on the surface of crack are extracted.

One is interested in the evolution of the contact pressure according to the axis Oy , on two lines (line in $x=0$ and line in $x=1$) of the surface of crack [Figure 8.2 -8.1.1-a]. It would normally be necessary to test the value of the contact pressure in each node of these 2 lines, but to reduce the number of tests, one can simply test the minimum and the maximum of the pressures on each line, which results in carrying out 4 tests.

Standard	identification of reference	Value of reference	Tolerance
Line in $x=0$: MAX (LAGS_C)	"ANALYTIQUE"	-106	4.0%
Line in $x=0$: MIN (LAGS_C)	"ANALYTIQUE"	-106	4.0%
Line in $x=1$: MAX (LAGS_C)	"ANALYTIQUE"	-106	4.0%
Line in $x=1$: MIN (LAGS_C)	"ANALYTIQUE"	-106	4.0%

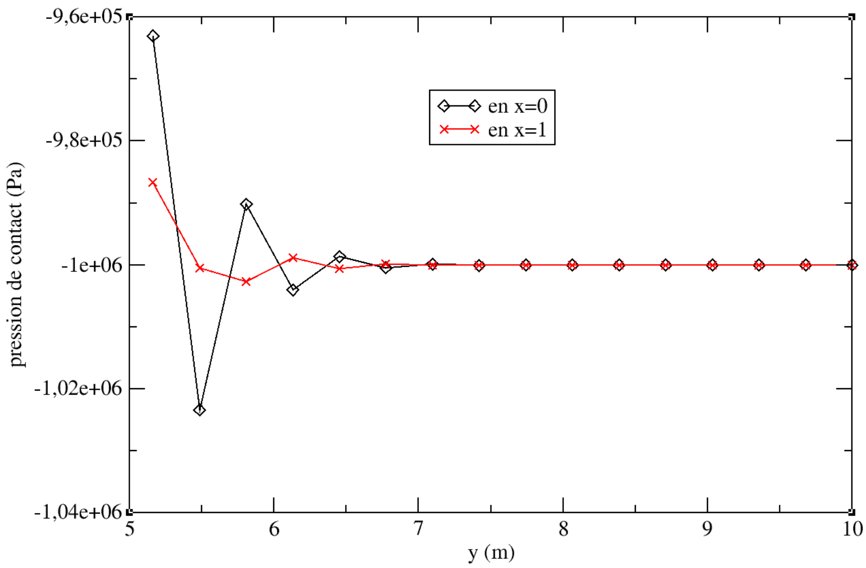


Figure 8.2 -8.1.1-a : Evolution of the contact pressure along the crack

8.1.2 compression room Rate of energy restitution

, the crack does not open, and rate of energy restitution as well as the stress intensity factors are normally null.

To ensure itself some, one tests the values of G room along the crack tip, for various contours of fields theta (the values of radius of the tori are those used for the modelization C) in taking into account a lissage of the type "LAGRANGE" .

To test all the nodes of the crack tip in only once, one tests the values min and max of G on all the nodes of the crack tip.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1: MAX (G)	"ANALYTIQUE"	0.0.5.0	10-4
Contour 1: MIN (G)	"ANALYTIQUE"	0.0.5.0	10-4
Contour 2: MAX (G)	"ANALYTIQUE"	0.0.5.0	10-4
Contour 2: MIN (G)	"ANALYTIQUE"	0.0.5.0	10-4
Contour 3: MAX (G)	"ANALYTIQUE"	0.0.5.0	10-4
Contour 3: MIN (G)	"ANALYTIQUE"	0.0.5.0	10-4
Contour 4: MAX (G)	"ANALYTIQUE"	0.0.5.0	10-4
Contour 4: MIN (G)	"ANALYTIQUE"	0.0.5.0	10-4
Contour 5: MAX (G)	"ANALYTIQUE"	0.0.5.0	10-4
Contour 5: MIN (G)	"ANALYTIQUE"	0.0.5.0	10-4
Contour 6: MAX (G)	"ANALYTIQUE"	0.0.5.0	10-4
Contour 6: MIN (G)	"ANALYTIQUE"	0.0.5.0	10-4

One by the operator tests also the value K_I of product POST_K1_K2_K3 at the first point of the crack tip.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1, not initial: KI	"NON_DEFINI"	0.0	10-6

8.2 Comments

contact pressures close to the crack tip present light disturbances, which are reduced as one moves away from the crack tip. That is due to the enrichment of displacement with singular functions. Indeed, the elements containing the crack tip are enriched by singular functions, but these NON-polynomial functions are not integrable exactly by a classical diagram of Gauss. A light inaccuracy on displacement, and thus on the jump of displacement through crack causes a light disturbance on contact pressures close to the crack tip. Far from the bottom, this enrichment is not present, and the displacements as well as contact pressures are perfectly concordant with the analytical solution.

The values of local rate of energy restitution are quasi-null, and this for any contour of field of theta. However, it is noted that the values of K_I are non-zero and 10 times less large than those obtained for the modelization C in opening. However, theoretically, the bilinear form $g(u,v)$ gives 0 if u is a field of displacement without singularity (like here the field of displacement in compression) and v the asymptotic field in mode I.

9 Modelization H: fissure X-FEM inclined in tension

In this modelization, the crack is not with a grid, it is represented by of level sets. The crack passes by the points to $y=0$ and $z=15$, its length is $2{,}5$, it is tilted of an angle of $45°$:

$$LSN=(z-L).\cos\alpha-y.sin\alpha$$

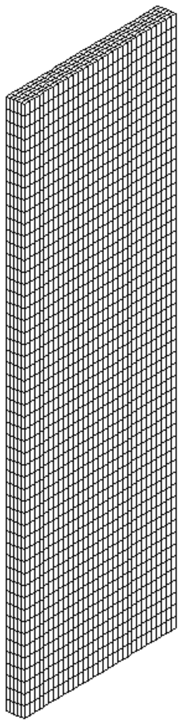
$$LST=(z-L-a.sin\alpha).\sin\alpha+(y-a.cos\alpha).\cos\alpha$$

with $\begin{cases} L=15 \\ a=2.5 \\ \alpha=45° \end{cases}$

In this modelization, the Young modulus is equal to $205000\,MPa$.

9.1 Characteristics of the mesh

the structure is modelled by a sane mesh, regular of $5\times31\times51$ HEXA8, respectively along the axes X, Y, Z [Appear 9.1-a]. The crack tip is in the middle of elements, as in the modelization C, but with a tilted crack.



Appear 9.1-a : Mesh 5*31*51 HEXA8

9.2 Boundary conditions and loadings

the loading applied is the same one as the loading n°1 of the modelization A, i.e. a uniform request in tension by a pressure imposed $\sigma=-10^6 Pa$ on the lower and higher sides. The crack being tilted, one will introduce a loading in mixed mode K_I and K_{II}

For a leaning crack of an angle α compared to the normal plane with the direction of request in pure tension, the coefficients of stress concentration in crack tip are:

$$K_{1(\alpha)}=K_{1(0)}.\cos(\alpha)$$
$$K_{2(\alpha)}=K_{1(0)}.\sin(\alpha)$$

where $K_{1(0)}$ is the coefficient resulting from a request into K_I pure ($\alpha=0$).

Thus, whatever the intensity of the loading (within the limits of application of the theories of the linear elastic mechanics of fracture LEFM), the direction of crack propagation depending only on the ratio K_I/K_{II} is [bib5]:

$$\beta=2\arctan\left[\frac{1}{4}.\left(\frac{K_I}{K_{II}}-sign(K_{II}).\sqrt{\left(\frac{K_I}{K_{II}}\right)^2+8}\right)\right] \quad \text{with} \quad \frac{K_I}{K_{II}}=\frac{\cos(\alpha)}{\sin(\alpha)}$$

Thus, if $\alpha=45°$: $\beta=-53.13°=-0.927295\,rad$

9.3 Quantities tested and results

One tests the values of β along the crack tip for various contours of fields theta.
According to the geometry, the higher radius of integration must remain lower than $a*\cos\alpha$. Radius the lower and superiors of the torus are thus parameterized according to the values of a and α . The values of the ratios $rayon/(a*\cos\alpha)$ brought back between 0 and 1 are:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6.0,1.0,2
$R_{\inf}/(a*\cos\alpha)$			0,4.0,1.0,3			0,4
$R_{\sup}/(a*\cos\alpha)$	1	1	1.0,7.0,7			0,8

Table 9.4 - G : Contours of the field theta

What gives us with $a=2,5$ and $\alpha=45°$:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
R_{inf}	0,1768	0,3536	0,7071	0,1768	0,5303	0,7071
$R_{\sup}$	1,7678	1,7678	1,7678	1,2374	1,2374	1,4142

Table 9.4 - H : Contours of the field theta

to test all the nodes of the crack tip in only once, one on all the tests the minimal and maximum values β of points of the crack tip.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1: MAX (β)	"ANALYTIQUE"	-0,9273	6.1%

Contour 1: MIN (β)	"ANALYTIQUE"	-0,9273	6.1%
Contour 2: MAX (β)	"ANALYTIQUE"	-0,9273	6.1%
Contour 2: MIN (β)	"ANALYTIQUE"	-0,9273	6.1%
Contour 3: MAX (β)	"ANALYTIQUE"	-0,9273	6.1%
Contour 3: MIN (β)	"ANALYTIQUE"	-0,9273	6.1%
Contour 4: MAX (β)	"ANALYTIQUE"	-0,9273	6.1%
Contour 4: MIN (β)	"ANALYTIQUE"	-0,9273	6.1%
Contour 5: MAX (β)	"ANALYTIQUE"	-0,9273	6.1%
Contour 5: MIN (β)	"ANALYTIQUE"	-0,9273	6.1%
Contour 6: MAX (β)	"ANALYTIQUE"	-0,9273	6.1%
Contour 6: MIN (β)	"ANALYTIQUE"	-0,9273	6.1%

9.4 Comments

the results are stable whatever the contour, but it are however higher than the expected results. That comes owing to the fact that the ratios K_I/K_{II} are not rigorously equal to 1.

The purpose of the test being primarily to validate the computation of the angle β of propagation in CALC_G compared to the values of K_I and K_{II} obtained, the test is conclusive.

10 Modelization I: Fissure X-FEM - surface Forces on edges (mode I)

This modelization is identical to the modelization E, but the loading of Dirichlet is replaced here by a loading of Neumann. Only the mode I is requested.

10.1 Characteristics of the mesh

The mesh is identical to that of the modelization E.

10.2 Boundary conditions and loadings

One wishes to apply the same loading as that of the modelization E, i.e. a loading in pure I mode. Instead of imposing the value of displacement, one imposes surface forces on the sides of structure, corresponding to the analytical statements of the stresses in mode I.

It is pointed out that the pure I mode (i.e. $K_I=1$, $K_{II}=0$ and $K_{III}=0$), the stress state is the following:

$$\sigma_{11}=\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$
$$\sigma_{12}=\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\sin\frac{\theta}{2}\cos\frac{3\theta}{2}$$
$$\sigma_{22}=\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1+\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right)$$

In our case, the reference defined is such that:

$$e_1=-e_y$$
$$e_2=e_z$$
$$e_3=-e_x$$

The surface forces to apply are then the following ones:

- on the left side face (of outgoing norm $-e_y$): $\begin{cases} FY=-\sigma_{yy}=-\sigma_{11} \\ FZ=-\sigma_{yz}=\sigma_{12} \end{cases}$
- on the right side face (of outgoing norm e_y): $\begin{cases} FY=\sigma_{yy}=\sigma_{11} \\ FZ=\sigma_{yz}=-\sigma_{12} \end{cases}$
- on the upper face (of outgoing norm e_z): $\begin{cases} FY=\sigma_{yz}=-\sigma_{12} \\ FZ=\sigma_{zz}=\sigma_{22} \end{cases}$
- on the lower face (of outgoing norm $-e_z$): $\begin{cases} FY=-\sigma_{yz}=\sigma_{12} \\ FZ=-\sigma_{zz}=-\sigma_{22} \end{cases}$

The rigid modes are blocked thanks to the points A , B and C [Appear 1.1-a]:

- the point A is blocked according to the 3 directions,
- the point B is blocked along the axis Oz ,
- the point C is blocked along the axes Ox and Oz .

In fact, as the number of elements along the axis Oz is odd, the points A , B and C do not coincide with nodes of the mesh. One thus uses nodes located just above and below these points to impose of the equivalent linear relations:

Are $A1$ (respectively $B1$ and $C1$) the node right below point A (B and C).
Are $A2$ (respectively $B2$ and $C2$) the node right above point A (B and C).

One writes the 6 following linear relations to block the rigid modes:

$$DX^{A1}+DX^{A2}=0$$
$$DY^{A1}+DY^{A2}=0$$
$$DZ^{A1}+DZ^{A2}=0$$

$$DZ^{B1}+DZ^{B2}=0$$

$$DX^{C1}+DX^{C2}=0$$
$$DZ^{C1}+DZ^{C2}=0$$

10.3 Quantities tested and results

One tests the values of K_I along the crack tip, for various contours of fields theta.
The values of radius lower and higher of the torus are the following ones:

	Crown 1	Contour 2	Crowns 3
$Rinf$	0.666	1	1
Rsup	1.666	2	3

Table 10.4 - E

to test all the nodes of the crack tip in only once, one tests the values min and max of K_I on all the nodes of the crack tip.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	1	6.0%
Contour 1: MIN (KI)	"ANALYTIQUE"	1	6.0%
Contour 2: MAX (KI)	"ANALYTIQUE"	1	6.0%
Contour 2: MIN (KI)	"ANALYTIQUE"	1	6.0%
Contour 3: MAX (KI)	"ANALYTIQUE"	1	6.0%
Contour 3: MIN (KI)	"ANALYTIQUE"	1	6.0%

10.4 Comments

the results are stable for any selected contour.
They make it possible to validate option CALC_K_G_F for the elements 3D X-FEM .

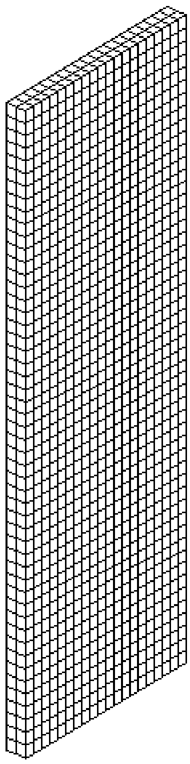
11 Modelization J: Fissure X-FEM - Propagation in the plane with the method simplex

the crack is that of the modelization B.

the Young modulus is equal to $205000\,MPa$.

11.1 Characteristics of the mesh

the structure is modelled by a sane mesh, regular of $2\times20\times51$ HEXA8, respectively along the axes x, y, z [Appear 11.1-a]. The crack tip is inside an element as in the modelization C [Figure 4.1 -4.1-a].



Appear 11.1-a : Mesh 2*20*51 HEXA8

11.2 Boundary conditions and loadings

the loading applied is the same one as the loading n°1 of the modelization A, i.e. a uniform request in tension by a pressure imposed $\sigma=-10^6\,Pa$ on the lower and higher sides. The crack is requested in pure K_I mode.

The advance of crack imposed on each call to PROPA_FISS is the following one: $\Delta a=0.5$

The lengths of crack to each call to PROPA_FISS are thus:

Initial state: $a_0=4.9$
Iteration 1: $a_1=5.4$
Iteration 2: $a_2=5.9$

The stress intensity factors are given by: $K_I = -P \cdot \sqrt{\pi a} \cdot f\left(\frac{a}{b}\right)$

with:

$$f\left(\frac{a}{b}\right) = \left(\frac{2b}{\pi a}\tan\frac{\pi a}{2b}\right)^{0.5} \cdot \frac{0.752 + 0.37 \cdot \left(1 - \sin\frac{\pi a}{2b}\right)^3 + 2.02\frac{a}{b}}{\cos\frac{\pi a}{2b}}$$

$b = 10$
$P = 10^6$

From where:

$K_{I0} = 1.07418.10^7$
$K_{I1} = 1.33139.10^7$
$K_{I2} = 1.67342.10^7$

11.3 Quantities tested and results

One tests the values of K_I along the crack tip, for contour $[1.;4.]$.
To test all the points of the crack tip in only once, one on all the tests the minimal and maximum values K_I of points of the crack tip.

Standard	identification of reference	Value of reference	initial
State Tolerance: `MAX (KI)`	"ANALYTIQUE"	1.07417689 107	5.0%
initial State: `MIN (KI)`	"ANALYTIQUE"	1.07417689 107	5.0%
Iteration 1: `MAX (KI)`	"ANALYTIQUE"	1.33138926 107	5.0%
Iteration 1: `MIN (KI)`	"ANALYTIQUE"	1.33138926 107	5.0%

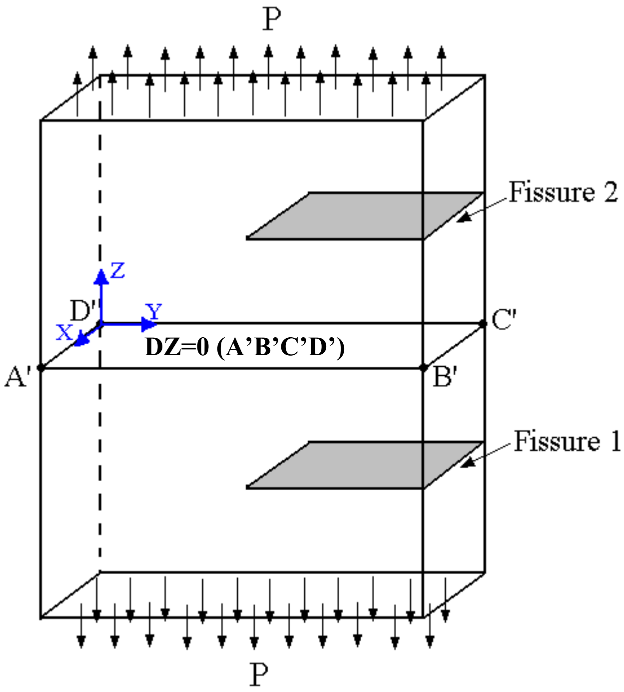
11.4 Comments

the values obtained are relatively close to the expected values. However, it is noted that the value of K_I calculated by Aster is lower than the analytical solution of reference. That probably comes from values K_{II} thenull ones instead of being strictly equal to 0. Moreover, it is observed that while propagating on very few time step (here 3) these values K_{II} the undesirable ones are all the more present (ratio $K_I / K_{II} = 6000$ at the initial state, 2600 after the 1st iteration. That does not imply inevitably that the crack is propagated "badly", because a deviation of crack of its initial plane can be caught up with by one K_{II} which rectifies its trajectory.
The first approach of the crack propagation XFEM in Aster makes it possible nevertheless to validate the complete process of propagation of level sets.

12 Modelization K: Fissure X-FEM – multi-cracking in tension

12.1 Geometry

In this modelization two cracks are present in structure [Appear 12.1-a] in order to test the features of multi-crackings in the case of a structure presenting of emerging cracks multiple.



Appear 12.1-a : Diagram of the modelization K

the geometry of initial structure (modelization A) was modified by doubling the height of the plate 3D . The plane $z=0$ thus became symmetry plane. There is thus a plate 3D for which two cracks identical and symmetric compared to the plane $z=0$ are introduced by functions of levels in the same way as for the modelizations presented before.

12.2 Characteristics of the mesh

the structure is modelled by a sane mesh, regular composed of $5\times31\times102$ HEXA8, respectively along the axes x, y, z . One observes that compared to the modelization C, the number of elements according to the direction z doubled, in agreement with the geometrical modification mentioned above. Of this way, the crack tips are, as for the modelization C, in the center of elements and the plane of crack does not correspond to sides of elements [Figure 4.1 -4.1-a].

12.3 Boundary conditions and loadings

As for the modelization C, a distributed pressure is imposed on the sides lower and higher of structure what has like effect the opening of two cracks in mode I, in a symmetric way. With this intention boundary conditions are imposed on the nodes belonging to median plane ($z=0$). Thus displacement following the direction z (degree of freedom DZ) is blocked for all these nodes. Moreover, to prevent motions of the rigid body, two other displacements were blocked for the nodes corresponding to the points A' and B' of Appear 12.1-a.

12.4 Quantities tested and results

One tests the values of K_I along the crack tip, for two cracks and various contours of fields theta whose values of radius inf and $\sup$ the torus same as those are considered for the modelization C [Table 4.4 - C]. One will be interested only in contours 2 and 3.

One uses lissage "LAGRANGE" and to test all the nodes of the only one crack tip times, one test the values min and max of K_I on all the nodes of the crack tip.

For crack 1:

Standard	identification of reference	Value of reference	Tolerance
Crowns 2: MAX (KI)	"ANALYTIQUE"	5.081166724 106	10.0%
Contour 2: MIN (KI)	"ANALYTIQUE"	5.081166724 106	10.0%
Contour 3: MAX (KI)	"ANALYTIQUE"	5.081166724 106	10.0%
Contour 3: MIN (KI)	"ANALYTIQUE"	5.081166724 106	10.0%

For crack 2:

Standard	identification of reference	Value of reference	Tolerance
Crowns 2: MAX (KI)	"ANALYTIQUE"	5.081166724 106	10.0%
Contour 2: MIN (KI)	"ANALYTIQUE"	5.081166724 106	10.0%
Contour 3: MAX (KI)	"ANALYTIQUE"	5.081166724 106	10.0%
Contour 3: MIN (KI)	"ANALYTIQUE"	5.081166724 106	10.0%

12.5 Comments

One observes that the results for two cracks are identical, which one expected because of symmetric geometrical and kinematical. In more one finds the same values numerical as those found for the modelization C (only one positioned crack in the same way compared to the mesh).

13 Modelization L: Fissure X-FEMs - Forces surface on edges (modes II and III)

Even modelization that the modelization I, but in mode II and III.

In this modelization, the Young modulus is equal to $100\,MPa$.

13.1 Characteristics of the mesh

The mesh used is that of the modelization I.

13.2 Boundary conditions and loadings

13.2.1 Mode II

One wishes to simulate the opening of crack in pure II mode. For that, one applies to all the sides of structure a loading in surface forces corresponding to the analytical statements of the stresses in mode II.

It is pointed out that the pure II mode (i.e. $K_I=0$, $K_{II}=1$ and $K_{III}=0$), the stress state is the following:

$$\sigma_{11}=\frac{-1}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\left(2+\cos\frac{\theta}{2}\cos\frac{3\theta}{2}\right)$$
$$\sigma_{12}=\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}\left(1-\sin\frac{\theta}{2}\sin\frac{3\theta}{2}\right) \qquad \text{éq 13.2.1-1}$$
$$\sigma_{13}=\frac{1}{\sqrt{2\pi r}}\sin\frac{\theta}{2}\cos\frac{\theta}{2}\cos\frac{3\theta}{2}$$

In our case, the reference defined is such that:

$$e_1=-e_y$$
$$e_2=e_z \qquad \text{éq 13.2.1-2}$$
$$e_3=-e_x$$

surface forces to apply are then the following ones:

- on the left side face (of outgoing norm $-e_y$): $\begin{cases} FY=-\sigma_{yy}=-\sigma_{11} \\ FZ=-\sigma_{yz}=\sigma_{12} \end{cases}$
- on the right side face (of outgoing norm e_y): $\begin{cases} FY=\sigma_{yy}=\sigma_{11} \\ FZ=\sigma_{yz}=-\sigma_{12} \end{cases}$
- on the upper face (of outgoing norm e_z): $\begin{cases} FY=\sigma_{yz}=-\sigma_{12} \\ FZ=\sigma_{zz}=\sigma_{22} \end{cases}$
- on the lower face (of outgoing norm $-e_z$): $\begin{cases} FY=-\sigma_{yz}=\sigma_{12} \\ FZ=-\sigma_{zz}=-\sigma_{22} \end{cases}$

The rigid modes are blocked thanks to the points A, B and C [Appear 1.1-a]:

- the point A is blocked according to the 3 directions,

- the point B is blocked along the axis Oz ,
- the point C is blocked along the axes Ox and Oz .

In fact, as the number of elements along the axis Oz is odd, the points A , B and C do not coincide with nodes of the mesh. One thus uses nodes located just above and below these points to impose of the equivalent linear relations:

Are $A1$ (respectively $B1$ and $C1$) the node right below point A (B and C).
Are $A2$ (respectively $B2$ and $C2$) the node right above point A (B and C).

One writes the 6 following linear relations to block the rigid modes:

$$DX^{A1}+DX^{A2}=0$$
$$DY^{A1}+DY^{A2}=0$$
$$DZ^{A1}+DZ^{A2}=0$$

$$DZ^{B1}+DZ^{B2}=0$$

$$DX^{C1}+DX^{C2}=0$$
$$DZ^{C1}+DZ^{C2}=0$$

13.2.2 Mode III

One wishes to simulate the opening of crack in pure III mode. For that, one applies to all the sides of structure a loading in surface forces corresponding to the analytical statements of the stresses in mode III .

It is pointed out that the pure III mode (i.e. $K_I=0$, $K_{II}=0$ and $K_{III}=1$), the stress state is the following:

$$\sigma_{13}=\frac{-1}{\sqrt{2\pi r}}\sin\frac{\theta}{2} \qquad \sigma_{23}=\frac{1}{\sqrt{2\pi r}}\cos\frac{\theta}{2}$$ **éq 16 - 3**

In our case, the reference defined is such that:

$$e_1=-e_y \qquad e_2=e_z \qquad e_3=-e_x$$ **éq 16 - the 4**

surface forces to apply are then the following ones:

- on the left side face (of outgoing norm $-e_y$): $FX=-\sigma_{xy}=-\sigma_{13}$
- on the right side face (of outgoing norm e_y): $FX=\sigma_{xy}=\sigma_{13}$
- on the upper face (of outgoing norm e_z): $FX=\sigma_{xz}=-\sigma_{23}$
- on the lower face (of outgoing norm $-e_z$): $FX=-\sigma_{xz}=\sigma_{23}$
- on the frontal face (of outgoing norm e_x): $\begin{cases} FY=\sigma_{xy}=\sigma_{13} \\ FZ=\sigma_{xz}=-\sigma_{23} \end{cases}$

- on the back face (of outgoing norm $-e_x$): $\begin{cases} FY=-\sigma_{xy}=-\sigma_{13} \\ FZ=-\sigma_{xz}=\sigma_{23} \end{cases}$

The rigid modes are blocked thanks to the points A , B and C [Appear 1.1-a]:

- the point A is blocked according to the 3 directions,
- the point B is blocked along the axis Oz ,
- the point C is blocked along the axes Ox and Oz .

In fact, as the number of elements along axis OZ is odd, the points A , B and C do not coincide with nodes of the mesh. One thus uses nodes located just above and below these points to impose of the equivalent linear relations:
Are $A1$ (respectively $B1$ and $C1$) the node right below point A (B and C).
Are $A2$ (respectively $B2$ and $C2$) the node right above point A (B and C).

One writes the 6 following linear relations to block the rigid modes:

$$DX^{A1}+DX^{A2}=0$$
$$DY^{A1}+DY^{A2}=0$$
$$DZ^{A1}+DZ^{A2}=0$$

$$DZ^{B1}+DZ^{B2}=0$$

$$DX^{C1}+DX^{C2}=0$$
$$DZ^{C1}+DZ^{C2}=0$$

13.3 Quantities tested and results

the goal of this modelization is to test the imposition of surface forces of the edge elements X-FEM containing the crack tip. Those intervene at the time of the imposition of FY and FZ on the sides frontal and back of structure.

One tests the values of K_I , K_{II} and K_{III} along the crack tip, for various contours of fields theta.
The values of radius inf and $\sup$ the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
Rinf	2	0.666	1	1	1.2.1	
Rsup	4	1.666	2	3	4.3.9	

Table 12.4 - I

to test all the nodes of the crack tip in only once, one on all the tests the values minimum and maximum K_I of nodes of the crack tip.

13.3.1 Tests of K_I, K_{II} and K_{III} mode II

13.3.1.1 Values of K_I

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 1: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 2: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 2: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 3: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 3: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 4: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 4: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 5: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 5: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 6: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 6: MIN (KI)	"ANALYTIQUE"	0.0	0.02

13.3.1.2 Values of K_{II}

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	1.0	1.5%
Contour 1: MIN (KI)	"ANALYTIQUE"	1.0	1.5%
Contour 2: MAX (KI)	"ANALYTIQUE"	1.0	1.5%
Contour 2: MIN (KI)	"ANALYTIQUE"	1.0	1.5%
Contour 3: MAX (KI)	"ANALYTIQUE"	1.0	1.5%
Contour 3: MIN (KI)	"ANALYTIQUE"	1.0	1.5%
Contour 4: MAX (KI)	"ANALYTIQUE"	1.0	1.5%
Contour 4: MIN (KI)	"ANALYTIQUE"	1.0	1.5%
Contour 5: MAX (KI)	"ANALYTIQUE"	1.0	1.5%
Contour 5: MIN (KI)	"ANALYTIQUE"	1.0	1.5%
Contour 6: MAX (KI)	"ANALYTIQUE"	1.0	1.5%
Contour 6: MIN (KI)	"ANALYTIQUE"	1.0	1.5%

13.3.1.3 Values of K_{III}

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 1: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 2: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 2: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 3: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 3: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 4: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 4: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 5: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 5: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 6: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 6: MIN (KI)	"ANALYTIQUE"	0.0	0.02

13.3.2 Tests of K_I, K_{II} and K_{III} mode III

13.3.2.1 Values of K_I

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 1: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 2: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 2: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 3: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 3: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 4: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 4: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 5: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 5: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 6: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 6: MIN (KI)	"ANALYTIQUE"	0.0	0.02

13.3.2.2 Values of K_{II}

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 1: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 2: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 2: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 3: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 3: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 4: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 4: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 5: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 5: MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 6: MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 6: MIN (KI)	"ANALYTIQUE"	0.0	0.02

13.3.2.3 Values of K_{III}

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	1.0	8.0%
Contour 1: MIN (KI)	"ANALYTIQUE"	1.0	8.0%
Contour 2: MAX (KI)	"ANALYTIQUE"	1.0	8.0%
Contour 2: MIN (KI)	"ANALYTIQUE"	1.0	8.0%
Contour 3: MAX (KI)	"ANALYTIQUE"	1.0	8.0%
Contour 3: MIN (KI)	"ANALYTIQUE"	1.0	8.0%
Contour 4: MAX (KI)	"ANALYTIQUE"	1.0	8.0%
Contour 4: MIN (KI)	"ANALYTIQUE"	1.0	8.0%
Contour 5: MAX (KI)	"ANALYTIQUE"	1.0	8.0%
Contour 5: MIN (KI)	"ANALYTIQUE"	1.0	8.0%
Contour 6: MAX (KI)	"ANALYTIQUE"	1.0	8.0%
Contour 6: MIN (KI)	"ANALYTIQUE"	1.0	8.0%

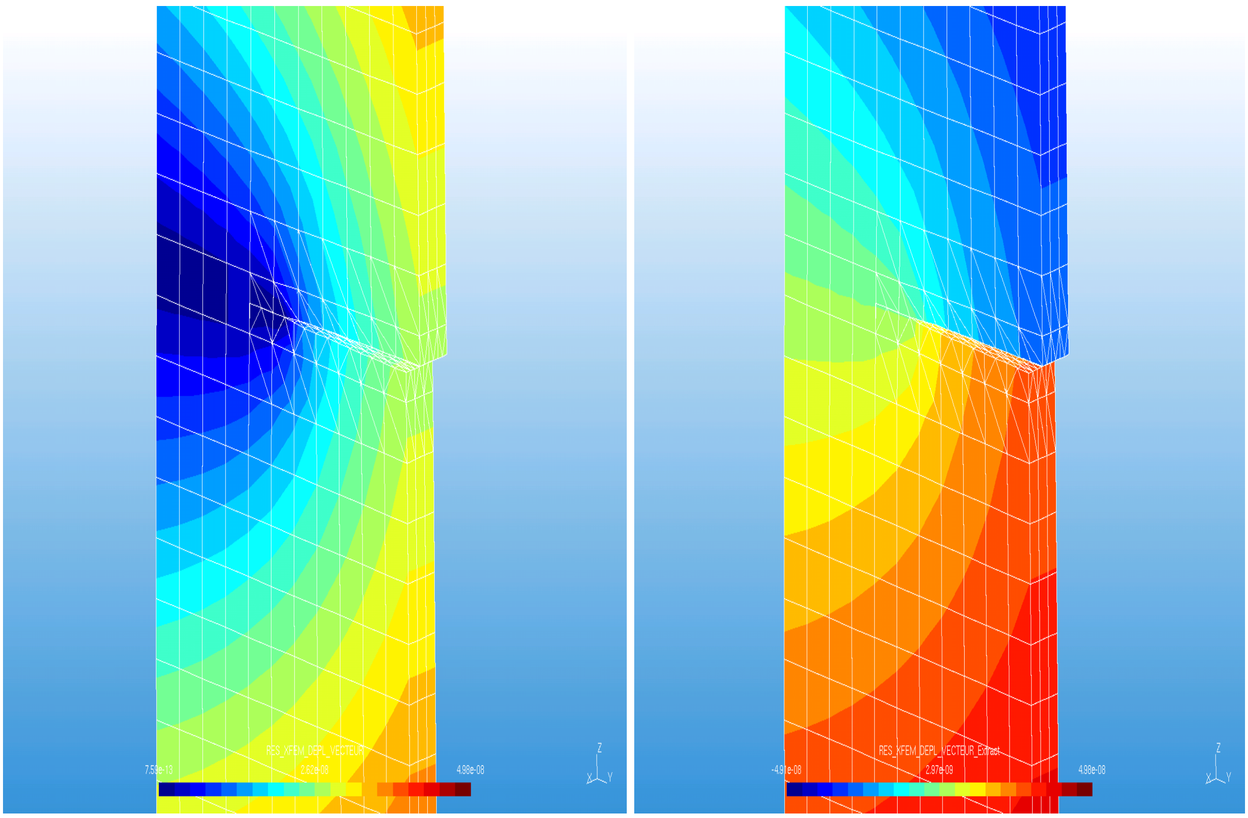
13.3.3 Tests on the coordinates of the nodes

One tests the min/max of the coordinates of the nodes of the upper lip belonging to the face of right.

Standard	identification of reference	Value of reference	Tolerance
MAX (COOR_Y)	"ANALYTIQUE"	10.0	1e-7%
MIN (COOR_Y)	"ANALYTIQUE"	10.0	1e-7%
MAX (COOR_Z)	"ANALYTIQUE"	15.0	1e-7%
MIN (COOR_Z)	"ANALYTIQUE"	15.0	1e-7%

13.3.4 Remark on the visualization of the field of displacement

Notices concerning the visualization of the field of displacement with operators POST_MAIL_XFEM and POST_CHAM_XFEM. When one visualizes the field of total displacement $\sqrt{u_x^2+u_y^2+u_z^2}$, it is necessary to pay attention to what could be interpreted like a "bug" in the visualization [Appear 13.3.4-a 13.3.4-a], but which is in fact completely normal. One and the sees a discontinuity of displacement between a classical element two elements X-FEM with dimensions. This is explained by the fact why, here, only u_x is non-zero; what means that total displacement that one visualizes is worth in fact $\sqrt{u_x^2+u_y^2+u_z^2}=|u_x|$. This function is not linear: there is thus a difference between a linear approximation (on the classical element) and a linear approximation per pieces (on the two elements X-FEM).
This "bug" disappears when a linear quantity is visualized, for example u_x [Appear 13.3.4-a 13.3.4-a].



Appear 13.3.4-a : visualization of displacement (magnitude on the left, component DX on the right)

13.4 Comments

the results are stable for any selected contour.

14 Modelization M: Fissure X-FEM - surface Forces on edges (modes II and III) – geometrical enrichment

This modelization is exactly the same one as the modelization L. the only difference is that the zone of enrichment in crack tip now has a size fixed by the user, it is not thus more restricted with only one elements layer in crack tip.

14.1 Enrichment in crack tip

the nodes being at a distance from the crack tip equal or lower than a certain criterion are enriched by the singular functions. This criterion is selected equal to the double of the usual criterion [bib4]. Here, it is worth $2\,m$.

14.2 Quantities tested and results

One tests the values of K_I , K_{II} and K_{III} along the crack tip, for various contours of fields theta. The values of radius inf and $\sup$ the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
Rinf	2	0.666	1	1	1.2.1	
Rsup	4	1.666	2	3	4.3.9	

Table 13.3 - J

to test all the nodes of the crack tip in only once, one tests the values min and max of K_I on all the nodes of the crack tip.

14.2.1 Values of K_I

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: `MAX (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 1: `MIN (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 2: `MAX (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 2: `MIN (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 3: `MAX (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 3: `MIN (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 4: `MAX (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 4: `MIN (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 5: `MAX (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 5: `MIN (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 6: `MAX (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 6: `MIN (KI)`	`"ANALYTIQUE"`	0.0	0.02

14.2.2 Values of K_{II}

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: `MAX (KI)`	`"ANALYTIQUE"`	0.0	0.02
Contour 1: `MIN (KI)`	`"ANALYTIQUE"`	0.0	0.02

Contour 2 : MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 2 : MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 3 : MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 3 : MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 4 : MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 4 : MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 5 : MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 5 : MIN (KI)	"ANALYTIQUE"	0.0	0.02
Contour 6 : MAX (KI)	"ANALYTIQUE"	0.0	0.02
Contour 6 : MIN (KI)	"ANALYTIQUE"	0.0	0.02

14.2.3 Values of K_{III}

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1 : MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 1 : MIN (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 2 : MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 2 : MIN (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 3 : MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 3 : MIN (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 4 : MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 4 : MIN (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 5 : MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 5 : MIN (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 6 : MAX (KI)	"ANALYTIQUE"	1.0	3.0%
Contour 6 : MIN (KI)	"ANALYTIQUE"	1.0	3.0%

14.3 Comments

the results are stable for any selected contour.
These results highlight the contribution of key word RAYON_ENRI. Without this key word (modelization L), the mistakes made on K_{III} are it order of 10%. While informing this key word, this error is brought back to less 5%. However, that has a cost. The number of degrees of freedom nouveau riches is increased, which weighs down computations. For example, the time spent in STAT_NON_LINE doubled between the modelizations L and M).

14.4 Notice

In this case, key word FISS_XFEM of the command DEFI_GROUP is used in all the cases in order to make sure of its correct operation.

15 Modelization N: Fissure X-FEM in tension - quadratic elements

This modelization is exactly the same one as the modelization C. the only difference is that the finite elements used are quadratic elements instead of linear elements.

15.1 Quantities tested and results

One tests the values of K_I along the crack tip, for various contours of fields theta, exits of the command CALC_G.
The values of radius inf and $\sup$ the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
Rinf	2	0.666	1	1	1.2.1	
Rsup	4	1.666	2	3	4.3.9	

Table 14.2 - K

to test all the nodes of the crack tip in only once, one tests the values min and max of K_I on all the nodes of the crack tip.

15.1.1 Values of KI for the lissage of the type "LAGRANGE"

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 1: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%
Contour 2: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 2: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%
Contour 3: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 3: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%
Contour 4: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 4: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%
Contour 5: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 5: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%
Contour 6: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 6: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%

15.1.2 Values of KI for the lissage of the type "LAGRANGE_REGU"

Identification	Standard of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 1: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%
Contour 2: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 2: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%
Contour 3: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 3: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%
Contour 4: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 4: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%
Contour 5: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 5: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%
Contour 6: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%

Contour 6 : MIN (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%

15.1.3 Values of KI for the lissage of the type "LAGRANGE_NO_NO"

Identification	Standard of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 1: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%

15.1.4 Values of KI for the lissage of the type "LAGRANGE_REGU"

Identification	Standard of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	1.0%
Contour 1: MIN (KI)	"ANALYTIQUE"	$1.1202664\ 10^{7}$	2.0%

15.2 Values tested resulting of the command POST_K1_K2_K3

One uses command POST_K1_K2_K3 to determine the value of K1. Maximum curvilinear X-coordinate is 1.5, and the number of cut nodes is 6.

One tests the value of K1 for the first point of the bottom.

Standard	identification of reference	Value of reference	Tolerance
K1	"ANALYTIQUE"	$1.1202664\ 10^{7}$	3.5%

15.3 Values tested resulting of the command POST_MAIL_XFEM

One uses command POST_MAIL_XFEM to generate the cracked mesh.
One tests the value of the sum of the absolute values of the Y-coordinates according to X, Y, Z. It is a test of NON-regression compared to the values obtained with version 10.2.17.

Standard	identification of reference	Value of reference	Tolerance
SOMM_ABS (COOR_X)	"NON_REGRESSION"	4.992 103	1e- 8%
SOMM_ABS (COOR_Y)	"NON_REGRESSION"	4.992 104	1e- 8%
SOMM_ABS (COOR_Z)	"NON_REGRESSION"	1.4976 105	1e- 8%

15.4 Values tested resulting of the command POST_CHAM_XFEM

One uses command POST_CHAM_XFEM result to generate the fields of adequate with the cracked mesh previously created.
One tests the value of the sum of the absolute values of displacements of the nodes of the mesh thus created. It is a test of NON-regression compared to the values obtained with version 10.2.17.

Standard	identification of reference	Value of reference	Tolerance
SOMM_ABS (DX)	"NON_REGRESSION"	8.312 10-3	$10^{4\%}$
SOMM_ABS (DY)	"NON_REGRESSION"	6.588	$10^{4\%}$
SOMM_ABS (DZ)	"NON_REGRESSION"	4.616	$10^{4\%}$

16 Modelization O: Crack with a grid - Volume forces

In this modelization, the crack is with a grid, and one uses the standard method of the finite elements to carry out computation. This modelization will be used as reference and will allow the comparison with the method X-FEM in the modelization P.

16.1 Caractéristiques of the mesh

the structure is modelled by a regular mesh composed of $5 \times 30 \times 50$ HEXA8, respectively along the axes X, Y, Z (see [Figure 2.1 - has]). Two superimposed surfaces are the lips of crack.

16.2 Boundary conditions and loadings

Two types of voluminal loadings (leading to same result) are studied here:

- A voluminal loading imposed on all structure with
FORCE_INTERNE $FX=0$ $FY=0$, $FZ=-78000$
- a loading of gravity with the key word PESANTEUR $=(10,0,0,-1)$, acceleration 10 in the direction $-z$ (it is pointed out that the density of structure is equal to 7800).

For each of the two loadings, the upper face of structure is clamped.

16.3 Quantities tested and results

One tests the values of K_I, K_{II} and K_{III} along the crack tip, for various contours of fields theta. The values of radius inf and $\sup$ the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
$Rinf$	2	0.666	1	1	1.2.1	
Rsup	4	1.666	2	3	4.3.9	

Table 15.55 - L

to test all the nodes of the crack tip in only once, one tests the min and max values of K_I on all the nodes of the crack tip. One has only the results of the first loading (with FORCE_INTERNE), those of the second are perfectly identical. The tests presented here are tests of non regression.

Values of K_I

Standard	Identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"NON_REGRESSION"	1.23 107	0.3%
Contour 1: MIN (KI)	"NON_REGRESSION"	1.23 107	0.3%
Contour 2: MAX (KI)	"NON_REGRESSION"	1.23 107	0.3%
Contour 2: MIN (KI)	"NON_REGRESSION"	1.23 107	0.3%
Contour 3: MAX (KI)	"NON_REGRESSION"	1.23 107	0.3%
Contour 3: MIN (KI)	"NON_REGRESSION"	1.23 107	0.3%
Contour 4: MAX (KI)	"NON_REGRESSION"	1.23 107	0.3%
Contour 4: MIN (KI)	"NON_REGRESSION"	1.23 107	0.3%
Contour 5: MAX (KI)	"NON_REGRESSION"	1.23 107	0.3%
Contour 5: MIN (KI)	"NON_REGRESSION"	1.23 107	0.3%
Contour 6: MAX (KI)	"NON_REGRESSION"	1.23 107	0.3%

Contour 6: MIN (KI)	"NON_REGRESSION"	1.23 107	0.3%

16.4 Comments

the results are stable for any selected contour. A mean value of $1.23D+07$ is taken as reference for computations with a crack $XFEM$ in the following modelization.

17 Modelization P: Fissure X-FEM - Volume forces

Even modelization that the modelization B, but with the loadings of the modelization O.
The modelization O is used as reference solution.

17.1 Characteristics of the mesh

the structure is modelled by a sane mesh, regular composed of $5\times30\times50$ HEXA8, respectively along the axes X, Y, Z in order to have the same number of elements as for the mesh of the modelization A (see [Figure 4.1 - has]). Thus, the plane of crack is in correspondence with sides d'HEXA8 and the crack tip with edges d'HEXA8. (see [Figure 3.1-a]).
This mesh is identical to that of the modelization B.

17.2 Boundary conditions and loadings

Two types of voluminal loadings (leading to same result) are studied here:
- a loading n°1: voluminal loading imposed on all structure with FORCE_INTERNE $FX=0$ $FY=0$, $FZ=-78000$
- a loading n°2: a loading of gravity with key word PESANTEUR acceleration 10 in the direction $-z$ (it is pointed out that the density of structure is equal to 7800).

For each of the two loadings, the upper face of structure is clamped.

17.3 Enrichment in crack tip

the nodes being at a distance from the crack tip equal or lower than a certain criterion are enriched by the singular functions. One uses topological enrichment by default.

17.4 Results of the modelization P

One tests the values of K_I, K_{II} and K_{III} along the crack tip, for various contours of fields theta. The values of radius inf and ¿ the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
$Rinf$	2	0.666	1	1	1.2.1	
Rsup	4	1.666	2	3	4.3.9	

Table 17.5 - m

to test all the nodes of the crack tip in only once, one on all the tests the values minimum and maximum K_I of nodes of the crack tip. One has only the results of the first loading (with FORCE_INTERNE), those of the second are perfectly identical. The tests presented here are tests compared to a numerical reference solution with crack with a grid (modelization O).

Standard	**identification of reference**	**Value of reference**	**Tolerance**
Crowns 1: MAX (KI)	"AUTRE_ASTER"	1.23 107	6.0%
Contour 1: MIN (KI)	"AUTRE_ASTER"	1.23 107	6.0%
Contour 2: MAX (KI)	"AUTRE_ASTER"	1.23 107	6.0%
Contour 2: MIN (KI)	"AUTRE_ASTER"	1.23 107	6.0%
Contour 3: MAX (KI)	"AUTRE_ASTER"	1.23 107	6.0%

Contour 3: `MIN (KI)`	"AUTRE_ASTER"	1.23 107	6.0%
Contour 4: `MAX (KI)`	"AUTRE_ASTER"	1.23 107	6.0%
Contour 4: `MIN (KI)`	"AUTRE_ASTER"	1.23 107	6.0%
Contour 5: `MAX (KI)`	"AUTRE_ASTER"	1.23 107	6.0%
Contour 5: `MIN (KI)`	"AUTRE_ASTER"	1.23 107	6.0%
Contour 6: `MAX (KI)`	"AUTRE_ASTER"	1.23 107	6.0%
Contour 6: `MIN (KI)`	"AUTRE_ASTER"	1.23 107	6.0%

One also tests the values of G (more precisely the maximum of G along the crack tip) obtained by the command CALC_G, option CALC_G, for the voluminal loading as well as the loading of gravity. One restricts oneself with the 1st integration contour. One compared to the value of G obtained by the command CALC_G, option CALC_K_G.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1, loading n°1: `MAX (G)`	"AUTRE_ASTER"	826.143	0.1%
Contour 1, loading n°2: `MAX (G)`	"AUTRE_ASTER"	826.143	0.1%

One by the operator tests also the value K_I of product POST_K1_K2_K3 at the first point of the crack tip with the loading n°1:

Standard	identification of reference	Value of reference	Tolerance
Crowns 1, not initial: `KI`	"NON_REGRESSION"	1.3416697 10^{7}	0.1%

One uses command POST_MAIL_XFEM to generate the cracked mesh.
One tests the sum of the absolute values of the Y-coordinates following X, Y, Z of the nodes of the cracked mesh. It is a test of NON-regression compared to the values obtained with version 10.2.17.

Standard	identification of reference	Reference	Tolerance
`SOMM_ABS (COOR_X)`	"NON_REGRESSION"	4.743 10^{3}	1e-8%
`SOMM_ABS (COOR_Y)`	"NON_REGRESSION"	4.743 10^{4}	1e-8%
`SOMM_ABS (COOR_Z)`	"NON_REGRESSION"	1.4229 105	1e-8%

One uses command POST_CHAM_XFEM result to generate the fields of adequate with the cracked mesh previously created. One tests the sum of the absolute values of displacements of the nodes of the mesh thus created. It is a test of NON-regression compared to the values obtained with version 10.2.17.

Standard	identification of reference	Reference	Tolerance
`SOMM_ABS (DX)`	"NON_REGRESSION"	3.88717 10-4	1e-6%
`SOMM_ABS (DY)`	"NON_REGRESSION"	2.3488575	1e-6%
`SOMM_ABS (DZ)`	"NON_REGRESSION"	2.310308	1e-6%

17.5 Comments

the results are stable for any selected contour. The error is about 5% what is satisfactory. It is noticed however that it is only one comparison with a numerical method (crack with a grid) which in addition, if one has an analytical solution, provides results less precise than the method XFEM.

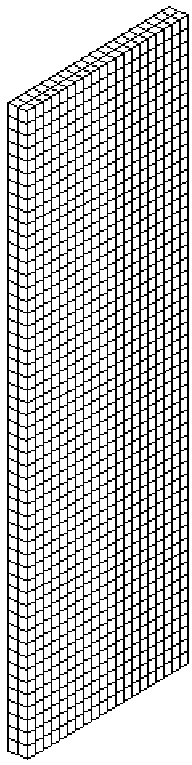
18 Modelization Q: Fissure X-FEM - Propagation in the plane with the method upwind

In this modelization, one takes again the modelization J and one uses the method upwind of operator PROPA_FISS to solve the equations of propagations. No auxiliary grid is used because the mesh of structure is very regular and it can thus be used directly.

The Young modulus is equal to $205000\,MPa$.

18.1 Characteristics of the mesh

the structure is modelled by a sane mesh, regular of $2\times20\times51$ HEXA8, respectively along the axes X,Y,Z [Appear 18.1-a]. The crack tip is inside an element as in the modelization C [Figure 4.1 -4.1-a].



Appear 18.1-a : Mesh 2*20*51 HEXA8

18.2 Boundary conditions and loadings

the loading applied is the same one as the loading n°1 of the modelization A, i.e. a uniform request in tension by a pressure imposed $P=-10^6 Pa$ on the lower and higher sides. The crack is requested in pure K_I mode.
The advance of crack imposed on each call to PROPA_FISS is the following one: $\Delta a=0.5\text{m}$

The lengths of crack to each call to PROPA_FISS are thus:

Initial state: $a_0=4.9\,m$
Iteration 1: $a_1=5.4\,m$

Iteration 2: $a_2=5.9\,m$

The stress intensity factors are given by: $K_I=-P\cdot\sqrt{\pi a}\cdot f\left(\frac{a}{b}\right)$

with:

$$f\left(\frac{a}{b}\right)=\left(\frac{2\mathrm{b}}{\pi a}\tan\frac{\pi a}{2b}\right)^{0.5}\cdot\frac{0.752+0.37\cdot\left(1-\sin\frac{\pi a}{2b}\right)^3+2.02\frac{a}{b}}{\cos\frac{\pi a}{2b}}$$

$$b=10\mathrm{m}$$

$$P=10^6\,Pa$$

From where:

$$K_{I0}=10.7418\,MPa\cdot\sqrt{m}$$
$$K_{I1}=13.3139\,MPa\cdot\sqrt{m}$$
$$K_{I2}=16.7342\,MPa\cdot\sqrt{m}$$

18.3 Quantities tested and results

One tests the values of K_I along the crack tip, for contour $[1.;4.]$.
To test all the points of the crack tip in only once, one on all the tests the minimal and maximum values K_I of points of the crack tip.

Standard	identification of reference	Value of reference	initial
State Tolerance: MAX (KI)	"ANALYTIQUE"	10.7417689	5.0%
initial State: MIN (KI)	"ANALYTIQUE"	10.7417689	5.0%
Iteration 1: MAX (KI)	"ANALYTIQUE"	13.3138925	5.0%
Iteration 1: MIN (KI)	"ANALYTIQUE"	13.3138925	5.0%

18.4 Comments

the values obtained are relatively close to the expected values. However, it is noted that the value of K_I calculated by Aster is lower than the analytical solution of reference. That is related to the mesh used which is not very refined.

19 Modelization R: Fissure X-FEM in tension - Mesh with pyramids

This modelization is exactly the same one as the modelization C. the only difference is that, prior to mechanical computation, one calls Homard to refine some meshes HEXA8. This process generates meshes PYRA5.

19.1 Quantities tested and results

One tests the values of K_I along the crack tip, for various contours of fields theta.
The values of radius lower and superior of the torus are the following ones:

	Crown 1	Contour 2	Crowns 3	Contour 4	Crowns 5	Contour 6
R_{inf}	2	0.666	1	1	1.2.1	
R_{sup}	4	1.666	2	3	4.3.9	

Table 7.3 - F

to test all the nodes of the crack tip in only once, one tests the values min and max of K_I on all the nodes of the crack tip.

Lissage "LAGRANGE" is used.

Standard	identification of reference	Value of reference	Tolerance
Crowns 1: MAX (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%
Contour 1: MIN (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%
Contour 2: MAX (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%
Contour 2: MIN (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%
Contour 3: MAX (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%
Contour 3: MIN (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%
Contour 4: MAX (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%
Contour 4: MIN (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%
Contour 5: MAX (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%
Contour 5: MIN (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%
Contour 6: MAX (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%
Contour 6: MIN (KI)	"ANALYTIQUE"	1.1202664 10^{7}	2.0%

The computation from K_I extrapolation of the jumps of displacements also carried out by the command POST_K1_K2_K3. One tests the value of K1 at the first point of the crack tip.

Standard	identification of reference	Value of reference	Tolerance
K_I to the first point of crack tip	"ANALYTIQUE"	1.1202664 10^{7}	4.0%

20 Summaries of the results

the purposes of this test are reached:

- to validate on a simple case the taking into account of singular enrichment in crack tip with the method X-FEM ;
- to validate the computation of the stress intensity factors (here only mode I) for the elements X-FEM , whatever the load (fixed or function);
- to validate the contact on a case of compression in mode 1 of closing;
- to validate the computation of K_I on a case of multi-cracking.
- to test the case of the imposed voluminal forces.

It will be retained that the use of the method X-FEM makes it possible to appreciably improve the accuracy of computation of K_I , and that this one increases when the zone of enrichment is not restricted with only one layer of elements in crack tip.