SSNV182 – The purpose of block with interface in contact rubbing with X-FEM

Summarized

This test is of the contact validating the taking into account on the lips of the crack, while being limited if the crack crosses structure completely. The contact is taken into account by the continuous method [bib1] adapted to the frame of the method X-FEM [bib2]. Two contact algorithms are tested: method of Augmented Lagrangian and penalized method [bib2].

This test brings into play a parallelepipedic block in compression modelled in $2\mathrm{D}$ and $3\mathrm{D}$. The interface the beam is represented by a level-set in the frame of the method X-FEM. One takes into account several angular positions of the interface: the interface follows the sides of the elements ($\theta=0°$) and cut interfaces it the elements ($\theta=26.56°$ in $3\mathrm{D}$ and $\theta=30°$ in $2\mathrm{D}$). By taking a coefficient of kinetic friction of sufficiently high Coulomb so that there is dependancy, one finds the solution of the same problem without interface. The modelizations D, E and F make it possible to also validate the method in $2\mathrm{D}$.

1 Problem of reference

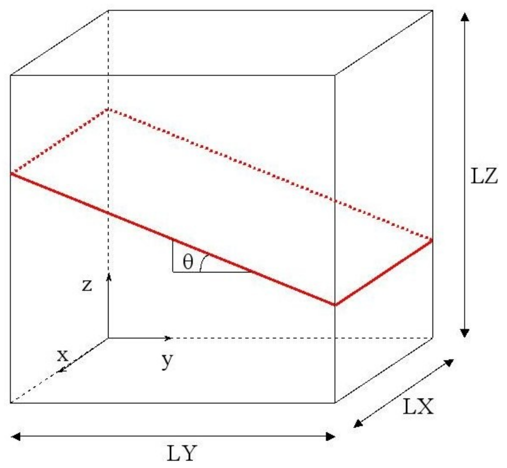
1.1 Geometry

the structure is a right at square base and healthy parallelepiped. Dimensions of the block (see [Appear -1.1-a]) are: $LX=5\,m$, $LY=20\,m$ and $LZ=20\,m$. It does not comprise any crack.

The interface is introduced by functions of levels (level sets) directly into the command file using operator `DEFI_FISS_XFEM` [U4.82.08]. The interface is present in the middle of structure by the means of its representation by the level sets. The level set norm (LSN) makes it possible to define a plane interface forming an angle θ with the plane Oxy by the following equation:

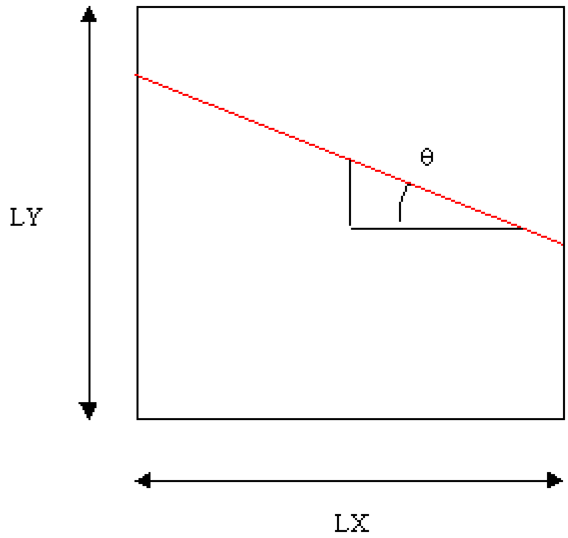
$$LSN=Z-(aY+b) \qquad \textbf{éq 1.1-1}$$

where a is the slope of the interface, is $a=-\tan(\theta)$ and $b=\frac{LZ}{2}-a\frac{LY}{2}$.



Appear -1.1-a : Geometry of the bar and positioning of the interface

In $2D$, one has an equivalent structure.



Appear 1.1-b **: Geometry and positioning of the interface in** $\mathbf{2D}$

1.2 Properties of the material

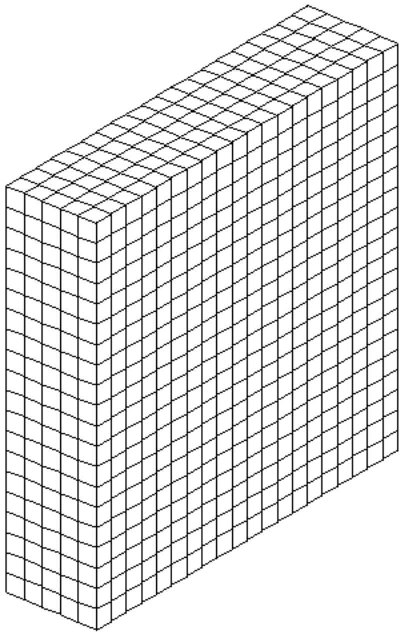
Modulus Young: $E=100\,MPa$
Poisson's ratio: $\nu=0$.

1.3 Boundary conditions and loadings

the nodes of the lower face of the bar are embedded and a displacement $UZ=-10^{-6}m$ is imposed on those of the upper face which corresponds to a loading in pressure along the axis z. Displacements along the axes x and y are blocked for the nodes of the upper surface. In $2\mathrm{D}$, displacement is carried out according to the axis y.

1.4 Characteristics of the mesh

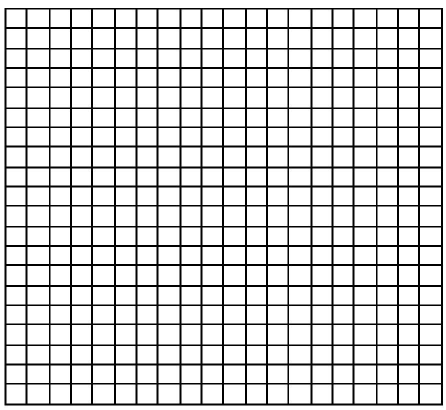
the structure is modelled in $3\mathrm{D}$ by a regular mesh composed of $5\times20\times20$ HEXA8 [Figure 1.4-a].



Appear -1.4-a : Mesh $3D$

This mesh is composed of linear finite elements. However, in the frame of the continuous method [bib1] with X-FEM [bib2], it is necessary to pass to a little special linear elements. These elements have linear shape functions and a quadratic mesh support. On these elements, the top nodes carry the unknowns of displacement, and the medium nodes carry the unknowns related to the contact. Moreover, when the interface follows the edge of an element, the top nodes of the element carry also the unknowns of contact.

For the benchmarks $2D$, the structure is modelled by a regular mesh composed of 20×20 QUAD4.



Appear -1.4-b : Mesh $2D$

1.5 Bibliography

1.MASSIN P., BEN DHIA H., ZARROUG Mr.: Elements of contacts derived from a continuous hybrid formulation, Handbook of reference of *the Code_Aster*, [R5.03.52]

2.MASSIN P., GENIAUT S.: Method X-FEM, Handbook of reference of *the Code_Aster*, [R7.02.12]

3. TARDIEU N., VAUTIER I., LORENTZ E.: Quasi-static nonlinear algorithm, Handbook of Reference of *the Code_Aster*, [R5.03.01]

4. DHATT G., TOUZOT G.: A presentation of the finite element method, Maloine ED., PARIS

2 Modelization a: interfaces right, method of Augmented Lagrangian

In this modelization, one represents a right interface, the angle θ is worth 0 then. The interface coincides with the sides of certain finite elements. The method of Augmented Lagrangian is used for the processing of the contact/friction.

2.1 Analytical resolution

When the adhering contact is taken into account on the interface, the problem is equivalent to that of a healthy bar in pure compression. The solution of the problem is that of the same problem without interface.
The stress in structure is:

$$\sigma_{zz}=E\frac{UZ}{LZ}$$ **éq 2.1-1**

and the value of the contact pressure on the interface is:

$$\lambda=\sigma_{zz}$$ **éq 2.1-2**

With the numerical values previously introduced $\lambda=-5.0\,Pa$.

2.2 Features tested

As it is announced to the preceding paragraph, it is necessary to activate friction. In operator `DEFI_CONTACT`, one stipulates FROTTEMENT='COULOMB' then, and the coefficient of kinetic friction is selected equal to 1.

Moreover, as of the first iteration of the active stresses, one makes the assumption that the points of contact have a contacting statute. This is possible by specifying CONTACT_INIT='OUI'.
If not, at the end of the first iteration, the contact not being activated, the higher block returns in the lower block but the two blocks did not become deformed. Their stress state is thus null, and it is then necessary to choose a total criterion (`RESI_GLOB_MAXI`) for the convergence of the algorithm of Newton - Raphson [bib3], criterion which is likely to be unsuited in the continuation of computations when the contact is activated.

To avoid that, and to have a relative criterion, one needs a non-zero stress state as of the first iteration, and thus to activate the contact as of the beginning.

The algorithm of the active stresses thus converges in an iteration.

2.3 Quantities tested and results

One tests the values of the normal pressure of contact after convergence of the iterations of operator `STAT_NON_LINE` and the loop on the active stresses. All the points of contact are tested, which correspond to the nodes of the mesh on the interface. It is checked that one finds well the values determined with [§2.1].

Identification	Reference	% Tolerance
`LAGS_C` for all the nodes of the interface	-5.00	1.00e-10

to test all the only one nodes times, one tests the minimum and the maximum of column.

2.4 Comments

This modelization X shows the possibilities of the continuous method of contact applied to the frame - FEM. The advantage is that the procedure of pairing is intrinsic with the method X-FEM since here, there are not really surface Master and slave considering whom one remains in small displacements.

3 Modelization b: interfaces leaning, method of Augmented Lagrangian

In this modelization, one represents a leaning interface. The angle θ is worth $\arctan(1/2)$, that is to say a slope α being worth $-1/2$. The interface does not coincide any more with the sides of the finite elements, and cuts the elements now. The norm with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}0\\1/\sqrt{5}\\2/\sqrt{5}\end{pmatrix},\qquad \tau=\begin{pmatrix}0\\2/\sqrt{5}\\-1/\sqrt{5}\end{pmatrix}$$ **éq the 4-1**

method of Augmented Lagrangian is used for the processing of the contact/friction.

3.1 Analytical resolution

the interface being leaning, it is likely to have sliding there. To avoid that, one forces the dependancy by choosing a coefficient of kinetic friction of sufficiently high Coulomb. Theoretically, it is enough to take:

$$\mu>\tan(\theta)$$ **éq 4.1-1**

Thus, the solution of the problem remains identical to that of the same problem without interface.
The stress in structure is always that of [éq 2.1-1], and the value of the contact pressure on the interface is function of the norm n to the interface:

$$\lambda=n.\sigma.n=n_z\sigma_{zz}n_z$$ **Q 4.1-2**

where n_z is the component following z of n .

The semi-multiplier of friction Λ is defined by:

$$r_\tau=\lambda\,\mu\,\Lambda$$ **éq 4.1-3**

With the density of tangential stress being written as follows:

$$r_\tau=(\tau.\sigma.n)\tau$$ **éq 4.1-4**

From where:

$$\Lambda=\left(\frac{1}{\mu}\frac{\tau.\sigma.n}{n.\sigma.n}\right)\tau=\left(\frac{1}{\mu}\frac{\tau_z}{n_z}\right)\tau$$ **éq 4.1-5**

One takes $\mu=1$.
With the numerical values previously introduced, $\lambda=-4.0\,MPa$ and $\Lambda.\tau=-0.5$.

3.2 Quantities tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of operator STAT_NON_LINE, the loop on the active stresses and the loop on the thresholds of friction. One tests the value of the multipliers of contact and friction at the points of contact of the mesh of visualization: the GROUP_NO of the points of contact is extracted by the key word PREF_GROUP_CO of the command POST_CHAM_XFEM.
It is checked that one finds well the values determined with [§3.1]. LAGS_F1 corresponds to the semi - multiplying of friction in the direction Ox (it is thus null), whereas LAGS_F2 corresponds to the semi-multiplier of following friction τ .

Identification	Reference	% Tolerance
LAGS_C for all the points of contact	-4.00	1.00e-8
LAGS_F1 for all the points of contact	0.00	1.00e-8
LAGS_F2 for all the points of contact	-0.50	1.00e-8

to test all the points of contact into only one times, one tests the value MIN and the value MAX of the column.

3.3 Comments

Let us specify that in this study, the key word CONTACT_INIT = "YES" makes it possible to begin the loop on the active stresses with an assumption of statute contacting for all the points of contact. That authorizes to take a relative criterion ("RESI_RELA_MAX") for the convergence of the iterations of Newton. Indeed, if one chooses CONTACT_INIT = "NON", at the time of the phase of prediction of Newton, the contact not being activated, the higher structure moves without becoming deformed, and that lower remains motionless. The stresses are then null and a relative criterion is not usable, only a total criterion is, whose value is left with the choice of the user. The problem is that this value can appear computation (active contact thereafter….) inadequate with the loadings and the stresses then concerned. Thus, it is to better provide to take a single relative criterion as of the beginning.

Moreover, the initial value of the threshold of friction was taken with -10^9 in order to being sure that one has dependancy as of the 1st iteration on the thresholds of friction.

4 Modelization C: interface right and under-integration, method of Augmented Lagrangian

the This modelization is identical to the modelization A, except that the diagram of numerical integration of the terms of contact changed.

In the modelization A, one uses a diagram of Gauss at 12 points by triangular facets of contact. In the modelization C, one only uses a diagram reduced to 4 points.

Indeed, the diagram must allow the exact integration of a constant field of pressure. The intégrande on the facet is then a students' rag procession in $x^i y^i$ with $i+j\leq 3$.

According to [bib4], a diagram with 4 Gauss points is enough.

4.1 Quantities tested and results

the same values are tested as for modelization A.

Identification	Reference	% difference
LAGS_C for all the points in contact	-5.00	0.20
LAGS_F1 for all the points of contact	0.00	1.00e-10
LAGS_F2 for all the points of contact	0.00	1.00e-10

to test all the nodes of the only one interface times, one test the values minimum and maximum.

4.2 Comments

This modelization shows that a diagram of integration reduced to 4 points makes it possible to pass the patch test where the solution in pressure is constant.

5 Modelization D: interface right in plane stresses, method of Augmented Lagrangian

In this modelization, one represents a right interface, the angle θ is worth 0 then. The interface coincides with the sides of certain finite elements.

In this modelization, the assumption of plane stresses is considered (although here, the Poisson's ratio being null, it does not have there a difference between stresses and plane strains).

The method of Augmented Lagrangian is used for the processing of the contact/friction.

5.1 Analytical resolution

the interface being right, and the compactness is uniaxial and normal with the interface, there is no possible sliding. The solution of the problem is that of the same problem without interface.
The stress in structure is:

$$\sigma_{zz}=E\frac{UY}{LY} \qquad \textbf{éq 8.1-1}$$

and the value of the contact pressure on the interface is:

$$\lambda=\sigma_{zz} \qquad \textbf{éq 8.1-2}$$

With the numerical values previously introduced $\lambda=-5.0\,Pa$.

5.2 Features tested

This case does not require the activation of friction. In operator DEFI_CONTACT, one stipulates FROTTEMENT='SANS' then.

Moreover, as of the first iteration of the active stresses, one makes the assumption that the points of contact have a contacting statute. This is possible by specifying CONTACT_INIT='OUI'.
If not, at the end of the first iteration, the contact not being activated, the higher block returns in the lower block but the two blocks did not become deformed. Their stress state is thus null, and it is then necessary to choose a total criterion (RESI_GLOB_MAXI) for the convergence of the algorithm of Newton - Raphson [bib3], criterion which is likely to be unsuited in the continuation of computations when the contact is activated.
To avoid that, and to have a relative criterion, one needs a non-zero stress state as of the first iteration, and thus to activate the contact as of the beginning.

The algorithm of the active stresses thus converges in an iteration.

5.3 Quantities tested and results

One tests the values of the normal pressure of contact after convergence of the iterations of operator STAT_NON_LINE and the loop on the active stresses. All the points of contact are tested, which correspond to the nodes of the mesh on the interface. It is checked that one finds well the values determined with [§5.1].

Identification	Reference	% difference
LAGS_C for all the nodes of the interface	-5.00	1.00e-10
LAGS_F1 for all the nodes of the interface	0.00	1.00e-5

to test all the only one nodes times, one tests the minimum and the maximum of column.

5.4 Comments

This modelization X shows the possibilities of the continuous method of contact applied to the frame - FEM in $2D$. The advantage is that the procedure of pairing is intrinsic with the method X-FEM since here, there are not really surface Master and slave for the processing of the contact since one remains in small displacements.

6 Modelization E: leaning interface in plane stresses, method of Augmented Lagrangian

In this modelization, one represents a leaning interface. The angle θ is worth $30°$. The interface does not coincide any more with the sides of the finite elements, and cuts the elements now. The norm with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}1/2\\ \sqrt{3}/2\end{pmatrix},\qquad \tau=\begin{pmatrix}-\sqrt{3}/2\\ 1/2\end{pmatrix}$$ **éq 10-1**

In this modelization, one considers the assumption of plane stresses (although here, the Poisson's ratio being null, it does not have there a difference between stresses and plane strains).

The method of Augmented Lagrangian is used for the processing of the contact/friction.

6.1 Analytical resolution

to avoid the sliding one takes $\mu=1$.

One must have:

$$\lambda=\boldsymbol{n}.\boldsymbol{\sigma}.\boldsymbol{n}=n_y \sigma_{yy} n_y$$ **éq 10.1-2**

when n_y is the component following y of n .

The semi-multiplier of friction Λ is defined by:

$$r_\tau=\lambda \mu \Lambda$$ **éq 10.1-3**

With the density of tangential stress being written as follows:

$$r_\tau=(\tau.\sigma.n)\tau$$ **éq 10.1-4**

From where:

$$\Lambda=\left(\frac{1}{\mu}\frac{\tau.\sigma.n}{n.\sigma.n}\right)\tau=\left(\frac{1}{\mu}\frac{\tau_y}{n_y}\right)\tau$$ **éq 10.1-5**

One takes $\mu=1$.
With the numerical values previously introduced, $\lambda=-3.75\,Pa$ and $\Lambda.\tau=1/\sqrt{3}$.

6.2 Quantities tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of operator `STAT_NON_LINE`, the loop on the active stresses and the loop on the thresholds of friction. One tests the value of the multipliers of contact and friction at the points of contact of the mesh of visualization: `the GROUP_NO` of the points of contact is extracted by the key word `PREF_GROUP_CO` of the command `POST_CHAM_XFEM`.

Identification	Reference	% difference
`LAGS_C` for all the points in contact	-3.75	0.01

LAGS_F1 for all the points of contact	5.77350E-01	0.01

to test all the points of contact into only one times, one tests the minimum and the maximum of column.

7 Modelization F: leaning interface in plane strains, method of Augmented Lagrangian

the This modelization is the same one as the modelization E, except the fact that one considers the assumption of plane strains. As the Poisson's ratio is null, that does not affect the results.

7.1 Quantities tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of operator STAT_NON_LINE, the loop on the active stresses and the loop on the thresholds of friction. All the points of contact are tested. One tests the value of the multipliers of contact and friction at the points of contact of the mesh of visualization: the GROUP_NO of the points of contact is extracted by the key word PREF_GROUP_CO of the command POST_CHAM_XFEM.

Identification	Reference	% difference
LAGS_C for all the points in contact	-3.75	0.01
LAGS_F1 for all the points of contact	5.77350E-01	0.01

to test all the points of contact into only one times, one tests the minimum and the maximum of column.

8 Modelization G: interface right, method penalized

In this modelization, one represents a right interface, the angle θ is worth 0 then. The interface coincides with the sides of certain finite elements. The penalized method is used for the processing of the contact/friction.

8.1 Analytical resolution

the analytical solution is the same one as that of modelization A.

8.2 Fonctionnalités tested

In operator DEFI_CONTACT, FROTTEMENT='COULOMB' then is stipulated, and the coefficient of kinetic friction is selected equal to 1.

Very high coefficients of penalization make it possible to approach the solution all the more precisely. The values of these coefficients however are limited by the conditioning of the stiffness matrix.

8.3 Quantities tested and results

One tests the values of the normal pressure of contact after convergence of the iterations of operator STAT_NON_LINE and the loop on the active stresses. All the points of contact are tested, which correspond to the nodes of the mesh on the interface. It is checked that one finds well the values determined with [§2.1].

Identification	Reference	% difference
LAGS_C for all the nodes of the interface	-5.00	1.00e-10
LAGS_F1 for all the nodes of the interface	0.	1.00e-10
LAGS_F2 for all the nodes of the interface	0.	1.00e-10

to test all the only one nodes times, one tests the value MIN and the value MAX of the column.

8.4 Comments

Realising of the important coefficients of penalization, the penalized method makes it possible to get the same results as the method of the Augmented Lagrangian one.

9 Modelization H: leaning interface, method penalized

In this modelization, one represents a leaning interface. The angle θ is worth $\arctan(1/2)$, that is to say a slope α being worth $-1/2$. The interface does not coincide any more with the sides of the finite elements, and cuts the elements now. The norm with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}0\\1/\sqrt{5}\\2/\sqrt{5}\end{pmatrix}, \qquad \tau=\begin{pmatrix}0\\2/\sqrt{5}\\-1/\sqrt{5}\end{pmatrix} \qquad \textbf{éq the 16-1}$$

penalized method is used for the processing of the contact/friction.

9.1 Analytical resolution

the analytical solution is the same one as that of the modelization B.

9.2 Functionalities tested

This case requires the activation of friction. In operator DEFI_CONTACT, one stipulates FROTTEMENT='COULOMB' then.

Very high coefficients of penalization make it possible to approach the solution all the more precisely. The values of these coefficients however are limited by the conditioning of the stiffness matrix.

9.3 Quantities tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of operator STAT_NON_LINE, the loop on the active stresses and the loop on the thresholds of friction. One tests the value of the multipliers of contact and friction at the points of contact of the mesh of visualization: the GROUP_NO of the points of contact is extracted by the key word PREF_GROUP_CO of the command POST_CHAM_XFEM.
It is checked that one finds well the values determined with [§3.1]. LAGS_F1 corresponds to the semi - multiplying of friction in the direction Ox (it is thus null), whereas LAGS_F2 corresponds to the semi-multiplier of following friction τ .

Identification	Reference	% difference
LAGS_C for all the points in contact	-4.00	5.00e-3
LAGS_F1 for all the points of contact	0.00	5.00e-3
LAGS_F2 for all the points of contact	-0.50	5.00e-3

to test all the points of contact into only one times, one tests the value MIN and the value MAX of the column.

9.4 Comments

Realising of the important coefficients of penalization, the penalized method makes it possible to get the same results as the method of the Augmented Lagrangian one.

10 Modelization I: interface right in plane stresses, method penalized

In this modelization, one represents a right interface, the angle θ is worth 0 then. The interface coincides with the sides of certain finite elements.

In this modelization, the assumption of plane stresses is considered (although here, the Poisson's ratio being null, it does not have there a difference between stresses and plane strains).

The penalized method is used for the processing of the contact/friction.

10.1 Analytical resolution

the analytical solution is the same one as that of modelization D.

10.2 Fonctionnalités tested

This case does not require the activation of friction. In operator DEFI_CONTACT, one stipulates FROTTEMENT='SANS' then.

Very high coefficients of penalization make it possible to approach the solution all the more precisely. The values of these coefficients however are limited by the conditioning of the stiffness matrix.

10.3 Quantities tested and results

One tests the values of the normal pressure of contact after convergence of the iterations of operator STAT_NON_LINE and the loop on the active stresses. All the points of contact are tested, which correspond to the nodes of the mesh on the interface. It is checked that one finds well the values determined with [§2.1].

Identification	Reference	% Tolerance
LAGS_C for all the nodes of the interface	-5.00	0.001
LAGS_F1 for all the nodes of the interface	0.00	0.001

to test all the only one nodes times, one tests the minimum and the maximum of column.

10.4 Comments

Realising of the important coefficients of penalization, the penalized method makes it possible to get the same results as the method of the Augmented Lagrangian one.

11 Modelization J: leaning interface in plane stresses, method penalized

In this modelization, one represents a leaning interface. The angle θ is worth $30°$. The interface does not coincide any more with the sides of the finite elements, and cuts the elements now. The norm with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}1/2\\ \sqrt{3}/2\end{pmatrix}, \qquad \tau=\begin{pmatrix}-\sqrt{3}/2\\ 1/2\end{pmatrix} \qquad \textbf{éq 20-1}$$

In this modelization, one considers the assumption of plane stresses (although here, the Poisson's ratio being null, it does not have there a difference between stresses and plane strains).

The penalized method is used for the processing of the contact/friction.

11.1 Analytical resolution

the analytical solution is the same one as that of the modelization E.

11.2 Functionalities tested

In operator DEFI_CONTACT, FROTTEMENT='COULOMB' then is stipulated, and the coefficient of kinetic friction is selected equal to 1.

Very high coefficients of penalization make it possible to approach the solution all the more precisely. The values of these coefficients however are limited by the conditioning of the stiffness matrix.

11.3 Quantities tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of operator STAT_NON_LINE, the loop on the active stresses and the loop on the thresholds of friction. One tests the value of the multipliers of contact and friction at the points of contact of the mesh of visualization: the GROUP_NO of the points of contact is extracted by the key word PREF_GROUP_CO of the command POST_CHAM_XFEM.

Identification	Reference	% Tolerance
LAGS_C for all the points of contact	-3.75	0.01
LAGS_F1 for all the points of contact	5.77350E-01	0.01

to test all the points of contact into only one times, one tests the MIN and the MAX of the column.

11.4 Comments

Realising of the important coefficients of penalization, the penalized method makes it possible to get the same results as the method of the Augmented Lagrangian one.

12 Modelization K: interface right in plane stresses, method of Augmented Lagrangian on a quadratic mesh

In this modelization, one represents a right interface, the angle θ is worth 0 then. The interface coincides with the sides of certain finite elements.

In this modelization, the assumption of plane stresses is considered (although here, the Poisson's ratio being null, it does not have there a difference between stresses and plane strains).

The contact friction is treated with quadratic elements X-FEM $P2P1$, i.e. carrying the degrees of freedom of displacement on all the nodes and the lagranges of contact friction on the nodes tops.

12.1 Analytical resolution

the analytical solution is the same one as that of modelization D.

12.2 Fonctionnalités tested

This case does not require the activation of friction. In operator DEFI_CONTACT, one stipulates FROTTEMENT='SANS' then.

12.3 Quantities tested and results

One tests the values of the normal pressure of contact after convergence of the iterations of operator STAT_NON_LINE and the loop on the active stresses. All the points of contact are tested, which correspond to the nodes of the mesh on the interface. It is checked that one finds well the values determined with [§2.1].

Identification	Reference	% Tolerance
LAGS_C for all the nodes of the interface	-5.00	1.00e-10

to test all the only one nodes times, one tests the minimum and the maximum of column.

12.4 Comments

This valid test:

- the computation of the stiffness matrix (the good shift during the filling of the matrix because the nodes do not increase the same number of degrees of freedom)
- the computation of the contact matrixes (integration on a SEG3 with Gauss points)

It does not make it possible to validate under cutting since the interface coincides with the sides of the elements.

13 Modelization L: leaning interface in plane stresses, method of Augmented Lagrangian on a quadratic mesh

In this modelization, one represents a leaning interface. The angle θ is worth $30°$. The interface does not coincide any more with the sides of the finite elements, and cuts the elements now. The norm with the interface is noted n and the tangent vector is noted τ :

$$n=\begin{pmatrix}1/2\\\sqrt{3}/2\end{pmatrix},\qquad \tau=\begin{pmatrix}-\sqrt{3}/2\\1/2\end{pmatrix}$$ **éq 20-1**

In this modelization, one considers the assumption of plane stresses (although here, the Poisson's ratio being null, it does not have there a difference between stresses and plane strains).

The contact friction is treated with quadratic elements X-FEM $P2P1$, i.e. carrying the degrees of freedom of displacement on all the nodes and the lagranges of contact friction on the nodes tops.

13.1 Analytical resolution

the analytical solution is the same one as that of the modelization E.

13.2 Functionalities tested

In operator `DEFI_CONTACT`, FROTTEMENT='COULOMB' then is stipulated, and the coefficient of kinetic friction is selected equal to 1.

13.3 Quantities tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence of the iterations of operator `STAT_NON_LINE`, the loop on the active stresses and the loop on the thresholds of friction. One tests all the points of contact, knowing that the points of contact are nodes tops of the elements cut by the interface.

Identification	Reference	% Tolerance
`LAGS_C` for all the points of contact	-3.75	0.01
`LAGS_F1` for all the points of contact	5.77350E-01	0.01

to test all the points of contact into only one times, one tests the value MIN and the value MAX of the column.

13.4 Comments

This valid test:

- the computation of the stiffness matrix (the good shift during the filling of the matrix because the nodes do not increase the same number of degrees of freedom)
- the computation of the contact matrixes (integration on `a SEG3` with Gauss points)
- under cutting (configurations in right interface and elements on right board)

the linear solver MUMPS detects a singularity in the matrix for the node $\mathrm{N252}$ on the component DY . This problem already arose on many other cases tests X-FEM in contact. The readjustment of the level set norm makes it possible to improve conditioning of the matrix but to the detriment of a too great error of discretization of the level set. One thus keeps the option of deactivation of the detection of singularity of the solver before suggesting a more satisfactory solution.

14 Modelization M: leaning interface in 3D, method of Augmented Lagrangian on a quadratic mesh

We take again characteristics identical to the modelization B, put except for the mesh which is quadratic. For recall, the interface is thus tilted of an angle θ being worth $\arc\tan(1/2)$, that is to say a slope α being worth $-1/2$. The modelization is three-dimensional, the interface not coinciding with the sides of the finite elements. The norm $\boldsymbol{n}$ and the tangent vector τ with the interface are given by (éq the 4-1).

The method of Augmented Lagrangian is used for the processing of the contact rubbing. The rubbing contact is treated with quadratic `elements` XFEM `P2P1`, i.e. carrying the degrees of freedom of displacement on all the nodes and the “lagranges” of contact/friction on the nodes tops.

14.1 Analytical resolution

the analytical solution is the same one as that of the modelization B.

14.2 Functionalities tested

In operator `DEFI_CONTACT`, FROTTEMENT=' COULOMB' `then is stipulated`, and the coefficient of kinetic friction is selected equal to 1. This test is used to test contact `XFEM` on the quadratic meshes (attribute `CONTACT=' P2P1'` in command `MODI_MODELE_XFEM`) in the three-dimensional case. Friction is activated, therefore it is also tested (`FROTTEMENT=' COULOMB'`).

14.3 Quantities tested and results

One tests the values of the normal pressure of contact and the semi-multiplier of friction after convergence. All the points of contact are tested. Let us recall that in this discretization “`P2P1`” the degrees of freedom of contact are carried by the nodes tops of the elements cut by the interface: it is thus in these points that one tests the values.

Identification	Reference	% Tolerance
`LAGS_C` for all the points of contact	-3.75	0.01
`LAGS_F1` for all the points of contact	5.77350E-01	0.01

to test all the points of contact into only one times, one tests the value MIN and the value MAX column.

14.4 Comments

This test P2P1 validates `the modelization` in `3D` namely:

- the computation of the stiffness matrix (the good shift during the filling of the matrix because the nodes do not increase the same number of degrees of freedom),
- the computation of the contact matrixes.

At the present time, one under-cutting quadratic elements `3D` with linear elements, quadratic under-cutting, possibly with curved sides, not being yet available. In the same way, the facets of contact are `TRI3`. This test thus does not constitute a validation of subelements or facets of contact individuals.

15 Summaries of the results

the purposes of this test are reached:

- It is a question of showing the feasibility of the taking into account of the contact rubbing on the lips of crack with the continuous method adapted to the frame X-FEM. Only the case of a crack crossing structure completely was considered (interface).
- The cases where the interface follows the border of the elements ($\theta=0°$) and where the interface cut the elements ($\theta=26.56°$ in $3D$ and $\theta=30°$ in $2D$) were validated.
- The method was validated in $2D$ for $P1P1A$ and $P2P1$ in $3D$ for $P1P1A$.
- The method was validated at the same time with the method of Augmented Lagrangian and the method penalized for the processing of the contact/friction.